YuMi Deadly Maths

Statistics and Probability

Prep to Year 9

Prep to Year 9: Resource 7 – **Statistics and Probability**

QUT YuMiDeadly
*Growing community
through education*

YuMi Deadly Maths

Prep to Year 9

Statistics and Probability

VERSION 3, 20/12/19

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The YuMi Deadly Centre acknowledges the traditional owners and custodians
of the lands in which the mathematics ideas for this resource were developed,
refined and presented in professional development sessions.

DEVELOPMENT OF THIS BOOK

This version of the YuMi Deadly Maths Statistics and Probability book is a modification and extension of a book developed as part of the Teaching Indigenous Mathematics Education (TIME) project funded by the Queensland Department of Education and Training from 2010–12. The YuMi Deadly Centre acknowledges the Department's role in the development of YuMi Deadly Maths and in funding the first version of this book.

YUMI DEADLY CENTRE

The YuMi Deadly Centre is a research centre within the Faculty of Education at QUT which is dedicated to enhancing the learning of Indigenous and non-Indigenous children, young people and adults to improve their opportunities for further education, training and employment, and to equip them for lifelong learning.

"YuMi" is a Torres Strait Islander Creole word meaning "you and me" but is used here with permission from the Torres Strait Islanders' Regional Education Council to mean working together as a community for the betterment of education for all. "Deadly" is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life.

YuMi Deadly Centre's motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates the YuMi Deadly Centre's vision: *Growing community through education*.

The YuMi Deadly Centre can be contacted at ydc@qut.edu.au. Its website is http://ydc.qut.edu.au.

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ABOUT YUMI DEADLY MATHS

From 2000–09, researchers who are now part of the YuMi Deadly Centre (**YDC**) collaborated with principals and teachers predominantly from Aboriginal and Torres Strait Islander schools and occasionally from low socio-economic status (SES) schools in a series of small projects to enhance student learning of mathematics. These projects tended to focus on a particular mathematics strand (e.g. whole-number numeration, operations, algebra, measurement) or on a particular part of schooling (e.g. middle school teachers, teacher aides, parents). They resulted in the development of specialist materials but not a complete mathematics program (these specialist materials can be accessed via the YDC website, http://ydc.qut.edu.au).

In October 2009, YDC received funding from the Queensland Department of Education and Training through the Indigenous Schooling Support Unit, Central-Southern Queensland, to develop a train-the-trainer project, called the **Teaching Indigenous Mathematics Education** or **TIME** project. The aim of the project was to enhance the capacity of schools in Central and Southern Queensland Indigenous and low SES communities to teach mathematics effectively to their students. The project focused on Years P to 3 in 2010, Years 4 to 7 in 2011 and Years 7 to 9 in 2012, covering all mathematics strands in the Australian Curriculum: Number and Algebra, Measurement and Geometry, and Probability and Statistics. The work of the TIME project across these three years enabled YDC to develop a cohesive mathematics pedagogical framework, **YuMi Deadly Maths**, that covers all strands of the *Australian Curriculum: Mathematics* and now underpins all YDC projects.

YuMi Deadly Maths (**YDM**) is designed to enhance mathematics learning outcomes, improve participation in higher mathematics subjects and tertiary courses, and improve employment and life chances. YDM is unique in its focus on creativity, structure and culture with regard to mathematics and on whole-of-school change with regard to implementation. It aims for the highest level of mathematics understanding and deep learning, through activity that engages students and involves teachers, parents and community. With a focus on big ideas, an emphasis on connecting mathematics topics, and a pedagogy that starts and finishes with students' reality, it is effective for all students. It works successfully in different schools/communities as it is not a scripted program and encourages teachers to take account of the particular needs of their students. Being a train-the-trainer model, it can also offer long-term sustainability for schools.

YDC believes that changing mathematics pedagogy will not improve mathematics learning unless accompanied by a whole-of-school program to challenge attendance and behaviour, encourage pride and self-belief, instil high expectations, and build local leadership and community involvement. YDC has been strongly influenced by the philosophy of the Stronger Smarter Institute (C. Sarra, 2003) which states that any school has the potential to rise to the challenge of successfully teaching their students. YDM is applicable to all schools and has extensive application to classrooms with high numbers of at-risk students. This is because the mathematics teaching and learning, school change and leadership, and contextualisation and cultural empowerment ideas that are advocated by YDC represent the best practice for **all** students.

YDM is now available direct to schools face-to-face and online. Individual schools can fund YDM in their own classrooms (contact ydc@qut.edu.au or 07 3138 0035). This Statistics and Probability resource is part of the provision of YDM direct to schools and is the seventh in a series of resources that fully describe the YDM approach and pedagogical framework for Prep to Year 9. It focuses on teaching statistics and probability and covers tables and graphs, probability, and statistical inference. It overviews the mathematics and describes classroom activities for Prep to Year 9. Because YDM is largely implemented within an action-research model, the resources undergo amendment and refinement as a result of school-based training and trialling. The ideas in this resource will be refined into the future.

YDM underlies three projects available to schools: YDM Teacher Development Training (TDT) in the YDM pedagogy; YDM AIM training in remedial pedagogy to accelerate learning; and YDM MITI training in enrichment and extension pedagogy to build deep learning of powerful maths and increase participation in Years 11 and 12 advanced maths subjects and tertiary entrance.

Contents

1 Purpose and Overview

Statistics and probability are crucial everyday life skills and important mathematical topics. Most decision-making of modern society is based on statistics and probability. In advertising, politics and economics, samples are organised, survey questions developed, answers sought, results tabulated and organised, and predictions displayed with averages and graphs to show distributions, relationships and trends before decisions are made on such questions as, *What do people want in a car? Should Queensland have daylight saving*? Many computer banks are filled with the raw data on which such decisions will be made.

Large amounts of raw data are incomprehensible; statistical techniques such as tables, graphs and measures of central tendency are indispensable tools for comprehending the raw data on which decision-making is based. Measures of central tendency such as means and medians supply a framework with which to describe what happens. Graphs supply a visual way of presenting the range of alternatives available and of indicating the density of interest (e.g. most popular/likely). Probability is equally important because nearly all life decisions involve uncertainty, and thus are made on possibilities and probabilities. Situations in real life can be of a predictable or random nature. Probability involves the measurement of the likelihood of events in chance processes.

This book divides the material for statistics and probability into three chapters as below and as represented by the figure on right.

1. **Tables and graphs –** covering data gathering and representation, and tables, charts and graphs as visual ways of presenting alternatives and indicating density of interest (e.g. most likely, most popular).

2. **Probability** – covering chance as attribute and fraction, measurement of the likelihood of events in chance processes, and experimentation and its relation to inference.

3. **Statistical inference** – covering analysis and interpretation of data, measures of central tendency and distribution such as *means*, *medians* and *deviation,* complex representations (e.g. box and whisker graphs), and decision-making from data.
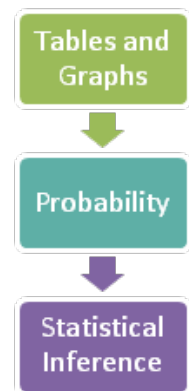
This chapter provides purposes for, and an overview of, this Statistics and Probability book. It covers connections and big ideas (section 1.1), sequencing (section 1.2), teaching and cultural implications (section 1.3) and overview of book (section 1.4). Please note that, although Probability is a separate chapter from the two Statistics chapters, probability activities provide rich sources of activities for statistics and most inferences involve probability.

1.1 Connections and big ideas

This section covers the knowledge that can allow a teacher to assist students to see mathematics knowledge as a structure of recurring ideas, of connections and big ideas.

1.1.1 Connections

Statistics and probability activities involve counting responses to questions and assigning numeric values to chance events for analysis. Subsequently, number concepts of counting, fractions and percent are inherently linked to statistics and probability activities.

Table and graph statistics activities involve counting responses to questions and assigning numeric values to chance events for analysis. Subsequently, number concepts of counting, fractions and percent are inherently linked to statistics and probability activities. Table and graph statistics is also highly visual and relies on

classifying, organising and summarising data into rows and columns and using this data to draw rectangles, lines and dots within a Cartesian framework or to divide a circle into sectors.

The data for the tables and graphs can be in: (a) a category form such as colour; (b) an ordinal form such as short, normal and tall; or (c) an interval form such as height in centimetres. The data can also be in: (a) discrete and discontinuous form like cost to nearest dollar; or (b) continuous form like mass in kg to decimal places. It is also the case that the gathering of data, particularly interval and continuous data, involves some acts of measuring attributes such as distance and temperature.

Thus, statistics is connected to the other mathematical strands **as on right** and in terms of the following detail.

1. **Pre-number and Number** – predominantly classifying and patterning, counting, fractions and percent but also including operations.

2. **Algebra/Number sense** – basic ability to notice relationships and comparisons; to recognise the relative magnitudes of the comparisons and relationships; and to understand continuous, discrete and linear relationships.

3. **Geometry** – rectangular and circular shapes, Cartesian coordinates, square tessellation (e.g. patterns of rows and columns), angle, and visualisation.

4. **Measurement** – understanding of attributes, order and measurement with respect to distance, mass, capacity, area, volume, time and temperature.
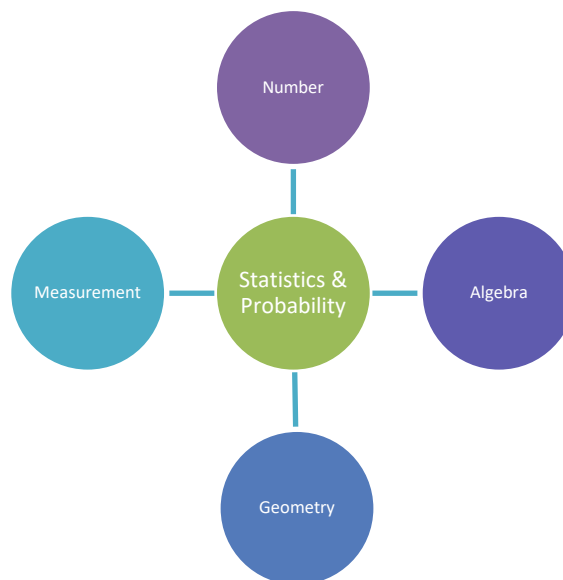
Weaknesses or lack of conceptual understanding in one of these connected topics, particularly counting, fractions and percent, will hamper students' progress in tables and graphs. Where students are demonstrating lack of success in the statistics of tables and graphs, it is necessary to ascertain whether the difficultly lies with the statistical concepts and processes or with the numerical, algebraic, geometrical and measurement processes attached to their representations.

1.1.2 Big ideas

Big ideas are mathematical ideas that underlie topics and recur across the years of schooling. They can be concepts, principles, or strategies (as on right). They can be global or more specific (global big ideas are relevant to nearly all mathematics ideas). They can also relate to teaching.
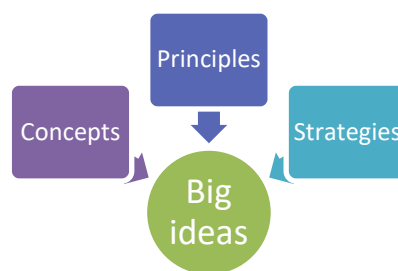
Global big ideas

1. **Chance vs certainty**. In arithmetic, problems have **certain** answers, that is, 4+7=11. However, in statistics, decisions can be made in terms of **chance** or **uncertainty**. The data is not absolute; it shows that there are more options and thus predicts the best chance for an outcome. It is important that students know when they are in certain and when they are in chance situations.

2. **Accuracy vs exactness**. This goes hand-in-hand with the first big idea. In arithmetic, answers can be calculated exactly. However, in measures and in drawing inferences from data, there is sometimes no exactness, there is only being as accurate as possible or as required.

3. **Interpretation vs construction**. It is essential to be able to interpret data but this can be assisted by learning how to construct data.

Concept big ideas

1. **Matrix structure**. In tables and graphs, data is gathered and combined into rows and columns, and mostly drawn on paper which has a Cartesian structure (the rows and columns of a matrix).

2. **Notion of unit** (and commonality of unit). Units must be selected that effectively measure the concept under focus and the same size units from the same starting point must be used in comparisons and trends.

3. **Scale**. Components of representations must have commonality in scale and not be stretched or squashed or truncated.

4. **Multiplicative structure**. Comparisons should be understood in a multiplicative sense (such as percentages) and the significance of their comparisons and trends should be understood in a multiplicative sense.

Principle big ideas

1. **Variation and uncertainty**. Statistics and probability do not deal with exactness (this follows on from global big idea 2). Activities tend to provide information that varies (in terms of who gathered it and the time/place of the gathering) and is uncertain (will have a range of numbers that are possible – called error, i.e. instead of length being 210.5 m it will be 210.5 m ± 0.25 m).

2. **Centrality of context**. Instead of providing ideas that are generic (i.e. hold for all situations), statistics and probability tend to provide answers that are particular to the context being looked at.

3. **Integration of information**. Probability and statistics differ from arithmetic in that their focus is wider, often using large data sets and integrating many ideas. This makes, in particular, inferences holistic, requiring information to be integrated. Arithmetic tends to break problems into parts.

4. **Relation between sample and population**. Decisions made from samples can be considered generic or universal if the relationship between sample and population allows generalisation, otherwise they are contextual.

5. **Efficacy of models and simulations**. Inference is assisted by use of models and simulations as these allow variation and uncertainty to be taken into account.

6. **Formulae**. Probability contains formulae for probability when in situations of equally likely outcomes (e.g. $P(A) = \frac{number\ giving\ A}{total\ number}$). Statistics contains a series of formulae for mean, mode, median, mean deviation and standard deviation (see Chapter 4).

Strategy big ideas

1. **Data driven/Complex thinking**. Statistics problem solving and investigations should be based on data and, therefore, are often examples of complex thinking.

2. **Evidence-based/Inferential thinking**. Statistics problem solving and investigations should be driven by evidence from the data and, therefore, are often examples of inferential thinking.

Teaching big ideas

A teaching big idea is a way of approaching the teaching of mathematics, a technique or an instructional strategy, that works across many year levels and many mathematics topics. The Reality–Abstraction–Mathematics–Reflection (RAMR) model (see section 1.3) is basically a framework of big teaching ideas.

1.2   Sequencing

Although statistics and probability are connected topics within the curriculum and have features in common, it is difficult to combine them under a consistent set of big teaching ideas or sequences. In order to overcome this difficulty and in the interests of clarity, the remainder of this book will address statistics and probability separately in the three chapters – tables and graphs, probability, and statistical inference. However, as stated earlier, because probability activities provide a rich source for statistics activities, these topics should be used together to provide students with clear ideas of the links between the topics – as is discussed across the three chapters.

