Rigid motion, congruent triangles and proof

Original lesson by Beth Menzie

Objective

In this lesson, students will identify the reasons that triangles are congruent, using SAS, ASA, SSS, and AAS, by closely examining diagrams and 'givens. '

Big idea

Using the method of flowchart proofs, students begin to develop the skills necessary to understand and create congruent triangle proofs.

Using rigid motion to prove triangles are congruent

15 MINUTES

Rigid motions, proofs, and puzzled faces

To begin, I let the students know that we are going to use our knowledge of rigid motion, to write proofs that two triangles are congruent. As an opening, I plan to discuss the meaning of the word congruent, and how rigid motions might be used to show congruence. We will also discuss the meaning of congruence with respect to the parts of a triangle.

I plan to take things as far as explaining that we are looking for a series of isometries that will map each side and each angle of one triangle onto another. I'll say, 'if we can find these isometries, we will have proven that the triangles are congruent. Another way of saying this is that one triangle is an image of the other.'

My students often want to know why we are going to do this. If they ask, I'll say, 'in this unit, we are trying to determine the minimum amount of information that we need to know in order to prove congruence. They are usually satisfied by this response.

Practice with triangle congruence

25 MINUTES

To begin, we will practise identifying why two triangles are congruent. For this task, I will use a worksheet that I found online, provided by Kuta Software. But you can use anything similar.

I will do the first two or three problems with the students. Then, I will ask them to complete the worksheet in their groups, discussing the concepts as they go. When most students are finished, I plan to call on students to read their answers to a problem, working my way around the room systematically. When there is disagreement, I will ask students to explain their reasoning, beginning with the original respondent.

As we discuss answers, I will stress the importance of marking the diagrams. I want to make sure that my students recognise the importance of this step in solving a problem. It is difficult to discuss triangle congruence if congruent parts are not clearly and accurately indicated in the diagram. A 'rule' that I stress is that three sets of congruent parts must be marked each time to prove that two triangles are congruent. I have found that this is not always obvious to students; they will work with triangles that appear to be congruent without confirming their observation. The theme of carefully and fully annotating diagrams will be present throughout this unit.

SSS, SAS, ASA and AAS congruence

Part A

Here are pairs of triangles in different formations. State if the two triangles are congruent. It they are, state how you know.

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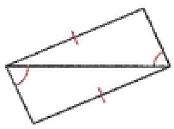

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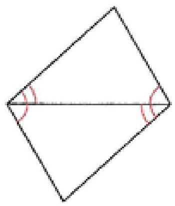

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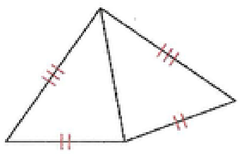

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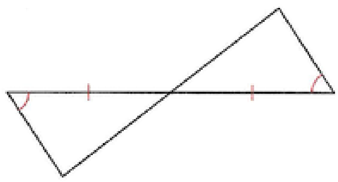

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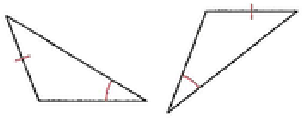

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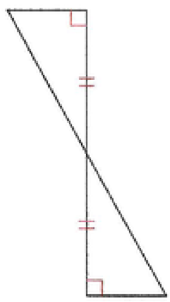

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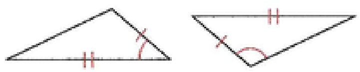

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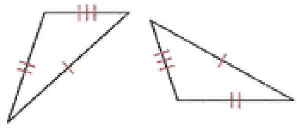

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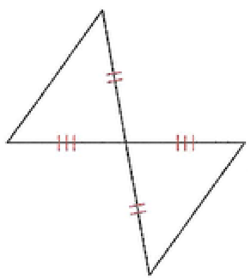

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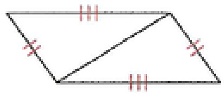
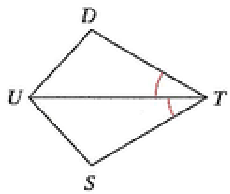


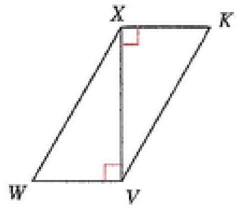
Part B

State what additional information is required in order to now that the triangles are congruent for the reason given.

