# Pythagoras’ theorem two ways

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| Year levelStrand(s)Lesson lengthCD Code: | 9Measurement60 mins[AC9M9M03](https://v9.australiancurriculum.edu.au/f-10-curriculum.html/learning-areas/mathematics/year-9/content-description?subject-identifier=MATMATY9&content-description-code=AC9M9M03&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick) |
| Lesson summary | In this lesson, students use Pythagoras’ theorem to calculate unknown side lengths in right-angled triangles after undertaking a step-by-step method of working through visual and algebraic proofs simultaneously. This will help students develop deeper connections between algebra and measurement in this context. |
| Learning intention | * We are learning to use Pythagoras’ theorem to calculate unknown side lengths in right-angled triangles.
* We will prove Pythagoras’ theorem using measurement skills and knowledge.
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| Success criteria | By the end of this lesson, students can: * prove Pythagoras’ theorem by creating a set of three squares
* identify the need to use Pythagoras’ theorem as one element of a solution to a broader spatial problem.
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| Why are we learning about this? | Pythagoras’ theorem can be used as a tool to solve mathematical problems in an extraordinary number of fields, including design, architecture, building, engineering and computers. If you want to find the shortest distance between two points and get there faster, use Pythagoras’ theorem. |
| Prerequisite student knowledge and language | Prior to this lesson, it is assumed that students have knowledge of working with algebra, linear relationships, number sense, as well as the Pythagoras’ theorem and scalability of right-angled triangles.  |
| **Resources** | Teacher’s slides (PowerPoint)Exit ticket (Word)Cardboard or paper cut into squares 10 cm × 10 cmBlank paper (optional) |

Curriculum information

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| Achievement standard | Students apply Pythagoras’ theorem and use trigonometric ratios to solve problems involving right-angled triangles. |
| Content description(s) | Solve spatial problems, applying angle properties, scale, similarity, Pythagoras’ theorem and trigonometry in right-angled triangles ([AC9M9M03](https://v9.australiancurriculum.edu.au/f-10-curriculum.html/learning-areas/mathematics/year-9/content-description?subject-identifier=MATMATY9&content-description-code=AC9M9M03&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick)) |
| General capabilitiesCross-curriculum priority | Numeracy* Understanding geometric properties ([Level 7](https://v9.australiancurriculum.edu.au/f-10-curriculum.html/learning-areas/mathematics/year-9/general-capability-snapshot?subject-identifier=MATMATY9&content-description-code=AC9M9M03&general-capability-code=N&element-code=NM&sub-element-index=0&sub-element-code=NMUGP&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick))
* Understanding units of measurement ([Level 10](https://v9.australiancurriculum.edu.au/f-10-curriculum.html/learning-areas/mathematics/year-9/general-capability-snapshot?subject-identifier=MATMATY9&content-description-code=AC9M9M03&general-capability-code=N&element-code=NM&sub-element-index=1&sub-element-code=NMUuM&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick))
* Proportional thinking ([Level 7](https://v9.australiancurriculum.edu.au/f-10-curriculum.html/learning-areas/mathematics/year-9/general-capability-snapshot?subject-identifier=MATMATY9&content-description-code=AC9M9M03&general-capability-code=N&element-code=NN&sub-element-index=0&sub-element-code=NNPrT&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick))

Critical and Creative Thinking* Interpret concepts and problems ([Level 6](https://v9.australiancurriculum.edu.au/f-10-curriculum.html/learning-areas/mathematics/year-9/general-capability-snapshot?subject-identifier=MATMATY9&content-description-code=AC9M9M03&general-capability-code=CCT&element-code=CCTANA&sub-element-index=0&sub-element-code=CCTANAA&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick))
* Draw conclusions and provide reasons ([Level 6](https://v9.australiancurriculum.edu.au/f-10-curriculum.html/learning-areas/mathematics/year-9/general-capability-snapshot?subject-identifier=MATMATY9&content-description-code=AC9M9M03&general-capability-code=CCT&element-code=CCTANA&sub-element-index=1&sub-element-code=CCTANAB&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=0&subjects-start-index=0&view=quick))
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| Areas of challenge | Some students may:* misidentify the correct sides (a, b, c) in relation to the right angle when applying Pythagoras’ theorem
* apply Pythagoras’ theorem in non-right-angled triangles.