This means that we will look at sequencing within each of the three chapters and sequencing diagrams will form part of the beginning of each chapter. Thus, in this section we will summarise the major sequences that appear in each of the three chapters.

1.2.1   Sequencing in tables and graphs

The activity in the tables and graphs chapter has been divided into a sequence of five imperatives:
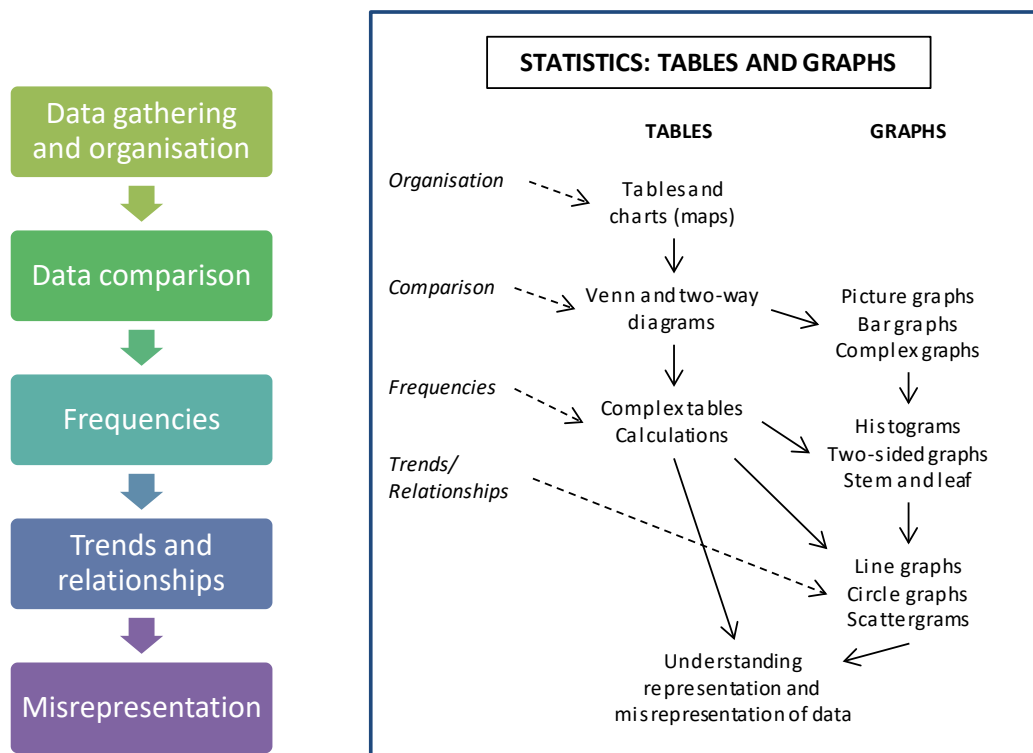
Forms of charts, tables and graphs have been assigned to each of these imperatives. These components have been ordered and this order is the basis of the sequencing in the diagram above right.

Tables and charts become more complex as the graphs become more complex and vice versa. However, there are ways of connecting more complex tables and graphs to their simpler forms.

1.2.2   Sequencing in probability

The sequence suggested by YDM is to start with classification and to end with applications – this sequence moves through five stages and eight skills as shown in the figure below.

Perceive and identify attribute	• Skill 1: Students can correctly classify an event as impossible, possible or certain

Compare and order likelihood of outcomes	• Skill 2: Students can list all possible outcomes of a chance process • Skill 3: Students can list all possible outcomes in consecutive trials of a chance process, with and without replacement • Skill 4: Students can state which outcome is most likely in a single trial of a chance process

Compare and order situations for desired outcome	• Skill 5: Students can correctly choose in what situation an event is more (or most) likely to occur

Measure probabilities	• Skill 6: Students can correctly assign a numerical probability to an event (and use this number for skills 4 and 5) • Skill 7: Students can estimate the numerical probability of an event using "simulation" methods

Apply probability in inferences	• Skill 8: Students can use probability skills and understanding to make informed decisions and predictions

1.2.3 Sequencing in statistical inference

The development of statistical inferences goes through three steps as below:

Statistical literacy	• Focuses on utility and purpose of tools; interprets, critiques, debates and judges.

Statistical reasoning	• Focuses on reasoning and making sense; utilises data, graphs and statistics information to understand the situation.

Statistical thinking	• Focuses on the why and how of statistical investigations; understands distributions; infers, creates and sees things as big ideas.
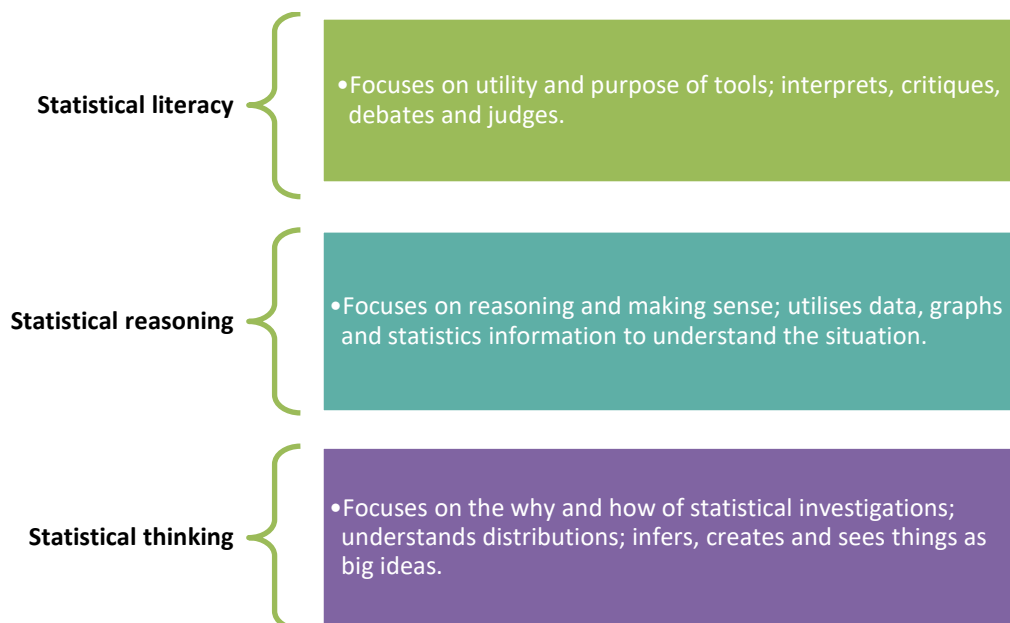
For the purposes of the statistical inference chapter, we have divided the focus of the inferential statistics into a **five-step sequence** from early years to later years, as shown on right.

The reason for these five stages is to look at statistical literacy development in the first section, begin the movement to statistical reasoning in the second section, and then start to build towards statistical thinking in the next two sections of the chapter. We end at looking at the role of misrepresentation in statistics (i.e. "how to lie with statistics") that has led to the statement "there are lies, damned lies, and statistics".

Finally under sequencing, we look at the types of questions, tasks or projects that can be set across the years of inference (Years 3 to 9). We break these problem types into four levels:
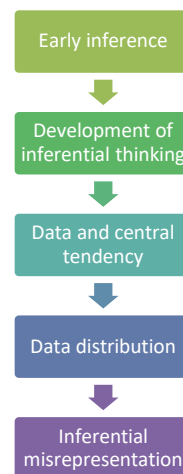
Level A: Simple – one uncertainty, e.g. *Do most students have brown eyes?*

Level B: Multiple – two or more uncertainties, e.g. *Do tall children run faster?*

Level C: Extended – two or more uncertainties plus need for other maths/science knowledge, e.g. *What year level has the healthiest lunch?, What is the best design for a loopy aeroplane?*

Level D: Complex – all of Level C plus differences between types, e.g. *Do typical Year 7 students eat healthy cereals?*

1.3 Teaching and cultural implications

This section looks at teaching and cultural implications, including the Reality–Abstraction–Mathematics–Reflection (RAMR) framework and the impact of Western statistics and probability teaching on Indigenous and low SES students.

1.3.1 Teaching implications

As well as taking into account connections, big ideas and sequencing, the YDM pedagogy is built around the RAMR framework. This is described in Chapter 3 of the YDM Overview book and briefly discussed here. It is also worth looking in more detail at the role of representations and models as these are used across statistics and probability.

YDM sees mathematics teaching as comprising three components – *technical* (handling materials), *domain* (the particular pedagogies needed for individual topics) and *generic* (pedagogies that work for all mathematics). Interestingly, and fortunately, the domain section is not as complicated as it could be because mathematical ideas that are structurally similar can be taught by similar methods. For example, methods for teaching number topics like fractions and percent can be used in probability because probabilities can be expressed as fractions or percentages. There are also some generic teaching methods that hold for any topic.

The **RAMR framework** (see figure below) is the basis of lesson planning in YDM because of the generic teaching ideas contained in the framework, and is applicable to the teaching of statistics and probability. For a start, it grounds all mathematics in reality and provides many opportunities for connections, flexibility, reversing, generalisations and changing parameters, as well as body → hand → mind. The idea is to use the framework and all its components throughout the years of schooling and this will help prevent learning from collapsing back into symbol manipulation and the quest for answers by following procedures.
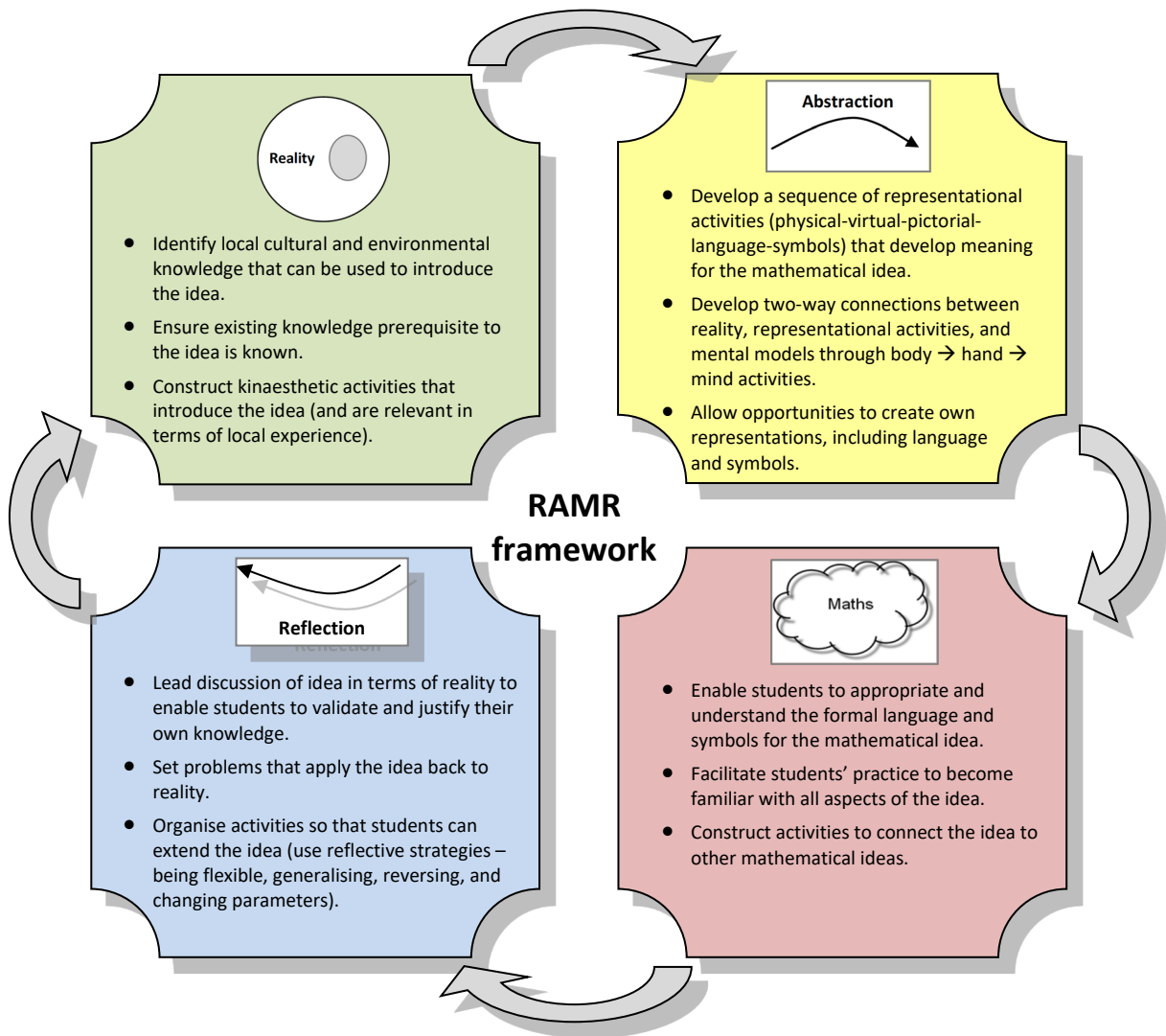
The YuMi Deadly Maths RAMR framework

Tables and graphs, probability, and statistical inference will all be introduced by the Rathmell cycle, and through abstraction. However, probability is the area which will require a more traditional emphasis on abstraction and the topic needing the most understanding of models and representations. For example, a sample space can have its components related to the area model or the set model. As students find area models easier to interpret than set models, spinners should be used before discrete materials such as marbles in developing probabilistic reasoning.

Spinners can be partitioned into equal (*units)* or non-equal (*gross*) segments. As well, like parts can be arranged so they are together (*contiguous*) or split apart (*noncontiguous*). Unit measurement is easier for young students to interpret than gross measurement, and contiguous is easier to interpret than noncontiguous (see below).
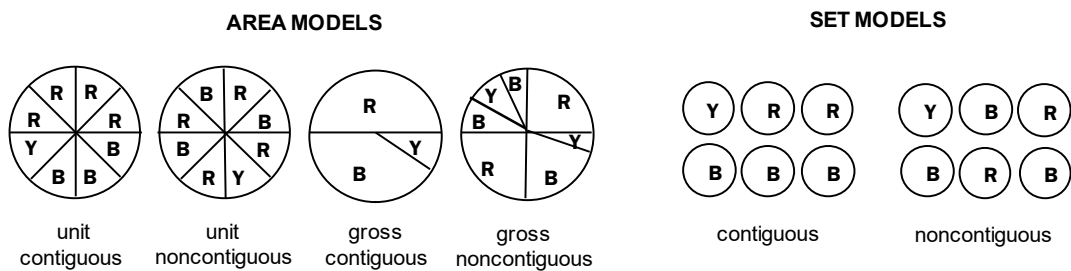
The representation of probabilistic concepts and processes should proceed from real models to concrete to pictures to symbols as for number.