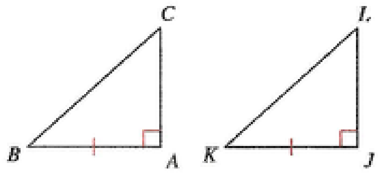
11) ASA



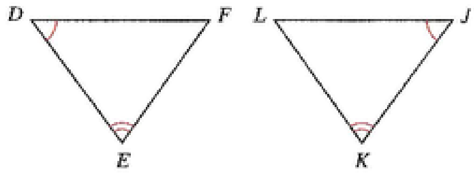
12) SAS



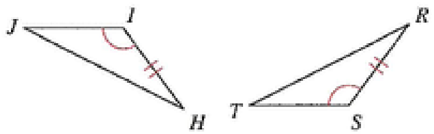
13) SAS



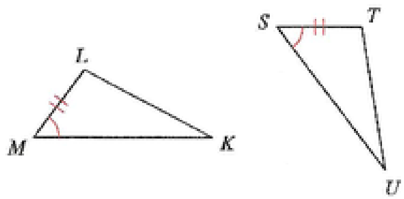
14) ASA



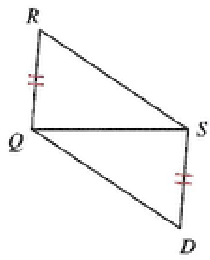
15) SAS



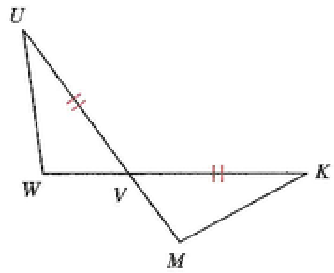
16) ASA



17) SSS



18) SAS



Let's try some proofs

35 MINUTES

I have copied the document entitled ‘Statements used in geometric proofs’ onto brightly coloured paper. (I have found that using coloured paper for important information helps my students to find these sheets quickly and drives home the fact that this is important information.) Much of the information on this sheet is familiar to my students. By placing it all on one sheet, however, the students can easily refresh their memories and remind themselves of their possible options.

After I hand out ‘Statements used in geometric proofs’, I give the class some time to look the statements over. I make sure that students locate the ways of proving triangles listed on the back. Then, I hand out the Beginning proofs document.

I have chosen to present these opening proofs as flowchart proofs. I have been using flowchart proofs in class for several years now as an introduction and have found that they are a great way to start. Each year I have experimented with them a little more. Now, I am pretty pleased with how my students develop their understanding from these visual proofs.

For today, each proof has been laid out for the students so that it is clear that they need to find three pairs of congruent triangle parts, and, to do so, they must focus on each given statement one at a time. When they have used up all the of given statements, but still need to prove another set of triangle parts congruent, I emphasise that they need to look closely at the diagram to determine if any other information is contained in the diagram (for example, vertical angles or a reflexive side).

We'll work on the first proof as a class. As we do I will emphasise the importance of reading and writing down each given, and marking the diagram. (I provide coloured pencils for the marking of the diagrams.) When I feel that the class has a good understanding of the first proof, I ask them to work on the second proof, and we take our time discussing and answering questions on it, proceeding in this manner, through all four proofs on the worksheet.

Mathematics Hub

Statements used in geometric proofs

Properties

1. Addition property: if equal quantities are added to equal quantities, their sums are equal.
2. Subtraction property: if equal quantities are subtracted from equal quantities, their differences are equal.
3. Substitution property: any quantity may be substituted for its equal.
4. Reflexive property: a quantity is equal/congruent to itself.
5. Partition: the whole is equal to the sum of its parts.

Definitions:

1. Congreuent segments are equal in length.
2. Congruent angles are equal in measure.
3. A midpoint divides a line segment into 2 congruent segments.
4. An angle bisect or divides an angle into 2 congruent angles.
5. The bisect or of a segment divides a segment into 2 congruent segments.
6. Perpendicular lines form right angles.
7. Vertical angles are the opposite angles formed by a pair of intersecting lines.
8. Supplementary angles are two angles whose sum is 180 degrees.
9. Complementary angles are two angles whose sum is 90 degrees.
10. Corresponding parts of congruent triangles are congruent (CPCTC).

Theorems involving angles

1. Vertical agles are congruent.
2. All right angles are congruent.
3. If two sides of a triangle are congruent, then the angles opposite these sides are congruent.,
4. If two angles of a triangle are congruent, then the sides opposite these angles are contruent.

Theorems involving parallel lines

1. If 2 parallel lines are cut by a transversal, the alternate interior angles are congruent.
2. If 2 parallel lines are cut by a transversal, the corresponding angles are congruent.
3. If 2 parallel lines are cut by a transversal, the same side interior angles are supplementary.

