Teacher’s notes and Egg-cellent solutions

In pairs student must choose whether the following probability experiments are:

with replacement – is the result or item from the first pick available in the second pick

without replacement – once an item has been picked it is gone and no longer available in the second pick.

|  |  |  |
| --- | --- | --- |
| Luca draws a coloured marble from a bag, replaces it and draws another marble.**WITH REPLACEMENT** | Mrs Johnson has a digital spinner to choose which students answer questions in her maths class. The student’s name is removed from the spinner after they answer.**WITHOUT REPLACEMENT** | Ms Khan uses lolly sticks with students’ names to choose which students answer questions in her maths class. Lolly sticks are always returned to the jar.**WITH REPLACEMENT** |
| Amelia picks 3 chocolates from a box to eat.**WITHOUT REPLACEMENT** | Noah is listening to music and puts his favourite album on shuffle **WITH / WITHOUT REPLACEMENT1** | Mr Woo selects 2 students from his class to send to the school office**WITHOUT REPLACEMENT** |
| Kai is playing egg roulette with his friend Bowen.**WITHOUT REPLACEMENT** | Nira is a biologist who is counting sharks to see how many have been tagged. She catches 1 shark at a time, checks for a tag and releases it.**WITH REPLACEMENT** | Sofia is playing bingo with her Nona and draws out 3 numbers at a time**WITHOUT REPLACEMENT** |

Notes

1 Music shuffle algorithms were originally designed to be random (WITH REPLACEMENT) but users complained that the same songs kept repeating and it wasn’t ‘random enough’. The developers have now changed their algorithms, so songs don’t often repeat, ironically making them less random. Hence, shuffles are now more like a WITHOUT REPLACEMENT scenario, although not perfectly.

Egg roulette – exploring the probabilities

Having prior knowledge about the game, think back to the video.

1. What was the probability that Miriam chose a cooked egg on her first pick?

Answer: $\frac{9}{12}=\frac{3}{4}$

1. Given that Miriam chose a cooked egg, what was the probability that Denise chose a cooked egg on her first pick?

Answer: $\frac{8}{11}$ Note that both the number of cooked eggs and total number of eggs have decreased by 1 because of Miriam’s pick.

In your pair complete the table listing the sample space for the first two eggs picked. For result RR, RC, CR, CC shade the boxes a different colour, where R = raw and C = cooked. Then answer the questions below,

Note: Explain that a table or array like this is an excellent way of listing and identifying all the outcomes in a sample space for a two-stage probability experiment.

|  |
| --- |
| **Second egg selected – Denise** |
| Raw1 | Raw2 | Raw3 | Cooked1 | Cooked2 | Cooked3 | Cooked4 | Cooked5 | Cooked6 | Cooked7 | Cooked8 | Cooked9 |
| **First egg selected – Miriam** | Raw1 |  | RR | RR | RC | RC | RC | RC | RC | RC | RC | RC | RC |
| Raw2 | RR |  | RR | RC | RC | RC | RC | RC | RC | RC | RC | RC |
| Raw3 | RR | RR |  | RC | RC | RC | RC | RC | RC | RC | RC | RC |
| Cooked1 | CR | CR | CR |  | CC | CC | CC | CC | CC | CC | CC | CC |
| Cooked2 | CR | CR | CR | CC |  | CC | CC | CC | CC | CC | CC | CC |
| Cooked3 | CR | CR | CR | CC | CC |  | CC | CC | CC | CC | CC | CC |
| Cooked4 | CR | CR | CR | CC | CC | CC |  | CC | CC | CC | CC | CC |
| Cooked5 | CR | CR | CR | CC | CC | CC | CC |  | CC | CC | CC | CC |
| Cooked6 | CR | CR | CR | CC | CC | CC | CC | CC |  | CC | CC | CC |
| Cooked7 | CR | CR | CR | CC | CC | CC | CC | CC | CC |  | CC | CC |
| Cooked8 | CR | CR | CR | CC | CC | CC | CC | CC | CC | CC |  | CC |
| Cooked9 | CR | CR | CR | CC | CC | CC | CC | CC | CC | CC | CC |  |

1. Explain why the boxes on the diagonal have been shaded out in grey.

Answer: Because egg roulette is without replacement. Note that students may struggle with this question. Choose an example like Raw1, Raw1. Explain that for each cell to be an outcome means having both the options at the header of the row and column as true, but once Raw egg1 has been picked it is not available for the second pick so Raw1, Raw1 is not a possible outcome.

1. How many outcomes are in the whole sample space (count the boxes in the table)

Answer: There are $132=12×11$ outcomes as there are 12 possible choices for the first pick, but only 11 remaining for the second pick. Students who answer $144=12×12$ have forgotten to exclude the greyed-out diagonal ie they are modelling a with replacement scenario

1. How many outcomes are CC (both eggs cooked)?

Answer: There are 72 outcomes of CC. The two blue shaded triangles could be combined to make an $8×9$ rectangle.