Probability is an application of fractions (i.e. part/whole). Therefore similar models (area, set) can be used to represent sample space. The same sequence of models should be kept in mind (area before set). In establishing the representation, language and symbolism of probability, Payne and Rathmell's (1977) model should be followed (see right).

REAL-WORLD PROBLEM
(related to student's culture)

Mardi needs to get a blue marble to score a point in a game. What chance does she have of getting, without looking, a blue marble from the bag below?

1 ↑ ↓ 2

REPRESENTATION

3 4   7 8

LANGUAGE
1 chance out
of 5 chances   5 6   **SYMBOL**
$\frac{1}{5}$

1.3.2 Cultural implications

As well as connections, big ideas, sequencing and the RAMR cycle, the YDM pedagogical approach takes account of culture, involves parents and community and is based on whole-school programs of change.

Statistics and probability are inherently linked to understanding of number and fractions. Essentially statistics serve to enumerate data and present it in a form for analysis which may be summarised in measures of central tendency or presented graphically. Probability assigns a numeric value to the likelihood of an event. As such, cultural implications for number can be extended to these strands of mathematics. However, there are also specific cultural implications for each of these domains of mathematics.

Statistics are used to summarise, describe and represent data for analysis. Data is frequently gathered by research. Students may be exposed to statistics that have been collated and reported that are not representative of their cultural group but instead based on extreme cases that exist in any cultural group. Where any given group has been widely researched, or has experienced the effects of colonisation such as Aboriginal and Torres Strait Islander groups, statistics may exist that paint extremely negative pictures of the group. Where statistics have been reported, represented and disseminated in negative contexts, use of these figures as examples can be detrimental or distressing to these students. While it is important for students to realise and be able to interpret cases of misrepresentation of statistics, this should be done sensitively.

Probability describes likelihood of chance events and is frequently linked to gambling. While this makes a very real-world link for some students and can be an area of mathematics where students excel, teachers need to be sensitive to initiatives and feeling within their local community. In some communities cards are outlawed in an attempt to deal with local issues. As a result, activities within these communities should be conducted in such a way that playing cards are not needed and gambling games minimised. Check with local Elders and community groups or councils.

1.4 Overview of book

This section provides an overview of the contents of the book and describes how teaching ideas are provided through activity plans and investigations as well as RAMR lessons.

1.4.1 Structure of the book

The book has four chapters and three appendices:

Chapter 1: Purpose and overview – describing purpose, connections and big ideas, sequencing, and teaching and cultural implications.
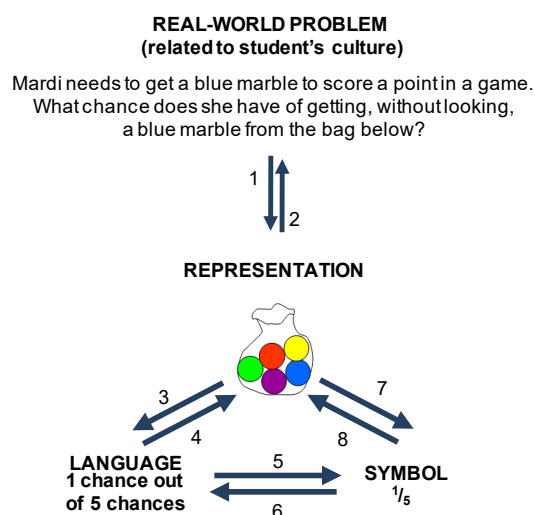
Chapter 2: Tables and graphs – covering data gathering and organisation into tables and charts, data comparison through picture and bar graphs, frequency tables and graphs, methods of showing trends and relationships, and statistical misrepresentation.

Chapter 3: Probability – following the five stages for probability learning and focusing on the eight skills, with sections covering probabilistic situations, outcomes from probability events, desired event, probability as a fraction, experimental probability, and inference.

Chapter 4: Statistical inference – looking at early inference, development of inferential reasoning, measures of central tendency, data distribution, and inferential misrepresentation.

Chapter 5: Teaching framework for Statistics and Probability.

Appendix A: Types of tables and graphs – summarises the various different types of tables, charts and graphs.

Appendix B: Extra material for tables and graphs – provides resources and activities to use in conjunction with Chapter 2.

Appendix C: Extra material for probability – provides additional resources, games, activities and rich tasks for probability.

1.4.2   Activity plans

In the chapters, the teaching ideas are sometimes described by activity plans, that is, general descriptions, activities and reflections. These do not follow the RAMR model advocated for YDM teaching. They provide brief descriptions of classroom activities without giving detail of what would be included in a RAMR-designed lesson.

RAMR lessons are effective lessons because they are based on the actual histories and characteristics of students, and therefore the best lessons are written by teachers and are different for different schools and different teachers. This book provides ideas for the teacher to translate to their students, and recommends that you, the teacher, turn them into RAMR lessons by using your knowledge of your students. Interestingly, sometimes less detailed activity plans can be the most effective in providing teaching ideas to teachers. Also, statistics appears to be a mathematics area that is global in perspective. Everyday real activities (reading tables, understanding graphs, following football statistics) are its focus and, therefore, this focus appears to be omnipresent, rendering reality a part of all four stages in RAMR.

Finally, although this book is divided into chapters and sections, it is important to take every opportunity **to teach across chapters and sections**. For example, if practising tallying for section 2.2, this would mean immediately using the tally material to construct tables and charts and, where appropriate, construct picture and bar graphs for section 2.3 or line graphs and scattergrams for section 2.5. In other words, don't stay within a section of a chapter when there are opportunities to use material from one section in another section. This is because statistics appears to be less sequential and more integrative than arithmetic, and to reflect an investigatory approach to teaching. This lower emphasis on sequencing means that it is often **better to teach by rich tasks that cut across the sections** and not step-by-step through the sections.

Special note: Show how graphs are constructed and read but ensure that electronic means of constructing graphs is a major part of the tables and graphs topic.

1.4.3   Investigations

This book uses an approach to teaching based upon the notion of Renzulli (1977) that mathematics ideas that extend and enrich students should be developed through three stages.

1. **Stage 1: Motivate the students** – pick an idea that will interest the students and will assist them to engage with mathematics.
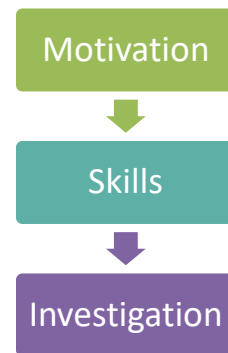
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2. **Stage 2: Provide prerequisite skills** – list and then teach all necessary mathematics ideas that need to be used to undertake the motivating idea.

3. **Stage 3: Provide integrating investigations** – end the teaching sequence by setting students an open-ended investigation to explore.

This means that the sections of each chapter will have two foci:

- **activities** that form the basis of RAMR lessons – starting from motivating situations in the students' world and then building necessary concepts and skills; and

- **investigations** that integrate the ideas and connect to other sections in a problem-solving approach.

Thus, the book combines two approaches to teaching:

- **structural/RAMR teaching of activities** that lead to the discovery and abstraction of mathematical concepts and skills (processes, strategies and procedures) starting from the world of the students; and

- **integrating investigations** that allow students an opportunity to solve problems and build their own personal solutions, and which give opportunities to combine knowledge across sections and chapters.

This chapter is the first of the three main teaching chapters. It begins teaching statistics and probability by looking at the various forms of tables, charts and graphs. The focus of this teaching is to:

(a)   organise and represent data efficiently in ways that enable outcomes such as comparisons, trends and relationships to be easily seen;

(b)   construct and read a variety of tables/charts and graphs (e.g. bar graphs, picture graphs, line graphs, circle graphs, stem and leaf graphs, and scattergrams);

(c)   recognise forms of tables, charts and graphs most effective for different outcomes (e.g. comparison, trends, relationships to whole); and

(d)   understand how different representations can be used to misrepresent data.

The chapter covers sequencing for tables and graphs (section 2.1), data gathering and organisation – tables and charts (section 2.2), data comparison – picture and bar graphs (section 2.3), frequencies – tables, calculations, histograms, two-sided and stem-leaf graphs (section 2.4), trends and relationships – lines, circles, and scattergrams (section 2.5), and misrepresentation – how to lie with statistics (section 2.6). **Appendix A** summarises and provides examples of the various different types of tables and graphs.
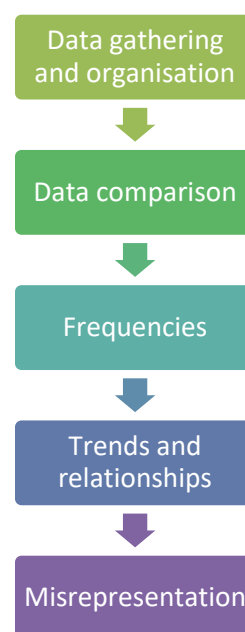
2.1   Sequencing for tables and graphs

Effective teaching of tables and graphs relies on appropriate sequencing of the content to be taught. This section looks at some recommended ways to order content so that learning is enhanced. Thus, this section provides background and overview for the chapter as it looks at sequencing of the sections in this chapter and particular sequencing for graphs.

2.1.1   Sequencing of chapter components

The chapter is based on a **series of five sections** that teach tables and graphs from Year P to Year 9. The sections are as shown in the diagram on right; they are grouped according to outcomes.

1.   **Data gathering and organisation – tables and charts**. This section covers a variety of tables (e.g. simple, regular, irregular) and charts (e.g. strip maps, branch maps).

2.   **Data comparison – picture and bar graphs**. This section covers simple comparison graphs (e.g. picture/block graphs, bar or column graphs, bar line graphs), complex comparison graphs (e.g. one-to-many picture graphs, pictograms, complex bar graphs) and charts (Venn and Carroll or two-way diagrams). Comparison is based on length (or height). The complex picture graphs involve scale.

3.   **Frequencies – tables, calculations, histograms, two-sided and stem-leaf graphs**. This section moves from gathering individual pieces of data to counting the number of times the same numerical data is present on its own or in groups. This leads to tables with many columns and tables which are used to calculate costs, and graphs which are based on frequencies. In the graphs, the length or height determines frequency, even in examples which appear to be area based.

Data gathering and organisation

↓

Data comparison

↓

Frequencies

↓

Trends and relationships

↓

Misrepresentation

4. **Trends and relationships – lines, circles, and scattergrams**. This section covers line graphs, circle graphs and scattergrams which are graphs designed to represent trends across data and relationships between some data and all the data. The trends are shown by length or height and the relationships by either length/height or angle (even in cases where they appear to be shown by area).

5. **Misrepresentation – how to lie with statistics**. This section provides an overview of how different tables and graphs show different representations and uses this understanding to explore how statistics can misrepresent data through how data is gathered and how it is presented in a graph.
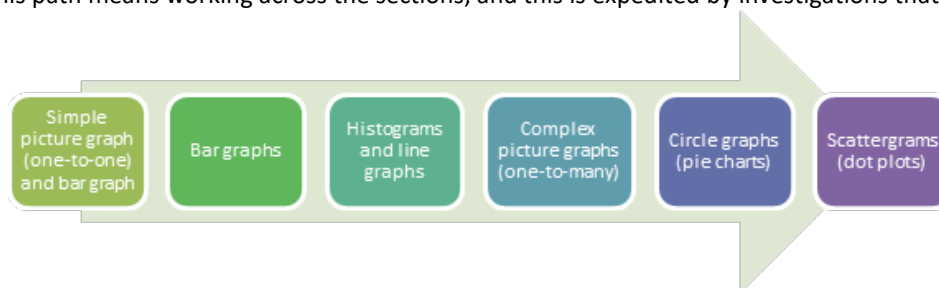
The relationship between outcomes and graphs is as follows:

- bar graphs – comparisons (e.g. which is larger or smaller?)
- line graphs – trends and comparisons
- histograms, two-sided and stem-leaf graphs – comparisons (of frequencies)
- circle graphs – comparisons and relationships (part-part and part-whole)
- scattergrams – trends and relationships.

Note: The drawing and reading of box plots is not part of this chapter. This method of graphing will be covered in Chapter 4.
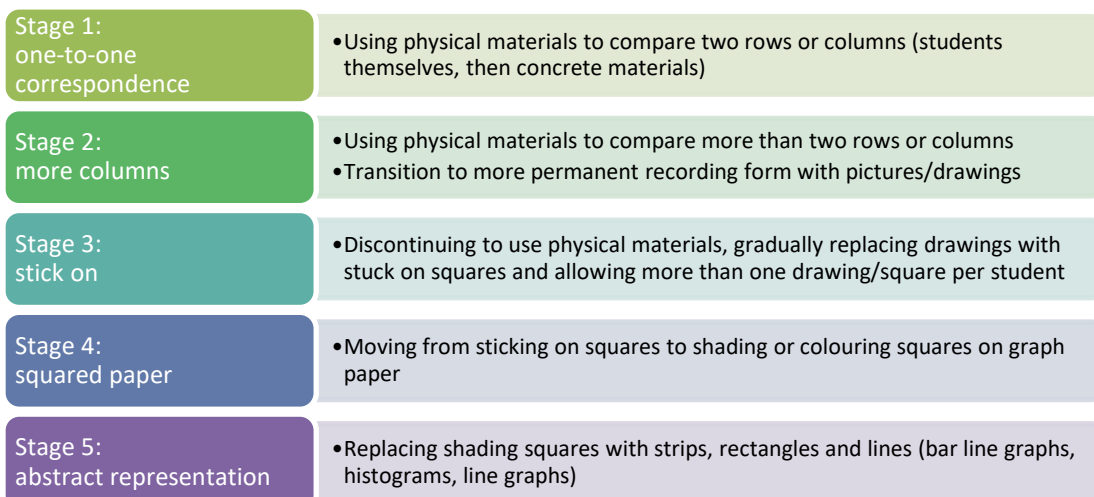
2.1.2 Sequencing of graph teaching

The various forms of graphs need to be introduced to students in a sequence whereby complexity and abstraction move from low to high as in the figure below. To connect graphs to the imperatives (i.e. comparison, frequencies, trends–relationships), this book has slightly changed this order for some graph types (notably complex picture graphs). This sequence is the basis of the overall sequencing in the diagram above. Following this path means working across the sections, and this is expedited by investigations that involve:

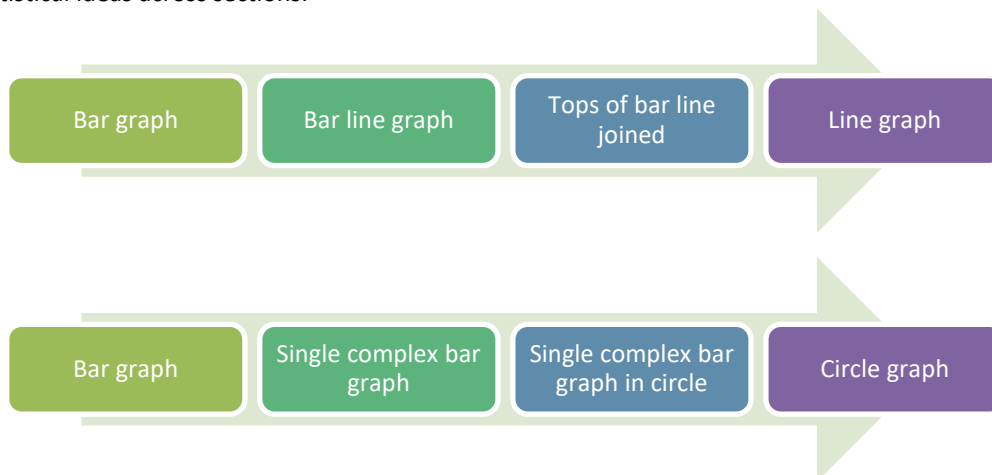| Simple picture graph (one-to-one) and bar graph | Bar graphs | Histograms and line graphs | Complex picture graphs (one-to-many) | Circle graphs (pie charts) | Scattergrams (dot plots) |

Forms of graphs and their teaching sequence

The development of understanding of the most common form of graph, the bar graph, should proceed through the stages described below. These stages act within section 2.3 that focuses on comparison and picture–bar graphs.