   And their converses:

4. If the alternate interior angles are contruent, the lines are parallel.
5. If the corresponding agles are contruent, then the lines are parallel.
6. If the same-side interior angles are supplementary, the lines are parallel.

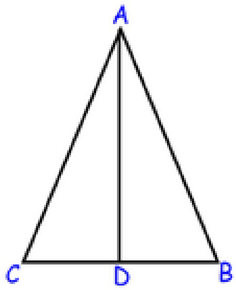
Theorems for proving triangles congruent

1. If the two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent.
   SAS ≅ SAS
2. If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.
   ASA ≅ ASA
3. If three side of one triangle are contruent to three sides of another triangle, then the triangles are congruent.

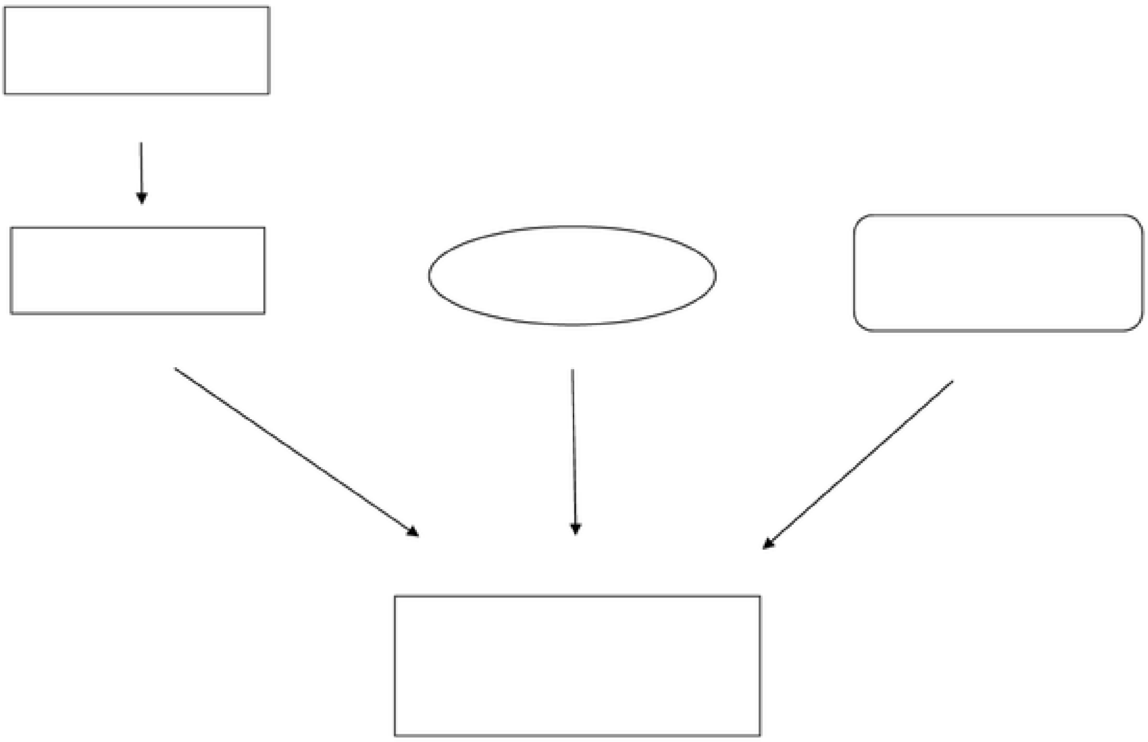
SSS ≅ SSS

4. If two angles and a non-included side of one triangle are congruent to two angles and the corresponding non-included side of another triangle, then the triangles are congruent.
   AAS ≅ AAS
5. If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and the corresponding leg of another triangle, the the triangles are congruent.
   HL ≅ HL