Students who answer $81=9×9$ have included the greyed diagonal and not taken account of the non-replacement

1. Miriam and Denise both got cooked eggs on their picks. Were they really lucky? Explain your answer by calculating and discussing the probability that this might happen.

Answer: Altogether there are 72 outcomes of 2 cooked eggs (CC) from a total of 132, so the probability that they get CC is $\frac{72}{132}≈54.5\%.$ This is in fact the most likely outcome, so they are not especially lucky.

Egg roulette – simulation

To simulate multiple games of egg roulette against the computer:

work in pairs, with one person running the simulation and the other recording the results (Swap roles as needed)

open the Excel spreadsheet

click the arrows to run a simulation.

Simulation 1: first two picks

You are going to see if you can be as lucky as Miriam and Denise and get no raw eggs in the first 2 picks of the game.

Record the result of each simulation from the yellow section labelled, ‘What is the result of the first two picks’ in the tally column of the table below.

|  |  |  |  |
| --- | --- | --- | --- |
| Result of first 2 picks | Tally | Frequency | Experimental probability (%) |
| Cooked – Cooked |  |  |  |
| Cooked – Raw |  |  |  |
| Raw – Cooked |  |  |  |
| Raw – Raw |  |  |  |

Run the simulation 50 times. Record your tally each time.

Complete the frequency column by counting the tally for each result.

1. Calculate the experimental probability (%) as $\frac{frequency}{50}×100$.

Answer: The activity has asked for probabilities to be calculated as percentages as this makes it easier and more intuitive to compare experimental and theoretical probabilities.

1. Compare your calculated probabilities with the table prepared in part I. Explain any differences that have arisen and suggest ways to reduce them.

The probabilities would be expected to be somewhat close to those in the table on page [x]. however, a trial size of 50 is relatively small and therefore there would be expected to be some variation from the expected values. Increasing the number of trials will give experimental probabilities closer to the theoretically expected values, which is an example of the law of large numbers.

Simulation 2: How many times will you get egged?

You are now going to play against the computer to see how many times you get egged over a whole game.

1. What is the sample space for how many raw eggs you get over the whole game?

Answer: Sample space is the set of possible outcomes. Here, sample space $=\left\{0,1,2,3\right\}$ raw eggs

1. Are these outcomes equally likely? If not, which outcome(s) do you expect to be most common and have a higher probability?

Answer: Students may be surprised to learn that these outcomes are not equally likely. Getting 0 or 3 eggs is much less likely that 1 or 2. There is a pattern though, as obtaining 0 or 3 raw eggs have the same probability (both 9.1%) – this can be explained because if you get 0 your opponent gets 3 and vice versa. Swapping the person does not change the probability. Similarly, getting 1 and 2 raw eggs have the same probability of 40.9% each.

Justifying these probabilities requires an advanced understanding of combinatorics, far beyond the level of most Year 9 students:

To solve this problem, we can use the formula for combinations from combinatorics. The formula for combinations is denoted as $\left(\begin{matrix}n\\r\end{matrix}\right)$ where $n$ is the total number of items, and$ r$ is the number of items being chosen. The formula is given by: $\left(\begin{matrix}n\\r\end{matrix}\right)=\frac{n!}{r!\left(n-r\right)!}$

As an example, we will find the probability of choosing all 3 raw eggs and 3 cooked eggs from the 9 available cooked eggs.

The total number of ways to choose 6 eggs from 12 is given by $\left(\begin{matrix}12\\6\end{matrix}\right)$.

The number of ways to choose 3 raw eggs from the 3 raw available is $\left(\begin{matrix}3\\3\end{matrix}\right)$ and the number of ways to choose 3 cooked eggs from the 9 available is $\left(\begin{matrix}9\\3\end{matrix}\right)$.

So, the probability of choosing all 3 raw eggs when choosing 6 eggs is given by:

$$P=\frac{\left(\begin{matrix}3\\3\end{matrix}\right)\left(\begin{matrix}9\\3\end{matrix}\right)}{\left(\begin{matrix}12\\6\end{matrix}\right)}=\frac{1×84}{924}≈0.091≈9.1\%$$

Run the simulation 50 times and looking at ‘How many raw eggs do you get?’ and record the results in the tally column in the table below, including the frequency and experimental probability, calculated as before:

|  |  |  |  |
| --- | --- | --- | --- |
| Number of raw eggs I got | Tally | Frequency | Experimental probability (%) |
| 0 |  |  |  |
| 1 |  |  |  |
| 2 |  |  |  |
| 3 |  |  |  |

1. Compare your calculated probabilities with your expectations in part b).

Answer: Students should find that their experiment reveals that the outcomes 1 or 2 raw eggs are far more likely to occur than 0 or 3 eggs and their probabilities should be somewhat close to those discussed in part b).

Exit ticket

****

A box of chocolates contains four chocolates that look identical but three are caramel (yum) and one is a peanut (yuk).

Use your knowledge of probability, with a diagram, to help me decide whether I should eat one or two chocolates.

Exit ticket

****

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