Stage 1: one-to-one correspondence	• Using physical materials to compare two rows or columns (students themselves, then concrete materials)
Stage 2: more columns	• Using physical materials to compare more than two rows or columns • Transition to more permanent recording form with pictures/drawings
Stage 3: stick on	• Discontinuing to use physical materials, gradually replacing drawings with stuck on squares and allowing more than one drawing/square per student
Stage 4: squared paper	• Moving from sticking on squares to shading or colouring squares on graph paper
Stage 5: abstract representation	• Replacing shading squares with strips, rectangles and lines (bar line graphs, histograms, line graphs)

Sequence for developing understanding of bar graphs (horizontal and vertical)

The development of understanding across other graphs may be facilitated with the following sequences. This is part of section 2.5 and the development of line graphs. They show the power that comes from integrating some statistical ideas across sections.

Bar graph	Bar line graph	Tops of bar line joined	Line graph

Bar graph	Single complex bar graph	Single complex bar graph in circle	Circle graph

Sequences for developing understanding of line graphs and circle graphs

2.2 Data gathering and organisation – tables and charts

This section covers the basis of the chapter – gathering data and recording it in an organised way in tables and charts. It will be the first of two sections that look at this gathering and organising. When we get to frequency distributions (section 2.4), we will come back and look again at tables.

2.2.1 Gathering data

It is very important to give students experience with a variety of data forms. In Chapter 4 on statistical inference we will look at the crucial relationship between objective or purpose and data choice. For example, when looking at whether under 25s are more dangerous drivers than over 50s, one has to take into account the number of kilometres driven each day. It may be possible that, in total, more under 25s are in accidents than over 50s but that this reverses when looking at accidents per km driven.

It is also important that students understand the pre-number ideas of sorting and classifying. Often this can be a language problem. With Indigenous students, this can be overcome by using local language such as *which mob belongs* – putting things with their right "mob" can be a useful way of introducing sorting.

Activities

A. Data sorting

Materials: Scissors, data set in **Appendix B1**

Instructions:

1. Cut out the squares. Keep the frame.

2. Group them anyway you like (be prepared to explain your grouping).

3. Count the number in each group.

4. Repeat above, gathering your own data using blanks next page – compare the two sets of data.

Note: This is rich data that can be used in the next section. So, if the students are interested, jump to the next section with this data, then return to this section for more data gathering.

B. Data selection

Instructions:

1. Consider the following contexts. Choose three types of data that you would like to find for each context.

2. Choose one context and collect data – put data in squares like in Activity A.

3. Give data to another student/group of students to sort.

CONTEXT	THREE TYPES OF DATA
Teachers	
Cars	
Football players	
Mobile phones	
Local animals	
Your own context	

Reflection

- What kinds of data are there?

- Is there data you can just see? What data do you need to measure?

- What if data is in terms of descriptions – how do we reduce it to one thing?

2.2.2 Constructing tables and charts

Tables and charts are often not explicitly taught – students are left to pick up what they are, and how they work, by chance. In this subsection, we provide some ideas for directly teaching charts and tables. However, we also provide ideas for experiences – to ensure that students experience a variety of them, from "Googled" directions to bus routes to timetables and so on. Experience can also be a good teacher.

Activities

A. Gathering data through clustering or grouping circles

Instructions:

1. Organise students into groups in circles on floor so that people can cluster around characteristics such as eye colour, gender, left or right handedness, and so on. A lot of different interpretations and comparisons can be made.

2. This data is recorded and the numbers of students in the clusters can be made into simple tables.

3. Students can be organised into columns with the same spacing and starting line so that this data leads to bar graphs. Clustering gives rich data to make into tables, graphs and comparisons.

4. This activity can be repeated for "hand" material – that is, data on cards or collections of logic blocks to be sorted by colour, size, shape, and so on – where the cards/blocks are placed into groups on a desk.

B. Constructing a table

A table is based on an array or matrix – columns and rows. Students need to become familiar with reading tables across and down. Tables require the following: (a) working out what will be in the columns – their names/titles, (b) working out what will be in the rows – their names/titles, and (c) placing or reading the information correctly in the cells.

Instructions:

Choose a context and a topic: e.g. hair colour. Make a table using the following steps.

1. **Choose the rows**. These are the focus of the table, for example, each student or the different hair colours (it depends on what you want the table to do, count hair colours or give you the hair colour of each person) – these are down the side of the table.

2. **Choose the columns**. This is where different things occur for the rows, for example, you might have a column for girls, boys, and total, or you might list the hair colours if these are not the rows – these are across the top of the table.

3. **Choose the cells**. These are what is put in the squares of the table, for example, the number of girls/ boys/total with that hair colour or a tick if the student in that row has that hair colour.

Thus a table is as follows:
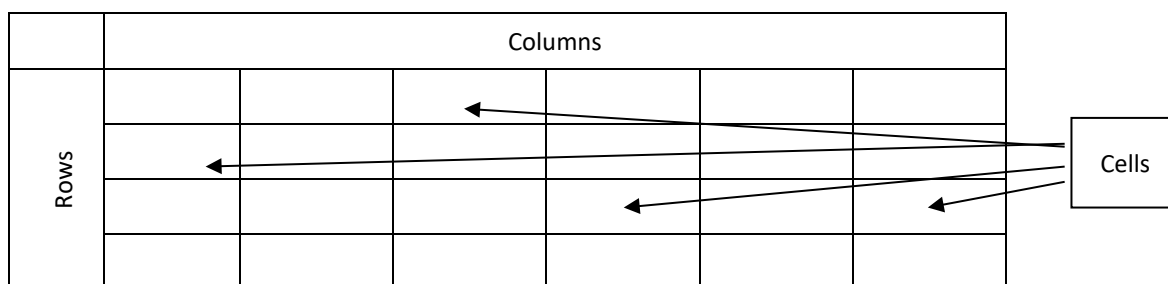
C. *Tallying and tables*

Tallying is a method of putting down one stroke | for each item and putting a cross line when moving from 4 to 5, 9 to 10 and any time we make a new 5, that is, 1 |, 2 ||, 3 |||, 4 ||||, 5 ⃥, 6 ⃥ |, 7 ⃥ || and so on. This means that 10 is ⃥ ⃥ and 17 is ⃥ ⃥ ⃥ ||.

Materials: Magazine, book or newspaper; pen; tally sheets as below; two dice; clothes pegs and line; 12 index cards with numbers 2 through 12 on them, arranged as in (3) below.

Instructions:

1. Construct tables and use them and tallying to complete the activities below.

   (a) Different hair colours in our class.

Hair colour	Blonde	Brown	Black	Red
Tally				
Total				

   (b) Different cars in the car park.

Type of car	Holden	Ford	Japanese origin	European origin
Tally				
Total				

2. Open the paper, book or magazine at any page, tally each letter on that page (or in an article on that page), and combine results into a total class tally. Questions – What are the most common and least common letters? What is the use of this information?

Letter	Tally	Total	Letter	Tally	Total	Letter	Tally	Total
A			J			S		
B			K			T		
C			L			U		
Continuing on to I			Continuing on to R			Continuing on to Z		

3. Toss the two dice 50 times, adding the two numbers for each throw. Place a clothes peg on the line hanging from the index card with the total on it for each throw. Questions – Which number has the most pegs? The least pegs? If we tossed the dice 1000 times, what would you expect to happen?

| 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
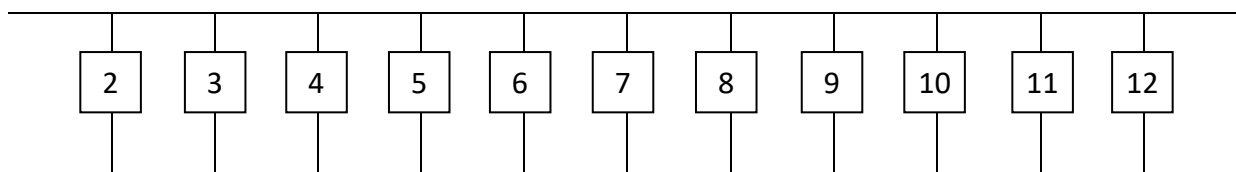
D.   Charts and tables

Materials: Packets, newspapers, magazines, manila folders, sticky tape/glue/scissors, logic blocks.

Instructions:

1. **Tables**. Collect everyday examples of tables (e.g. packets of cereal, papers, timetables, etc.). Get the students to experience tables with them. To do this – stick the best inside manila folders, place suitable titles on the outside of the folders, and make up work cards to go with the folders, which require the students to find information from the tables.

2. **Tables**. Construct tables which display: (a) the timetables for Year 1, 2 and 3 classes; and (b) the seating arrangements for your class.

3. **Strip maps**. Draw a strip map diagram of how you get from home to school – mark in major landmarks. Use a road map to draw a strip map from Brisbane to the Gold Coast (or other locations in your local area). Draw a strip map of a train line in your area.

4. **Strip map (timeline)**. Draw a timeline of what you did yesterday. Compare this with a friend.

5. **Strip map**. Use the following data to construct a strip map for Bus 2:

Stoodyville's bus timetable

	Bus 1	Bus 2	Bus 3
Monteyville	9:00	9:13	9:27
Rubber Road	9:11	9:35	9:51
Factory	9:21	9:55	
Fredsville	9:35	10:23	10:29
King Bridge		10:52	
Road's End		11:14	11:02

6. **Venn diagram**. Draw up a large Venn diagram in terms of red and large logic blocks. Place the blocks in their appropriate sections. How many blocks in each section after a complete set of logic blocks has been placed?

7. **Carroll or two-way diagram**. Draw up a large Carroll or two-way diagram in terms of triangular blocks and small blocks. Place the blocks in their appropriate sections. How many blocks in each section after a complete set of logic blocks has been placed?

Reflection

- What is data? What kind of data should we find? [Data can be ticks, words, tallies, numbers. We have to find the data that makes sense in terms of what we want the table to do.]

- Can data be used in more than one way? [Interestingly, data collected in one way can often be used in many ways in different tables.]

- What kind of data is best shown on tables? What kind of data is best shown in strip maps? Why do you sometimes see both at a bus, train or boat stop?

- Where would you use Venn or Carroll diagrams?

2.2.3 Reading tables and charts

The best way to learn to read tables and charts is to construct them – this is why this is the third subsection and not the first.

Activities

A. Reading tables

Instructions:

Consider the following table for trains – columns are stations, rows are trains, and cells are times. In **Appendix B2** Timetables, there is a description of **four ways** to read a table like that below. Read about these ways and then use them in the examples below. At the end, there is an example that involves more than just reading from the table.

Train	Start	Jackin	Karlin	Mont	Nanty	Ooptan
1	9:16 am	9:47 am	10:23 am	11:26 am	11:58 am	12:22 pm
2	9:45 am		10:43 am		12:06 pm	12:42 pm
3	10:07 am	10:39 am		12:09 pm		1:35 pm
4	10:54 am	11:28 am	12:03 pm	1:01 pm	1:42 pm	2:06 pm

1. Way 1: When does train 1 arrive at Karlin? Train 3 arrive at Ooptan? Train 4 arrive at Mont?

2. Way 2: What train arrives at Karlin 10.43 am? What train arrives at Ooptan at 1.35 pm?

3. Way 3: At what station does train 4 arrive at 11.28 am? At what station does train 1 arrive at 11.58 am?

4. Way 4: What train misses two stations and what are the stations?

5. If you miss train 1 and have to wait for the next train, how late will you be at Nanty? What about Mont?

6. Can you make up more examples like (5) above?

2.2.4 Reflection and investigation

Suggested reflections are as follows:

- What things can make tables hard to read? [Strange shapes, lack of titles and clear names for rows and columns, huge amounts of data.]

- What do you have to be good at to read a table? [Understanding vertical/horizontal and rows/columns, and being able to move eyes along rows/columns without jumping a row or column.]

- What should you look at before you start to find answers on a table? [Names of rows and columns, type of data in the cells, and missing cells.]

Suggested investigations are as follows:

- Take the data from Activity A in subsection 2.2.1 – the squares of data. Create as many forms of charts and tables with this data as you can. Make a poster called "The Power of Data" and put all your ideas in it in a creative and attractive way.

- What graphs could you make from these different charts and tables? Add these to your poster.

- Are there some charts and tables which cannot be made into graphs?

2.3 Data comparison – picture and bar graphs

This section covers another of the bases of this chapter: drawing and reading picture and bar graphs, which are designed to show comparison. Line graphs and circle graphs can be developed as an extension of bar graphs, so picture and bar graphs are an important part of the development of tables and graphs.