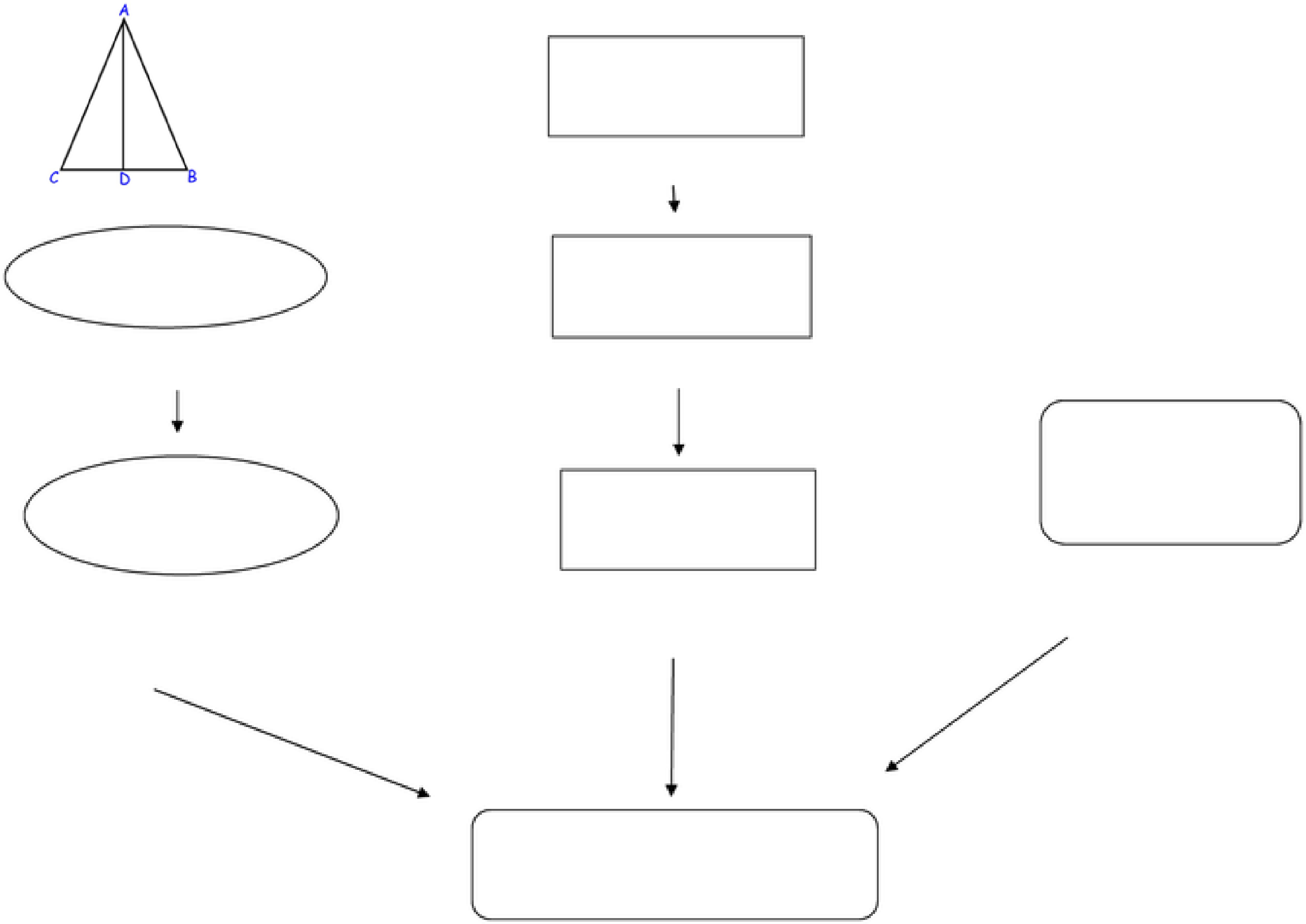
1.