Give students plenty of practice identifying side lengths in relation to angles before moving on to scaling between two triangles and also before introducing trigonometry.  |
| Strategies | [Classroom talks](https://www.mathematicshub.edu.au/plan-teach-and-assess/teaching/teaching-strategies/classroom-talks/)[Worked examples](https://www.mathematicshub.edu.au/plan-teach-and-assess/teaching/teaching-strategies/worked-examples/) [Mathematics investigation](https://www.mathematicshub.edu.au/plan-teach-and-assess/teaching/teaching-strategies/mathematics-investigation/)[Explicit teaching](https://www.mathematicshub.edu.au/plan-teach-and-assess/teaching/teaching-strategies/explicit-teaching/) |

Lesson structure

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| Learning hook15 mins | Download the teacher’s slides to complement the teaching of this lesson. Pre-prepare cardboard/paper square cut 10 cm × 10 cm for each student, or have students cut these out themselves. **Introduction*** Begin the lesson with slides 1–2 to introduce the lesson ideas, intentions and success criteria.
* Go through slides 3–5 as a class for a skills check on surd calculations and how they relate to solving Pythagoras’ theorem. Have students complete the calculations in their exercise books. Collect their books at the end of the lesson to note their progress and to see where further support is needed.

**Learning hook**Present slide 6 and pose the following ‘Two truths and a lie’ statements. * I was the leader of a cult who abstained from many pleasures and were sworn to secrecy.
* There is a statue of me in Greece with a giant triangle.
* I was the first person to refer to myself as a ‘philosopher’ (lover of wisdom).

Spend a few minutes as a class in discussion and then have students use their own knowledge of Pythagoras to replicate the activity. Encourage students to be creative, use the internet briefly to read about Pythagoras, or refer to what they know about right-angled triangles and Pythagoras’ theorem. They may use various proofs they may remember from the previous year or they may discover them in their search. |
| Explore40 mins | **Note:** The first part of this activity refers to slides 7–11. Explain to students that they will now continue learning about Pythagoras by connecting the visual proofs they learnt about in Year 8 with the specific algebraic steps; step by step they will simultaneously prove the theorem. Assure students they will be supported each step of the way as it first may appear challenging.**Step 1** (Slide 8)* Visual proof

Every student receives a square piece of card or paper thick enough to trace around. Students place the card on a blank piece of paper, or in their exercise books, and trace around the edges to create a second, equal-sized square.Explain that each side of the square must be split into two unequal lengths. This split must be replicated across all sides. Mark out the point at which these lengths meet on each side. Refer students to the diagram shown in the slide. Ask them to label one set of lengths as ‘*a*’ and the other as ‘*b*’.* Algebraic proof

Now ask students to determine an expression for the total area of the square in terms of *a* and *b*. **Step 2** (Slide 9)* Visual proof

Ask students to draw an edge between the inner endpoints of each pair of adjacent *a* and *b* lengths and to label these new lengths as *c*; then cut along them to create a set of four triangles and one square.* Algebraic proof

Ask students to determine an expression for the area of the four triangles and new, smaller square and to show that the combined area of these five shapes must be equal to the area of the large square determined in the previous step.**Step 3** (Slide 10)* Visual proof

Ask students to remove the inner square to create a blank space with an area that is equal to $c^{2}$. Have them move around the four triangles within the outer square to change the shape of the blank space from a single square to two smaller squares. * Algebraic proof