In section 2.2, we showed that charts and tables could be built around words and symbols like ticks as well as numbers. However, because of their nature, picture and bar graphs have to represent numbers because their diagrammatic representation as a graph shows difference as a comparison between lengths. Thus for comparison, we are restricted to data that are numbers and attributes of measures (e.g. 24 m or $31).

2.3.1 Introducing simple picture and bar graphs

As shown in the sequencing subsection 2.1.2, bar graphs are developed from picture graphs which are in turn developed in a sequence from simple two-way graphs. The sequence is as follows:

Stage 1: One-to-one correspondence – two columns – students, physical objects or pictures

Stage 2: More columns – students, physical objects or pictures

Stage 3: Stick on – using paper squares instead of students, physical objects and pictures

Stage 4: Squared paper stage – shading squares on graph paper

Stage 5: Abstract representation stage – drawings of rectangles on axes

Stage 6: Using lines for bars – replacing rectangles with a line

Activities

A.   People graphs

Instructions:

1. Use the Maths Mat to act out with students' bodies, bar graphs, one-to-one correspondence, and labels (e.g. more, less). A good example is to label columns on the mat with numbers 0, 1, 2 and so on. People line up, for example, based on the number of siblings in family when growing up.

2. Students place an object in the square they are in (e.g. recycled envelopes with names written on). Participants can step back to view the graph on the mat, but they still can identify their own position and view as a whole to bring out the language (e.g. bars, one-to-one correspondence).

3. Students then go back to graph paper and represent what is on the mat by colouring squares. Names are no longer the central focus – the focus is on the number in each column as the bar graph is completed.

4. The bar graph can be condensed to form a table. Note that this will require new labelling.

B.   Teaching principles of bar graphs

For bar graphs to be legitimate and provide easy observational comparison, they must have the following:

- **Appropriate labelling and scaling** – (a) two axes, (b) title for the graph, (c) titles for each axis, and (d) scales for each axis. The scales can be categorical (e.g. list of pets) for the horizontal axis but are normally interval (i.e. numbers) and discrete (only the counting numbers, 1, 2, 3, and so on, no decimal numbers).

- **Correct positioning of the pictures or squares** – (a) the same size pictures and squares, (b) the same spacing for the pictures and squares, (c) one-to-one correspondence, and (d) pictures and squares starting at the same point (0).

These principles of simple picture and bar graphs can (and should) be taught by activities such as the following.

Instructions:

1. Use the Maths Mat to act the following out with students' bodies: (a) same spacing, (b) one-to-one correspondence, (c) labelling, and (d) starting from the same point (0).

2. Use the Maths Mat to act out simple bar graphs.

3. Use the method called "torpedoing" (developed in the 60s and 70s by Bruner) to teach the graph principles. In this method, the teacher deliberately makes errors so that the students can argue that the teacher is wrong, and give the correct answers. **Appendix B3** has an example of a torpedoing activity to teach same size, equal spacing, one-to-one correspondence, and starting from zero.

4. Get to know how to use computers to draw all the types of bar graphs.

Reflection

- What makes an effective bar graph?
- What are the principles of a good bar graph?
- What are easy ways to mislead with a picture or bar graph?

2.3.2 More complex graphs

The sequence in section 2.1.2 takes bar graphs from materials via pictures to abstract rectangles. We now look at these abstract graphs.

Activities

A. *Moving pictorial to abstract*

Instructions:

1. **Appendix B4** shows the sequence from pictures to abstract bar graph. Read this sequence.

2. What questioning would you use at each of the steps in this sequencing – e.g. how do you move from pictures to squares, and squares to shading and shading to rectangles? What stays the same and what changes?

3. Gather some data from a group of people/students – have more than two columns (e.g. hair colour). Draw/construct four graphs showing the transition from pictures to abstraction as in Appendix B4.

B. *Recognising and fixing errors in bar graph presentation from pictures to abstract*

Once we get to abstract bar graphs, there is another principle to recognise, that the width of the graphs should be common, and that bars should not touch unless they represent touching intervals.

Instructions:

1. Look at the pictures of bar graphs in **Appendix B5**. What is wrong with the graphs?

2. Redraw the graphs in Appendix B5 so that they are more appropriate/correct.

3. If they are not already, turn each graph into its abstract form.

Reflection

- What is the sequence from start to abstraction for bar graphs?

- Are sequences like this useful for Year 8 students with little mathematics knowledge? Or can you go directly to the abstract form?

2.3.3 Reading comparison graphs

Graphs like picture and bar graphs are important because they visually show what/who has the most or least, and whether differences are large or small. Interestingly, although bar graphs are constructed by drawing rectangles of different heights, most people see the difference between heights multiplicatively. For example, it is easier to see that two bars are such that one is about double the other than to read off the numbers and see that one is 23 more than the other.

Activities

A. Construct and read

Instructions:

1. Organise the students to gather data that could be made into a bar graph from other students or from Internet sources. Place this data into a table.

2. Convert the table data into a bar graph. Then answer questions with regard to the graph. The students' experience in making the graph should assist them in answering questions.

3. **Appendix B6** contains a bar graph (it is the first graph) and questions. Get the students to gather data on rainfall for 2013 in their local area as for this graph. Ask the same questions. (*Note:* The percentage question should only be done if the students understand percent.)

4. Appendix B6 also has a simple picture graph about rainy days. Answer the questions for this graph. Look for other situations where a picture graph may be better than a bar graph.

5. Ask the students if there is anything about where they live that they would like to find out. Organise them to gather data and construct bar graphs. Get the students to answer questions about the graph.

B. Reading

Instructions:

1. Obtain examples of bar graphs from magazines, newspapers, promotion material, advertisements, Australian Bureau of Statistics, Bureau of Meteorology, and so on. Get the students to help find such graphs on the Internet.

2. Set questions for these graphs – help the students to be able to find information from the graphs and the corresponding tables.

3. Get the students to make up their own questions for other students to answer – question posing is an excellent way to teach question answering.

Reflection

- Are bar graphs relevant to students? How can we make them relevant?

- Students such as those in YDM schools have been engaged by gathering and graphing data that concerns their history and their present situation. What data interests your students? Data with regard to family? Data with regard to major historical events in community?

- Students could look at data that emerges from the country's colonial past – even that to do with injustice and invasion (e.g. Uncle Ernie Grant has found much statistical data from studying punitive expeditions against Indigenous people). They could also look at the present situation (e.g. average wages and unemployment rates as compared with non-Indigenous people). For example, the statistics associated with local community councils may be interesting. These examples can be very successful with regard to literacy and numeracy but have stirred up violence in some cases. Are they worth the risk?

- Could we look at controversial things in our schools? What safeguards would we need (e.g. permission and support of Elders)? Chris Sarra had students looking at substance abuse, violence in the home and life expectancy when at Cherbourg. Is this something only he could do?

- What about involving the students in their pre-post testing? Each student could graph their own data.

2.3.4   Investigation

Organise the students to undertake a study of their lives that sets up bar graphs. Make a poster on this. Ideas could be:

- A poster on Indigenous people in Australia – the rates of Indigenous people playing NRL and AFL, the rates of Indigenous people in various jobs, looking at a variety of jobs or a few particular vocations.

- A poster on mathematics improvement – give weekly tests on basic facts and so on and students graph each week their results (teachers in past YDC projects have done this very successfully).

- A poster on the local council – expenditure on various things across the last few years.

- A poster on native animals in the area – how many of each? – or of trees and grasses.

- A "care for country" poster looking at land degradation in the local area, or land usage.

2.4   Frequencies – tables, calculations, histograms, two-sided and stem-leaf graphs

This section moves on to more complicated/advanced tables and graphs – looking at how tables are used for solving problems and calculations (e.g. timetables and budgets), and looking at histograms and stem and leaf graphs. The focus is still comparison but the tables and graphs are more complicated.

There are three complications.

1. By their nature, picture and bar graphs are frequencies – they show how many students have blue eyes, or the mm of rainfall on Friday. However, things become more complicated when the x-axis is not in terms of category data (e.g. pets, or names of students) or ordinal data (e.g. months of the year) but rather in terms of interval data like height. It does not make sense to have the number of people of height 155 cm, because one will be 155.1 cm and another 154.6 cm. The answer lies in finding the number of people whose heights are between 154.5 and 155.5 cm. If this is done, then the rectangles touch each other and all the area is covered. These are called **histograms**.

2. The second complication is to look at bar graphs that go negative. An example of this is on election nights when looking at swings in terms of percentage votes from one party to the other. For the political parties, the rectangles/bars for some parties will be above the line showing a *swing to* while others will be below the line showing a *swing away*. Such graphs are called **two-sided complex bar or column graphs**. They can also be continuous like histograms.

3. The third complication is to use the graph to show the distribution of the data alongside the comparison. For example, data from heights could be placed on a vertical line, marked in 10 cm intervals, by writing the actual height for each student in each 10 cm interval, in a list small to large beside the interval. One can compare distributions by having different data on each side (e.g. girls vs boys). Such graphs are called **stem and leaf graphs**.

2.4.1 Constructing and using frequency tables/distributions

Here we look at timetables and budgets. Timetables were started in section 2.2 – we now look again at them and the ideas that are in **Appendix B2.**

Activities

A.   Table activities

Instructions:

1. Look again at **Appendix B2**. Examine the bus timetable below

Major stops	AM			PM		
Goodna Station	-	9.40	11.40	1.45	3.39	5.39
Redbank Station	-	9.46	11.46	1.51	3.45	5.45
Redbank Plaza	7.50	9.51	11.51	1.56	3.50	5.50
Riverview Primary Sch	7.55	9.56	11.56	2.01	3.55	5.55
Riverview Gardens	8.00	10.01	12.01	2.06	4.00	6.00
Dinmore Station	8.05	10.06	12.06	2.11	4.05	6.05
Bundamba Primary Sch	8.09	10.10	12.10	2.15	4.09	6.09
Booval Fair	8.12	10.13	12.13	2.18	4.12	6.12
Ipswich (Bell St)	8.19	10.20	12.20	2.25	4.19	6.19

2. Answer these questions:

    (a) What time would you need to catch the bus from Redbank Plaza in order to get to Bundamba Primary before school starts?

    (b) If the bus leaves Dinmore Station at 2.11 pm, what time would it arrive in Ipswich? If I was travelling from Redbank to Ipswich to arrive at the same time, what time would I have to leave?

3. Obtain actual train and bus timetables and set questions to be answered.

B.   Calculations and budgets

Instructions:

1. Look at **Appendix B7.** Examine how to set up the table to budget for the 4-day trip. Repeat this for a woman doing the trip. Make up your own 4-day trip costs.

2. Develop a list of items for the following situations:

    (a) things other than clothing that you need to take on a holiday; and

    (b) things that you might need to put in a new car to make it more livable/drivable.

3. Translate the two lists from (2) into budget tables as follows:

    (a) a table that determines the cost for these other items for a holiday of two weeks; and

    (b) a table that determines the cost of the extras across 6 months.

    Use the Internet to find values of items and work out the budget for each of the two situations.

Reflection

- What is important in setting up and interpreting timetables?

- What is important in working out a budget?

- How can you be sure that you have exhausted all possibilities for the budget?

2.4.2 Constructing and using complex picture graphs and histograms

Activities

A.   Picture graphs and scales

Instructions:

1. **Appendix B6** has a complex picture graph. It is the second graph under activity (4). Answer the questions. What is the scale in this exercise?

2. Answer, in particular, part (c) of this activity (4) in Appendix B6. The thing to note is that these scale picture graphs are useful when numbers are large.

3. What are the difficulties, the mistakes that might be made, with scale picture graphs?

4. Find examples of complex picture graphs with scales. Are they effective? Are they easy to follow? What are their scales?

B.   Histograms

Instructions:

1. **Appendix B8** describes how a histogram is developed. Read this appendix. Answer the questions.

2. Gather your own data on heights and repeat the process.

3. If class weights were between 44 kg and 91 kg, what intervals would you have for the weights if you only wanted around six intervals?

    [All weights are in range found by 91−44 = 47; 6 intervals means that interval length is 47÷6 = nearly 8, so length of 8; have to choose interval length of either 7 or 9 if need odd number for centralising which is 7 intervals, as 7×7 = 49>47 and 7×6 = 42<47, or 6 intervals, as 9×5 = 45<47 and 9×6 = 54>47. Of course, if it can tolerate even length, interval length of 8 will do.]

4. Gather data for continuous things like height or weight and develop other histograms. Follow the steps in Appendix B8.

5. Search Internet and magazines, etc., for examples of histograms and answer questions about them.

6. Use computers to construct histograms.

Reflection

- What is the problem of scale in complex picture graphs?

- As scale is a problem, why use scaled picture graphs? Are there situations where they are effective?

- Why do we need histograms when we have bar graphs? Do we need this special name? Are they just bar graphs with bars touching?

- Do we need odd length intervals? And if we do, when do we need them? Do we also need the zeros at each end (other than to make the polygon)?

2.4.3 Constructing and using two-sided complex bar graphs

These graphs are like bar–column graphs that go in either direction – positive or negative or comparing two sets of data.

Activities

A. Using people

Instructions:

1. Gather data on eye colour. Record the number of students for each colour separately for girls and boys. (*Note*: data could also be gathered around: *preferences* – favourite pet, favourite colour, politics; *places* – suburb where they live, where they were born; *what they have or own* – type of computer, type of phone, number of siblings; and so on.) It is normal for this graph that the data is not continuous. This data would be similar to that for a simple bar or column graph.

2. Using a strip of masking tape for the axis (usually vertical), mark sections by eye colour. Students stand to the side of the sections going out from the stem – girls on right and boys on left. This is a two-sided bar graph.

3. Get students to place a label where they are standing, step back and view what they have made, and then make a pen-and-paper copy of the graph. Discuss the language.
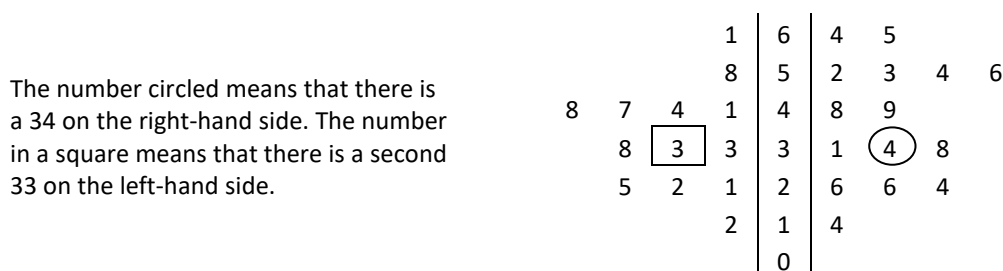
B. Constructing two-sided graphs

Instructions:

1. Choose a topic by which things can be characterised using positive and negative numbers and graph this. Normally this would be a horizontal axis with columns going off from either side. It should be noted that these graphs tend to have x-axis as discontinuous and the data tends to be categorical.