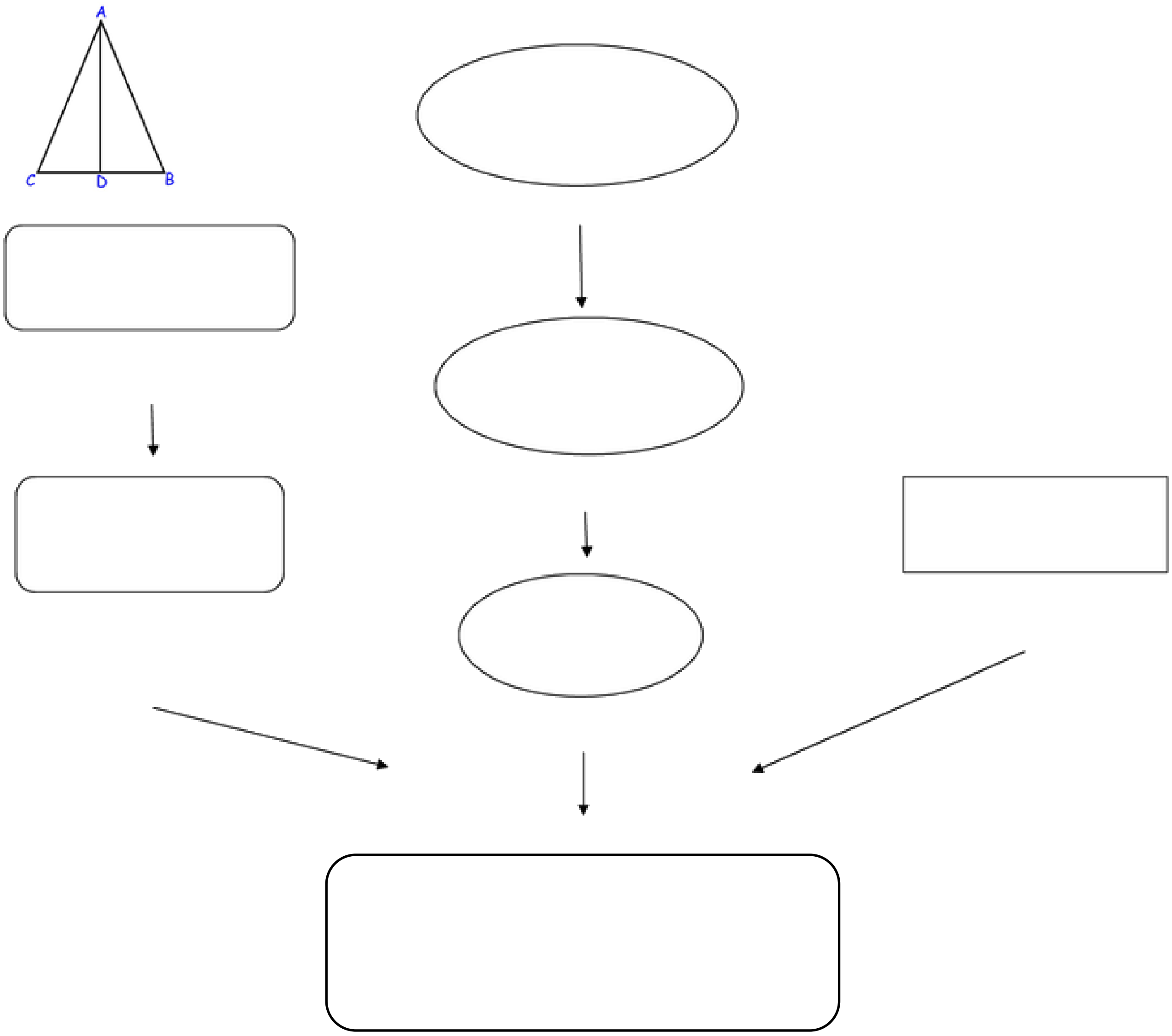
Given D is the midpoint of $\overline{CB}$, $\overline{AC} \cong \overline{AB}$ Prove $\Delta CAD \cong \Delta$ _______



2. Given D is the midpoint of $\overline{CB}$, $\overline{AD}$ is perpendicular to $\overline{CB}$ Prove $\Delta DCA \cong \Delta$ _______



3. Given $\overline{AD}$, bisects $\angle$CAB, $\overline{AD} \perp \overline{CB}$ Prove $\Delta ADC \cong \Delta$ _______



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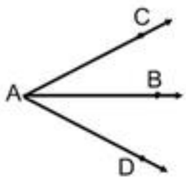
Homework task

5 MINUTES

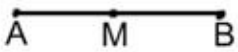
Homework gives my students an opportunity to review certain concepts and practise the skills that were covered in class today.

If the 'Given' says … what comes next?

$\overrightarrow{AB}$ bisects $\angle CAD$

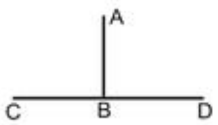


1.________________ because ____



M is the midpoint of $\overline{AB}$

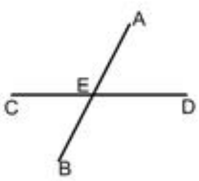
2.________________ because ____



$\overline{AB} \perp \overline{CD}$

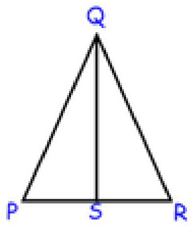
3.________________________ because ____

4.________________________ because ____



$\overline{AB}$ intersects $\overline{CD}$ at E

5.________________________ because ____



If two triangles share a side, then:

6.________________ because ____

If two triangles congruent, then:
(Hint: Use diagram from above.)

7. $\Delta PSQ \cong \Delta$________ because of one of **4** reasons: 1. ________________________

2. ________________________

3. ________________________

4. ________________________

Acknowledgement
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https://teaching.betterlesson.com/lesson/546136/rigid-motion-congruent-triangles-and-proof