Ask students to equate the two expressions they have determined to create an equation, that is: Area of outer square = area of four triangles + area of inner square. Then ask them to simplify the equation. Have students volunteer their equations and write them on the board or have students share their equations with their peers at their tables.**Step 4** (Slide 11)* Visual proof

Show that the blank space already identified is equal to $c^{2}$ and can be rearranged to be also equal to $a^{2}+b^{2}$. Therefore: $a^{2}+b^{2}=c^{2}$.* Algebraic proof

By cancelling out the $2ab$ on each side of the equation you should be left with $a^{2}+b^{2}=c^{2}$.Congratulate students on proving the famous Pythagoras’ theorem in two ways in the same number of steps. This is a perfect point to take questions and have a class discussion.**Apply** * So that students understand what they have just reasoned, reiterate that equations are mathematical **representations** of relationships, and that the famous Pythagoras’ theorem tells us that there is a relationship between the three sides of right-angled triangles.
* Slides 12–18: students are explicitly shown how to use the theorem to make calculations to find unknown side lengths. Do this as a class using the [Worked examples](https://www.mathematicshub.edu.au/plan-teach-and-assess/teaching/teaching-strategies/worked-examples/) and [Explicit teaching](https://www.mathematicshub.edu.au/plan-teach-and-assess/teaching/teaching-strategies/explicit-teaching/) strategies.
* Explain that the relationship can be applied to determine an unknown side length, provided we know the lengths of the other two sides. Now students will practise this application in two stages: first by finding the hypotenuse (*c*) and second by finding a shorter side (*a* or *b*) of a particular triangle. Note that students may not have made calculations with Pythagoras’ theorem before this lesson.

**Scale and similarity** (Slides 16 and 17)Identifying similar triangles is a key skill at this level and it is worth taking the time now to review this concept, with a particular focus on the RHS test. Alternate between slides 16 and 17 and ask students if they can spot the relationship between the triangles on each slide. The left triangle has been scaled by a factor of 3 and the right triangle has been scaled by a factor of 0.5 from the previous slide. Since students have already determined the missing side lengths in the previous activity, they can now use their knowledge of similarity to determine the missing lengths of on slide 17 without using the theorem. This is a good opportunity to reinforce that there can be simpler ways to tackle a problem if you already have prior information. While working through Pythagoras will give the same result, it is not necessary to perform these tedious calculations, given we have identified the triangles as similar.Ask students to draw an additional version of each triangle that has been scaled by a new factor and determine the missing length. These can be shared with the class and listed on the board. This can help to draw out the idea that an infinite number of triangles with the same shape that are merely scaled up or down. This links to the next slide about Pythagorean triples.**Pythagorean triples** (Slide 18)Students should already be familiar with this concept, but a brief review may also be necessary (see the Year 8 lesson, ‘Pythagorean triples’).The core idea is that given there is a relationship between the sides of a triangle, only certain sets of values will fit. These values will also naturally follow the rule that the sum of the square of the two smaller values will equal the square of the largest. When a set of three integers fits this rule we call them a Pythagorean triple.Given that students have now considered scaling, they can spot that these triples also come in scaled versions of one another. This could be an opportunity for a discussion on how many unique triplets exist, which could lead to further thoughts about prime numbers and infinity.Students can cement their understanding in this section by generating some scaled versions of other Pythagorean triples from the table shown on slide 18. |
| Summary and reflection5 mins | Today’s lesson was about making deeper connections with Pythagoras’ theorem. Ask students to volunteer the parts of the lesson that resonated with them. * Was it the first part where they took a simultaneous step by-step approach using visual and algebraic steps to prove the theorem?
* Did students connect more with applying the theorem in two ways to solve a measurement problem?
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| Assessment | The following exit ticket is suggested as an assessment opportunity. * Download and print the ticket from the What you need section of the online lesson for each student to complete in class or for homework.
* The exit ticket questions are also shown in the final slide of the downloadable teacher’s slides. Students could answer the questions in their exercise books if preferred.
* Review and provide feedback for each student.
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