    (a) One way to do this is to pick something where there are positive and negative numbers and graph students' scores or graph class averages. One possible example of this is NAPLAN scores above and below the state average.

    (b) A second way to do this is to choose something for data gathering where there is comparison with an average. For example, choose something for which there is census or other Australia-wide data, gather data from class, compare class averages with Australian averages, and graph positive and negative differences.

    (c) A third way to do this is to compare data from one time to another. For example, the values of shares could be compared from start to end of the year, as could house prices (with increases showing as positives and decreases as negatives). Also, you could have a series of devices to randomly generate numbers. These could be trialled twice and the first trials compared with the second. Another good example is pre-post data.

2. Choose a topic where you are comparing two sets of data on the same things (e.g. test results for boys and girls). These would usually be a vertical axis with horizontal bars for comparing two groups and a horizontal axis with vertical bars for positive and negative data. For example, the data for selecting cards (1 to 6), throwing a die, and spinning a 1-6 spinner could be as follows for 5 girls and 8 boys (the first row of data is the first trial and the second row of data is the second trial):

Cards		Die		Spinner	
Girls	Boys	Girls	Boys	Girls	Boys
1, 5, 4, 3, 2	5, 3, 5, 2, 1, 3, 4, 1	1, 5, 4, 5, 1	6, 4, 2, 5, 1, 1, 3, 2	3, 6, 3, 1, 1	4, 3, 4, 1, 5, 4, 2, 4
4, 6, 3, 4, 2	6, 2, 6, 3, 1, 4, 6, 2	5, 3, 6, 4, 4	5, 4, 3, 6, 1, 5, 4, 4	3, 1, 5, 6, 4	3, 2, 2, 6, 2, 2, 1, 2

(a) Average all students' first and second trials and draw a two-sided graph with horizontal axis and vertical bars of differences in the three averages (second – first). Repeat for six differences with boys and girls separately.

(b) Average boys' and girls' results separately for first trials and second trials, and draw a two-sided graph with vertical axis and horizontal bars with girls on right and boys on left for all six trials.

3. One common use of these graphs is the graphs shown in election-result coverage looking at voting patterns which show swings (positive and negative) for and against parties. Look these up and discuss them.

4. Search the Internet and other sources of information for two-sided graphs. Describe how they are made. Pose questions about them.

Reflection

- Where are two-sided graphs useful?

- How are they different to bar graphs? What is the use of this difference?

2.4.4   Constructing and using stem and leaf graphs

These are similar to the two-sided graphs above but show distribution within an interval. The data tends to be continuous.

Activities

A.   Using people

Instructions:

1. Choose a topic for which people have a measure. This could be age, street number of house, height, mass, and so on. For this example, we will choose height.

2. Measure the height of all students.

3. Using a strip of masking tape for the "stem" (normally vertical), mark sections as 0–9, 10–19, 20–29 and so on. Students stand to the side of the section in which their height belongs, going out from the stem – in order with lowest beside the stem. This is a one-sided, made-with-bodies stem and leaf graph. To make it two-sided, put females on one side and males on the other.

4. Get students to place a label where they are standing, step back and view what they have made, and then make a pen-and-paper copy of the stem and leaf graph. Discuss the language.

B.   Constructing two-sided graphs

Instructions:

The following is data from students' masses:

Girls: 37, 35, 56, 38, 41, 46, 62, 51, 43, 46, 31, 37, 60

Boys: 48, 51, 83, 76, 63, 64, 49, 58, 63, 72, 65, 62, 58, 70, 49

1. Use the data to draw a one-sided stem and leaf graph with all data together.

2. Use the data of boys and girls separately to draw a two-sided stem and leaf graph with boys on right and girls on left.

3. Which graph is best for what? Why?

   Note. You can use a very abstract form of stem and leaf graph where the stem just has the tens and the leaves have the ones. For example:

   The number circled means that there is a 34 on the right-hand side. The number in a square means that there is a second 33 on the left-hand side.

```
                  1 | 6  4  5
                  8 | 5 | 2  3  4  6
       8  7  4  1 | 4 | 8  9
       8 [3] 3  3 | 3 | 1 (4) 8
       5  2  1  2 | 6  6  4
               2 | 1 | 4
                  0
```

4. Search the Internet and other sources of information to find stem and leaf graphs.

5. Describe how they are made. Pose questions about them.

Reflection

- Where are stem and leaf graphs useful?
- How are they different to bar graphs? Is this difference useful?

2.4.5   Investigation

There are multiple opportunities here; some suggestions are as follows.

1. **Get students to interpret timetables**. Obtain a collection of timetables for buses, trains (from bus/train operators, or off the Internet) or something different (e.g. planes) and see if students can read them. Set questions from the timetables for students to answer. For example, questions could go from simple (e.g. *What time does train 3 get to whatever?*) to complex (e.g. *How long for train 3 is it from whatever to somewhere else? Is this the same for all trains? Why would these times be different?*).

2. **Set a rich timetable task**. Download the free Prevocational Maths materials from the YDC website (Vocational Learning resources). Two that come to mind are: *11.6 The Man from Hungary*, and *12.6 Rocking Around the World.*

3. **Set up budget situations**. Provide students with a variety of information for a variety of situations (e.g. different costs for mobile phones) and work with them to develop a table. Discuss how tables can be different (e.g. they can be square or rectangle; they can have missing sections), and perform different tasks (e.g. to compare different options, to describe different options, to ensure all options are covered, to calculate the costs involved).

4. **Set a budget rich task**. Once again the Prevocational Maths material from the YDC website has some good ideas. Two are: *11.2 The Big Day Out* and *11.4 Exchange Student*.

5. Finally, **planning a party** and **developing a family budget** are really worthwhile tasks – these are described in the free financial mathematics activities that can be found on the YDC website (Student Learning resources).

2.5 Trends and relationships – lines, circles and scattergrams

This section extends the focus of comparison to using comparisons to see trends and relationships – between parts and between a part and the whole. Trends are best seen in line graphs and relationships in circle graphs and scattergrams (or scatter plots), though looking at more than one line graph on the same axes also leads to relationships.

2.5.1 Constructing and using line graphs

Activities

A. Tables to bar to line graphs

Instructions:

1. **Appendix B9** describes how a bar graph can become a line graph. Read this appendix. Copy the bar graph and make the line graph.

2. Obtain some data and turn it into bar graphs, and then turn them into line graphs the same way as in Appendix B9.

3. Discuss how this process of finding the x-axis positions and going up the heights of the bar gives (x,y) positions that can lead to line graphs – that a table gives a series of two values that are the point in the line graph.

4. Do the activities (1) and (2) in **Appendix B10**. This shows how to shortcut from table to line graph.

B. Creating and using line graphs

Instructions:

1. Obtain data that needs to show trends (e.g. rainfall, payments) and construct line graphs.

2. Look up line graphs and determine questions for students to answer to show they can read them.

3. An example is in **Appendix B6**, second graph on the first page – answer the questions with the graph in the appendix.

4. Use computers to construct line graphs.

C. Relating line graphs to stories

Instructions:

1. Consider a line graph of the height of water in a bath; the water is turned on, and the height of the water increases; someone gets in, gets out and pulls the plug – what happens to the height of the water? What does a line graph look like of height against time?

2. Make up stories about the bath and then construct the graph. Take graphs and turn them into bath stories.

3. Try other stories, e.g. a car leaves home, meets someone, stops, then drives on – what does a graph of its distance from home against time look like?

These graphs can be complicated, with many lines, but there is always a story.

D. Relating line graphs to other line graphs

Instructions:

1. If there is more than one line graph, the two graphs can be compared. Find examples of this and compare – get two sets of data and put on same graph and compare.

2. Comparing across graphs adds an extra dimension; graphs of average monthly maximums and minimums can be made for a year for all cities – can we tell the city from the graph?

Reflection

- What is the difference between trends and comparisons?

- Why are lines important in trends?

- How many different ways can we see relationships in lines – within the lines, across lines and across graphs?

2.5.2 Constructing and using circle graphs

Activities

A. From bars to compound bars to circle graphs

Materials: Pictures of eyes to colour in, 2 cm graph paper, pole and streamers, sticky tape, scissors, paper circles, 100 beads – 10 blue, 10 red and so on.

Instructions:

Make up a simple set of data for a picture or bar graph. For example, get 20+ students to group themselves around eye colour.

1. Give each student a picture of an eye to colour the same as their eyes; get them to line up as a bar graph on the mat, place eyes where they are standing, then move off the mat to see the graph.

2. Colour in 2 cm graph paper squares with pencils to make a copy of the bar graph.

3. Re-form the human bar graph and then get bars to join end-on-end to make a single complex bar graph – place down coloured pictures of eyes and step back to see the whole complex bar.

4. Cut strips of 2 cm squares and join and colour in to make a copy of the complex bar.

5. Re-form the human complex bar, and move into an equally spaced circle around pole – people on ends of bars for each colour take a streamer and pull out to make a "maypole" – place pictures and streamers on ground and step back to see the circle graph.

6. Put paper copy of complex bar as a circle around the circle paper and draw lines to centre from end of bars of one colour to make a copy of circle graph – work out the fraction for each sector.

7. Put 100 beads around the circle graph and calculate percent of each sector. (*Note:* It is possible to show how to go from fraction to percent by this method.)

This method of data → bar graph → compound bar or strip → circle graph → percent is an excellent activity. It involves going from activity with body to paper work and back again.

B. Reading and constructing circle graphs

Instructions:

1. Complete the circle graph activity from **Appendix B6** activity (3) on the second page of the appendix.

2. Look for circle graphs on the Internet and in magazines, etc., and set questions to see if students can read them.

3. Complete the circle graph activities from **Appendix B10** – activities (3) to (5). Spend time turning percent into angle out of 360 and vice versa.

4. Gain data by setting up an activity to gather data from people, make a table and tally, draw a bar graph and then construct own circle graphs. (*Note:* Circle graphs are hard to construct unless using a computer.)

5. Do many examples of circle graphs on the computer.

Reflection

- Circle graphs use similar data to bar graphs; what is the difference in the relationships they show?

- Is it still important to do circle graphs by hand with protractors, or should we just use the computer?

2.5.3 Constructing and using scattergrams

Activities

A. Reading and constructing scattergrams

Instructions:

1. Complete the scattergram activities from **Appendix B10** – activities (6) to (10). Answer the questions.

2. Complete the scattergram activities from **Appendix B6** – activity (5), the last graph on the third page. Answer the questions.

3. Try to find scattergrams on the Internet and from reports and so on. Try to argue for relationships.

4. Gather data for scattergrams – look at things that may relate (e.g. hand span and height, arm span and foot length, and so on) and may inversely relate (e.g. hours watching TV and hours doing homework). Do not miss out on some data that does not relate.

5. Discuss outliers and special cases and their effect on relationships.

Reflection

- How close together do the dots have to be to show a relationship?

- Relate scattergram relationships to estimates of line of best fit.

2.5.4 Comparing graph types

The crucial skill to be gained by students from this chapter is to understand what types of representations (tables and graphs) most suit which data and which purpose.

Activities

A. Comparing different graphs

Instructions:

1. Take one set of data and show how it can be represented by different graphs.

2. Discuss which representation is most effective.

B. Relating graphs to data

Instructions:

1. **Appendix B11** has five sets of data. Look at this data.

2. Complete the tasks at the top of the data set.

3. Determine which graph type is best/better for each data set (if possible).

C.   Features of graphs – what are they good for?

Instructions:

1.   Copy the table from the last page of **Appendix B14**.

2.   Complete the table for each type of graph listed.

Reflection

- Can all graphs apply to all data?

- Is purpose more important than data type in determining what graph to use?

- If the above is the case, does this make graphs circular – the purpose selects them because they show the purpose in the best light?

2.5.5   Investigation

There are two suggested investigations:

1.   The first is a group activity involving cooperative problem solving. Groups of four receive four cards which have different representations of the same data – stem and leaf, box and whisker, raw unsorted data and bar graph. What do we get from each? (Mostly for discussion and language, to see what can be figured out.)

2.   The second is individual (though it could be group) and requires each student to develop a graph from a question or position. The task is described in **Appendix B12**.

2.6   Misrepresentation – how to lie with statistics

This section covers correct representation and misrepresentation. Ever since the statement "lies, damned lies and statistics", there has been an awareness that statistics can be manipulated to misrepresent. This is also true of graphs as we will now show.

2.6.1   Misrepresentation in gathering data

Activities

A.   Methods of data misrepresentation

The gathering of data can lead to misrepresentation in three ways:

(a)   the way questions are asked (leading questions);

(b)   the person who asks the questions (interviewer bias); and

(c)   who is chosen to answer the questions (sample bias).

Instructions:

1.   Look up these ideas and see what they mean. They can be powerful – a group once sent out an attractive blonde and an attractive brunette separately to ask 20 young men if "gentlemen preferred blondes" – the results were diametrically opposite between both interviewers.

2.   We will look at this more in section 4.6. However, if you wish to read about it in more detail, read Darrell Huff's well-known book, *How to lie with statistics*.

2.6.2 Misrepresentation in the construction of graphs

Activities

A.   Methods of graph misrepresentation

Instructions:

1.   Read **Appendix B13** (first page and top of the second page on line graphs). Complete activity 4.

2.   Read activity 5. See how pictures show a stronger presentation than bars.

3.   Bar graphs can also be made to misrepresent in the same way as line graphs – truncation, stretching vertically and narrowing the graph make differences look bigger, while not truncating, squashing down and widening make differences less. Try this out for a bar graph.

B.   Fixing up misrepresentations

Instructions:

1.   Read **Appendix B14**.

2.   Complete activities 1 to 7.

3.   List the inaccuracies; and redraw the graphs correctly.

Reflection

- Have you seen this type of thing in graphs? Search advertisements and reports (and the Internet) for examples.

- How much of misrepresentation is use of visuals and how much is manipulation of data?

2.6.3 Investigation

Below are three possibilities for further investigation:

1.   Look at something from the news. For example, it is commonly stated that China's economy is strong. Look this up on the Internet. Are there graphs that support this? Are there other graphs that show other possibilities? Look at the graphs and discuss what they show. Discuss also how the graphs could misrepresent the real situation.

2.   Look on the Internet for graphs that show change and how to prevent this change. For example, graphs of the concentration of members of audiences within a lecture show that concentration declines in the middle of a lecture. There are also graphs that show that an activity in the middle of the lecture causes concentration to rise. Find such graphs. What are they telling us? That taking breaks increases the amount learnt?

3.   Pick a statement like "cars are safer now than years ago" or "there is more crime now than 50 years ago". Gather data on these statements and present two posters that do their best to purport opposite views. Use your "how to lie" knowledge to make them both convincing!

3   *Probability*

In this chapter we look at how to teach probability. This requires some special approaches as probability deals with chance and uncertainty not with the exactness and certainty of arithmetic.

This chapter consists of the following sections: sequencing for probability (3.1), probabilistic situations (3.2 – Stage 1), outcomes from probability events (3.3 – Stage 2), desired event (3.4 – Stage 3), probability as a fraction (3.5 – Stage 4), experimental probability (3.6 – Stage 4), and inference from probability (3.7 – Stage 5). Each section will define the stages/skills that it covers, discuss approaches to teaching, including RAMR, and provide a variety of activities that build the ideas in the section.

Note: In probability, it is important to ask many questions because the calculation answer is just a pointer to the probability answer.

3.1   Sequencing for probability

In this section, we look at the sequence of stages and skills for probability, sequencing within teaching, and application to inference.

3.1.1   Stages and skills

The sequence suggested by YDM is to start with classification and to end with applications – this sequence moves through five stages and eight skills as below.

Perceive and identify attribute	•Skill 1: Students can correctly classify an event as impossible, possible or certain.
Compare and order likelihood of outcomes	•Skill 2: Students can list all possible outcomes of a chance process. •Skill 3: Students can list all possible outcomes in consecutive trials of a chance process, with and without replacement. •Skill 4: Students can state which outcome is most likely in a single trial of a chance process.
Compare and order situations for desired outcome	•Skill 5: Students can correctly choose in what situation an event is more (or most) likely to occur.
Measure probabilities	•Skill 6: Students can correctly assign a numerical probability to an event (and use this number for skills 4 and 5). •Skill 7: Students can estimate the numerical probability of an event using "simulation" methods.
Apply probability in inferences	•Skill 8: Students can use probability skills and understanding to make informed decisions and predictions.

3.1.2 Teaching sequences

The teaching sequence advocated is to follow the stages/skills above and to ensure that you:

- use area and set models;
- look at fair and unfair situations;
- vary the position of the outcomes (contiguous/noncontiguous);
- vary the number of outcomes; and
- include replacement and no replacement.

While doing this, have students' activities move from one step/one sample space problems to more than one step/more than one sample space problems. In each of these situations, it is important to do the following in order:

- list the outcomes;
- discuss the likelihood of an event occurring (certain, possible, impossible or yes/maybe/no); and
- compare two events to determine which is more/less likely to occur.

While moving into more than one step situations, it is likewise important to sequence activities as follows:

- two similar sample spaces (one sample space repeated);
- sample spaces with the same number of outcomes in each space (e.g. two spinners); and
- sample spaces with different numbers of outcomes in each sample space.

It is also important to spend time on events that have consecutive trials and to sequence them so that the events come from:

- one sample space;
- two or more similar sample spaces; and
- two or more different sample spaces.

It is also effective to ensure that each stage includes a variety of types of activities such as experiments, simulation trials, and games. However, when undertaking these activities, it is necessary to cover:

- symbolic representation;
- relationship to fraction; and
- comparison and order.

3.1.3 Inference

Finally, since probability is so much about life, it is important to ensure that probability enables students to make good decisions about their own lives. This means that teaching how to infer what probability says about life situations, the applications of probability, is an important outcome of probability teaching. As we shall see in the sections following, probability can enable people to have some control over daily activity and not have to leave the future to fate or luck.

3.2   Probabilistic situations

Skill 1: The student can correctly classify an event as impossible, possible or certain.

Big idea: Probabilistic vs deterministic.

This first teaching section deals with the first skill, that of classifying events as impossible, possible or certain; that is, recognising when situations involve chance. It is also, obviously, directly focusing on the big idea probabilistic vs deterministic because it is being able to recognise between the two. It is the focus of the early years of the Australian Mathematics Curriculum.

3.2.1   Defining Skill 1

Materials: Two dice, a bag, three counters (two red and one blue), pen and paper.

Directions: Answer these questions:

1. In a game in which two dice are rolled, is it impossible, possible, certain that I get a total which is:

    (a) 12?   (b) 1?   (c) less than 15?

2. A bag contains three counters, two red and one blue. I put my hand in and pick out two counters. Is it impossible, possible, certain that one of the counters picked is:

    (a) Blue?   (b) Red?

3. Set up both experiments. Try the activities. Do you get what you expected?
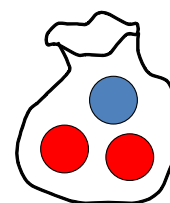
3.2.2   Teaching ideas

The following points are worth making:

1. This is an opportunity to start from the local context and culture of the students. Get students to discuss when events are uncertain and have possible outcomes. Most students have many experiences with chance, e.g. card games, sport, weather. Find out the important ones in your community.

2. Give the students experience of chance or probabilistic situations – here are some examples:

    (a) **Blindperson's Pick**. Blindfold the students and place a "present" in front of them for them to select. Have different coloured and patterned wrapping.

    (b) **Lucky Dip**. Students select counters, balls, blocks from a bag or box, or draw a card from a face down pack.

    (c) **Come in Spinner**. Students spin spinners with uneven sectors or with numbers repeated (or throw dice with number repeated).

    (d) **Dartboard**. Students throw darts (or blindly stab) at a chequered pattern board.

    (e) **Fishing**. Students fish with a magnet on a line into a box of cardboard fish (with paper clips on their noses) of different sizes and colours.

    (f) **Guessing game**. Students place fewer than 10 objects in a box for other students to guess the amount.

    (g) **Dropsy**. Students drop blocks over their shoulder into a square pattern of numbers.

3. As well as identifying chance situations, get the students to construct chance situations – this is the **interpretation-construction** big idea. Reverse everything as well – here is the situation, what is the uncertainty, the area of probability AND here is the area of probability, make up a situation.

4. Discuss characteristics of chance events – look at the focus, e.g. weather, sport, games; discuss whether chance involves activity or a question; consider whether there has to be more than one outcome, look at randomness and whether this is always present.

5. Spend time looking at a chance event – for example, buying a Lotto ticket or a scratchie. The question, *Will I win?*, always involves chance. Are there questions that involve Lotto tickets but do not involve probability – what about, *How much is a "Quickpick"?* So look at changing situations from chance to certainty and back again.

6. Spend time on a **word bank** for probability – make sure students understand all the probability words such as certain, possible, impossible, event, outcome and so on. Best to do this by setting fun probability activities and discussing what happens as the activities unfold. Listen to the students, this is your chance to pick up on local words and nuances.

3.2.3 Classroom activities

Here are a few ideas for classroom activities to help with Skill 1.

1. **Classifying events**. Create three areas (areas on the floor, desks, etc.) and label them *impossible, possible* and *certain.* Have a selection of events for students to place in the relevant area – students could make up events and take them to the position – stand in the position showing their situation. Ask students to write three stories, one for each area. Use local stories.

   Record where the stories go on a table divided into columns by headings impossible, possible and certain.

2. **Creating a language line**. Get two students to stand on left and right in front of room and hold a rope between them, the one on left with *impossible* on a sign and the one on the right with *certain* on a sign. Label the rope *possible*. Students make up situations, summarise them on pieces of paper, tell the other students and then, with other students' help, peg the paper on the rope/line where it best fits – possible ones can be placed halfway or towards one of the ends depending on what they are. For example, *I kicked the football and it flew away* is pegged at impossible, while *I kicked the football and it fell to the ground* is certain. However, *I kicked the football and it went 40 metres* is only possible and depending on the ability of the person can be placed between impossible and certain.

   Students could be asked to form small groups of three in which they will judge various events as certain, impossible or possible. Events should be localised in the school community. Some examples of situations/events could be: *It will rain tomorrow; drop a rock in water and it will sink; a flower seed planted today will flower tomorrow; the sun will rise tomorrow morning; if you ask someone who was the first Mayor of their town, they will know; you will have two birthdays this year*; and *you will be in bed by 10 pm*. This is a good opportunity to bring in students' identities and cultures. Students could be asked to write events in their families that are certain, possible and impossible. The use of local language could be encouraged.

   Note that this will lead to fractions in probability – impossible will become 0 and certain 1, while possible will be a fraction between 0 and 1.

3. **Continuum**. Using two colours of paper or card stock (say red and blue), cut out circles that are the same size. Cut a radius. Slide one onto the other and rotate one circle around the other to make a spinner base. Different positions of the spinner base could be discussed in terms of fair and unfair (e.g. if you state that red is two steps and blue is one, then fair is 2/3 blue). These spinners can be added to the language line as visual locators between impossible and certain.

   You can also use the spinner base to develop the notion of a probability continuum. Have students create a spinner base that is not fair (make your spinner base fair). Then give each student a paper clip and have them come out and place their spinner on a clothes line according to the chances of spinning blue. The

completed probability continuum will help students see that chance can be at different places on a continuum between impossible and certain.

4. **Three-dice throw**

Suitability: Year 4–7, 2–4 players. A useful activity to reinforce all basic number facts as well as probability.

Directions: Prepare a large copy of the game board below on cardboard. Obtain three dice and piles of different coloured counters for each student.

0	1	2	3	4	5	6	7
8	9	10	11	12	13	14	15
16	17	18	19	20	21	22	23
24	25	26	27	28	29	30	31
32	33	34	35	36	37	38	39
40	41	42	44	45	48	50	54
55	60	64	66	72	75	80	86
90	100	108	120	125	144	150	180

Rules:

- To begin play, each player in turn rolls or throws all three dice and the player with the smallest sum begins play. Play progresses clockwise around the table.
- Each player in turn throws the three dice. The player then uses the three displayed numbers to make any number which is uncovered on the board. They must use one or two operations on these displayed numbers. For example, suppose they throw a 3, 4 and 5. They could think: (3×4)+5, (3+4)×5, (3×4)−5, or 3×4×5.

- The player is then allowed to cover one resulting number on the board with one of their counters. They are not allowed to cover a number already covered. For example, suppose 60 was uncovered. Since 60 = 3×4×5, the player could then cover it.

- Scoring: For each number covered, the player scores one point plus an additional point for each box it touches which has been previously covered. For example, suppose a player throws a 5, 2 and 6; and 21 and 14 have been covered. Then the player could cover 22 = (5+6)×2 and then score 1 point + 2 points for touching the covered 21 and 14, i.e. 3 points for that turn.

- If a player cannot make an uncovered number on the board from their throw, play moves to the next player, with no scoring.

- The winner is the player with the most points, when no more numbers can be covered.

Variations: Change the scoring – 1 point for each marker (no additional points), or 1 point for each marker touching at least one other (no additional points and no points for isolating cover).

Add a "challenge": If a player feels that another player did not make the move that scored the most possible points, they may challenge that player. The player does not change the move, but the challenger gets the additional points the better move would have earned. If the challenger is incorrect, they lose one point.

5. **Footy kicks or Netball passes.** Take students out to sports fields or courts and attempt to score goals. Discuss the chance of getting a goal from a variety of distances and angles.

3.3   Outcomes from probability events

Skill 2: The student can list all possible outcomes of a chance process.

Skill 3: The student can list all possible outcomes in consecutive trials of a chance process.

Skill 4: The student can state which outcome is most likely in a single trial of a chance process.

Big idea: Contiguous vs noncontiguous.

This section looks at the three skills that deal with outcomes – determining them all, recognising what happens when they are repeated and picking most likely (and as well, equally likely and least likely). Likelihood of outcomes is one of the major bases of probability. However, for many students, it is tied up with beliefs, feelings of luckiness and misremembered experiences. There is a need to get students to see likelihood in mathematical terms, but the very nature of probability means that, when there are few trials, responses can represent low likelihoods. There needs to be a growing understanding of the need to ensure activity is random, and that there are many trials.

3.3.1   Defining Skills 2, 3 and 4

Skill 2

Materials: A die, a box of attribute blocks, a bag, pen and paper.

Directions:

1. Answer these questions:

    (a) When I roll a (common) die, what numbers could I get?

    (b) When I pick an attribute block from a bag, what colours and shapes could I get?

    (c) Peter is at a street corner, what are the possible directions for Peter to go?
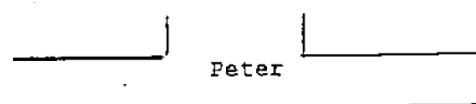
2. Set up experiments for (a), (b) and (c) above.

    (a) Roll the die and select a block. Do you get what you expected?

    (b) Draw the street corner with chalk on the floor. Walk the street. Which way can Peter go?

Skill 3

Materials: A die, a pentagon spinner, a bag with four attribute blocks, pen and paper.

Directions:

1. Answer these questions:

    (a) Last time I rolled a die, I got a six. If I roll it again, what numbers could I get?

    (b) When I spun this spinner last, I got a two. If I spin it again, what numbers could I get?
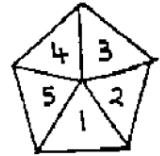
    (c) A shop sells three flavours of ice cream: vanilla, lime and chocolate. Yesterday Jane bought a lime ice cream. What flavours could she pick from today?

    (d) A bag contains four attribute blocks of different shape. Shane picks out the triangle but does not put it back in the bag. Now Jenny picks a piece. What possible pieces could she pick?
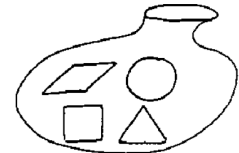
2. Set up and try experiments (a), (b), and (d). Did you get what you expected?

    (a) What changes in (d) would occur if the block was put back in the bag?

    (b) Why do people feel that the previous results will affect their next try?

Skill 4

Materials: A bag with three red and two blue counters, a hexagon spinner, pen and paper.

Directions:

1. Answer these questions:

    (a) A bag contains five red and two blue counters. Sally picks one counter from the bag. Are red and blue counters equally likely to be chosen? Which colour counter is most likely to be chosen?

    (b) I spin the spinner at right once. What numbers can I get? Which number is most likely?

    (c) David's mother quickly picks a party hat from the pile containing the following:
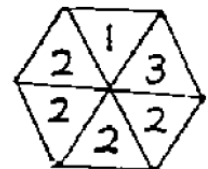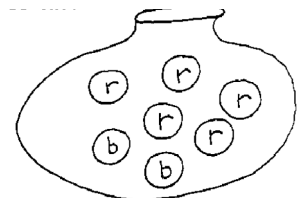
    How many sorts of party hats could she pick? Is she more likely to pick one of the two hats on the right?

2. Set up experiments (a) and (b). Trial them 10 times. Did you get what you expected?

3.3.2 Teaching ideas

For Skills 2, 3 and 4, we need to follow the sequence:

1. Look first at one-off trials/events and then list all outcomes – check that students are able to do this before moving on.

2. Move on to consecutive trials for the same events as in step (1) above – discuss if this changes the outcomes – for example, if four heads come up from four tosses of a coin, check that students do not believe that a tail is now more likely than a head in the fifth toss.

3. Look at replacement and no replacement (particularly for card activities and selecting things from a bag activities) – check that students know that replacement leaves things the same but no replacement does not.

4. Look at contiguous and noncontiguous models in counters and in spinners – check that students are not making mistakes when things are contiguous (e.g. thinking blue on right is more likely because it has two options).
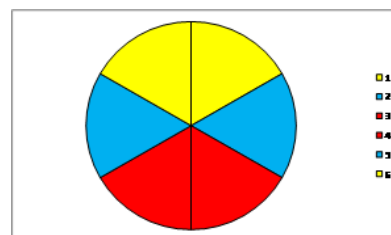
5. Go through each of these situations above but now look at likelihood of outcomes.

6. Compare and order different events in terms of the likelihood of the same outcome.

Note: When students are competent, it is possible to progress from qualitative comparisons of more or less likely to numeric comparison and ordering. Students should have experiences comparing two or more sample spaces with the same number of outcomes and two or more sample spaces with different numbers of outcomes but the same number of favourable outcomes to ensure robust understanding.

3.3.3 Classroom activities

1. **N-counters**

    Materials: 24 counters (12 of each colour), two dice, board as on right, two students or two groups of students to play.

    Directions: Students choose colour. Place all their counters anywhere on the board (can place multiple counters on the same number). In turn, students throw dice and add numbers, both students with a counter on that number remove **one** counter. Winner is first player to remove all counters.

	2	3	4	
5	6	7	8	9
	10	11	12	

2. **Counter Thief**. A variation of N-counters above. Played by two players. Use a 6×2 grid numbered 1 to 12. Each player has 12 counters and they place them anywhere on the grid. Players roll two dice, add the values and remove the counters from the corresponding grid square. With each roll of the dice, students need to keep a tally record of the dice total. At end of game, provided sufficient dice rolls have occurred, students will notice a bell curve from dice rolls.

3. **Red One**

    Materials: 20 counters: 8 blue, 6 green, 4 yellow, 2 red; group of students.

    Directions: The counters are spread out, students can choose a colour.

    Questions:

    (a) If you shut your eyes and choose, will the red be as likely as the other colours?

(b) How can you test this? Construct an experiment and test it!

(c) For what distribution of counters would the red be equally likely?

(d) Is there a way the counters could be set up so even with the present colours, red is equally likely? [Sort the colours into groups, number groups 1 to 4 – students select a number, then one counter from that group.]

4. **Compare and Order**

(a) The three containers on right all have the same number of counters. From which container is red most likely to be selected? [Middle container]

(b) The three containers on the right have different numbers of counters. From which container is red most likely to be selected? [Left container]

(c) Questions – Is the number of red counters important when the total number of counters is the same? Is the number of red counters important when the total number of counters is different? If not, what is important?
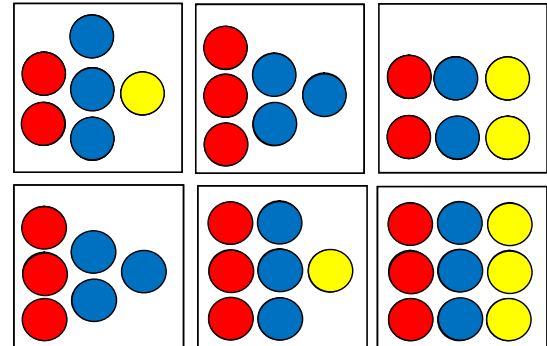
3.4 Desired event

Skill 5: The student can correctly choose in what situation an event is more likely to occur.

Big ideas: Deterministic vs probabilistic, concept of a fraction.

In this section, we turn around what was in section 3.3. Instead of waiting for the event to happen and predicting its outcome, we choose the outcome we would like and find the event that has most chance of enabling this outcome to happen. It is an act of **reversing**: instead of looking from event to outcome, we look from outcome to event. That is, from *teacher gives event* → *students choose most likely outcome* to *teacher gives most likely outcome* → *students choose event.*

This is a very important skill because, in life, we continually have in mind what we would like to happen and we search for ways to achieve it. For example, we want to have a good time, so what should we do? Or we want to obtain a job, so what should we do to maximise our chances?

3.4.1 Defining Skill 5

Materials: A bag with red and blue counters, pen and paper.

Directions:

1. Answer these questions:

(a) There are four bags of counters on the right. From which bag is Sue more likely to pick a blue counter?

(b) A lucky dip has two good prizes and three booby prizes, while a second has three good prizes and four booby prizes. Which lucky dip is most likely to give Jack a good prize?
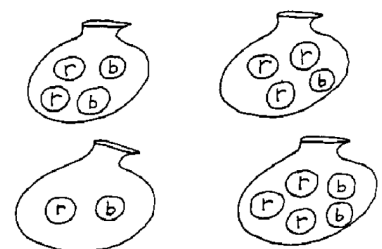
2. Set up an experiment:

(a) Trial each bag 10 times. Did you get what you expected?

3. I have been given these two spinners on right:
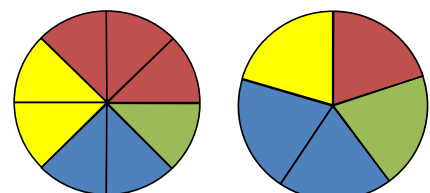
(a) I can pick the spinner and I can pick the colour. Then I spin the spinner and score a point each time my colour comes up.

(b) Which spinner and which colour should I choose to get to five points in the least number of spins?

4. Set up an experiment:

(a) Trial each spinner 10 times. Did you get what you expected?

3.4.2 Teaching ideas

The following points should be made.

1. Care should be taken to ensure students see how different this skill is to finding the most likely outcome of an event. This skill requires comparison of events in terms of likelihood of a certain outcome – sometimes when the events are very different.

2. This skill is about life – about what you do next to achieve what you want. So use it as an opportunity for your students to discuss what they want and the best ways they can see to achieve it. However, it is important not to assume things from the background of a teacher. Many students from Indigenous schools do not believe, with good reasons, that working hard at school will enable them to get a good job – they believe that it may work for others but not them. Do not apply your values onto the students – listen to what they say, and let them discuss activities that might best give them what they want.

3. It is also not important to tackle big ticket items. Simple things like what to do that night may be easier. How do they have fun? How do they stay safe?

4. This skill is the beginning of **inference** – using probability data from events to determine the best way to achieve wanted outcomes.

5. This inference is not made from ratio understanding but **fraction understanding** – not from comparing chances of wanted outcomes against other outcomes, but comparing chances of wanted outcomes against all outcomes.

6. In the modern world, Skill 5 can be excellently discussed in relation to computer games – what do you have to do to have the best chance of survival?

3.4.3 Classroom activities

1. **Lucky June problem.** June wanted to go on the end-of-year trip. Her father gave her five white and five black counters and two identical bags. He told her she could put counters in bags however she wanted but he would take the bags, mix them up and then she would have to choose the bag and one counter from it. If the counter was white she could go on the trip. If it was black she could not go on the trip.

   June was clever and so she chose a white and went on the trip. How did she place the counters in the bags to maximise her chance of getting a white?

   This is a simplified version of the famous "Lucky Prince" puzzle. To marry the Emperor's daughter and get half his kingdom, the Prince also has to select a white marble. He is given three identical barrels and 50 white and 50 black marbles. He can place the marbles as he wishes in barrels, but in the morning, they will be mixed around and he has to choose a barrel and then a marble from it.

   The Prince was also clever and won. So how did the Prince place his marbles to have the best chance of getting a white?

2. **Card Identities.** Many students will not need this discussion activity, but use it anyway to determine student knowledge of red and black cards, four suits, number and picture cards, number of cards in pack, 1/52 chance, etc.

Play card games like "21" where each player is dealt two cards. They total the cards and decide if they want another card to be dealt to them. They "bust" if the combined card total is greater than 21.

3. **Between the posts**

   (a) Deal two cards for a turn. Player looks at the cards and decides whether to call for another card.

   (b) If the call is made, it is because the player decides he/she has a reasonable chance of getting a third card that falls between the original two cards.

4. **Who wins**

   Materials: Place 13 counters in a line; two players or one group of players.
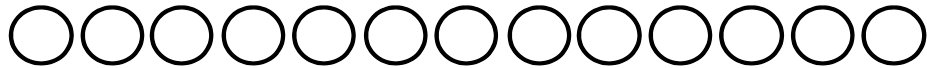
   Directions: Choose who goes first. In turn, remove one or two counters. Winner is the person who takes the last counter(s).

   Questions:

   (a) Who is most likely to win – first player or second player? Assume everyone plays well and no errors.

   (b) What if players removed only one counter each time? Who is most likely to win now? What if they remove one, two, or three counters each time?

   (c) Can you construct a game for nine counters where the first player is most likely to win? Construct a game for 10 counters where the first player is most likely to lose?

3.5   Probability as a fraction

Skill 6: The student can assign a numerical probability to an event.

Big idea: Theoretical vs frequentist.

This section begins the process of calculation with regard to probabilities and formally connects fraction and probability. Of the two options available, frequentist and theoretical, this section focuses on the theoretical.

3.5.1   Defining Skill 6

Materials: A coin, pen and paper.

Directions:

1. Answer these questions:

   (a) Ian tosses a coin. What outcomes are likely?

   (b) Are these outcomes equally likely?

   (c) What is the probability of a tail?

2. Set up an experiment:

   (a) Toss a coin 20 times. Did you get what you expected?

   (b) Check you did not fall into a rhythm and keep throwing in the same way.

3.5.2 Teaching ideas

The basis of this section is as follows.

1. Theoretical probability is based on equally likely outcomes. In this situation, the probability of what is wanted equals the number of outcomes giving what is wanted divided by the total number of outcomes. For example, to throw a die to get an even number means that the number of outcomes with the desired result is three while the total number of outcomes is six. Thus, the probability is $^3/_6$ or $^1/_2$.

2. Thus, probabilities are fractions and there is a need to follow the sequence given in **Appendix C1** which describes the activity and questions needed to connect probability to fraction. It is important to reverse all relationships that emerge and construct events for particular probabilities as well as interpreting given situations in terms of probability. Thus, we need event → probability **and** probability → event.

3. Ensure that students realise that fraction in probabilities go from 0 to 1 and not beyond 1. There are not probabilities of, say, 2¾! This means that it is useful to undertake the number-line activity where a string, rope or a line is drawn from 0 to 1 and students peg/place numbers onto it. Here the numbers can be fractions, percents, decimals and probability language such as "some chance", "possible", "probable" and so on.

4. To ensure that calculation of fraction probabilities includes all possible outcomes, it is important to build sample spaces of all possible outcomes. For example, if two dice are thrown and added, the sample space is as shown below. It means, for example, that the chance of getting a 7 is the number of outcomes giving 7 divided by the total number of outcomes = $^6/_{36}$ or $^1/_6$.

Two-dice sample space:

2	1,1
3	1,2 2,1
4	1,3 2,2 3,1
5	1,4 2,3 3,2 4,1
6	1,5 2,4 3,3 4,2 5,1
7	1,6 2,5 3,4 4,3 5,2 6,1
8	2,6 3,5 4,4 5,3 6,2
9	3,6 4,5 5,4 6,3
10	4,6 5,5 6,4
11	5,6 6,5
12	6,6

5. In the two-dice sample space above, if we want the probability of getting an odd number sum, we simply add the probabilities for 3, 5, 7, 9, and 11 because these events are disjoint (i.e. no outcomes in common). However, if events are overlapping, this is not so. For example, the probability of getting two numbers the same on the two dice and getting a score over 7 overlap. The results 4,4 , 5,5 , 6,6 are in both sets and so we cannot simply add the probabilities. We need to add the probabilities and then account for the overlap so that these results are not counted twice.

6. In complex sample spaces, there are often two or more stages – e.g. throw two dice, draw three cards, or throw a die and spin a spinner. In these situations a tree diagram such as that below helps with the sample space.

Toss a coin and throw a die:
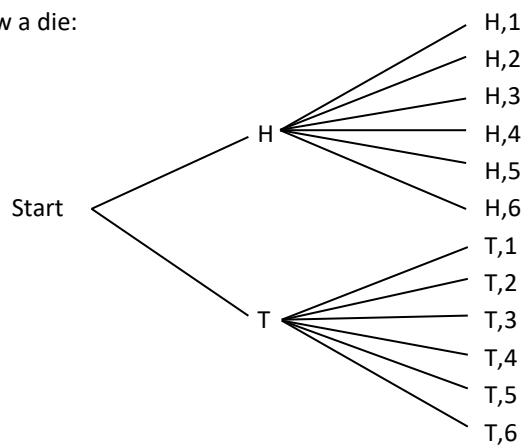
3.5.3 Classroom activities

1. **Tug of war**

   Materials: die, one counter, board as below, two players.

X						***						Y
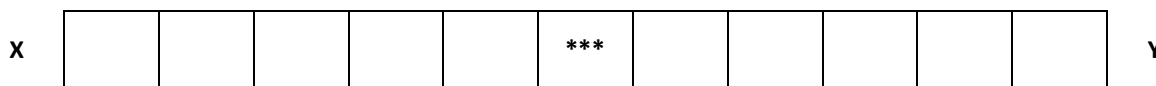
   Rules: Each player chooses an end (X or Y). The counter is place in position ***. Players in turn throw the die and move the counter the amount shown towards their end. The first player to reach his/her end wins.

   Questions (after many games):

   (a) Does it take a long time to get a winner? Why?

   (b) Which player is more likely to win? Is there an advantage in going first?

   (c) If the board was 100 squares long, would there ever be a winner?

2. **Planetfall**

   Materials: one coin, counters, board as below right, 2–6 players.

   Rules:

   (a) Players place counters (spaceships) at start (Earth).

   (b) Players in turn toss the coin and move left if heads and right if tails.

   (c) Players score one point for reaching Fronsi, two points for reaching Plenhaa or Hecto and three points for reaching Gelbt or Actur.

   (d) The first player to make 10 points wins.
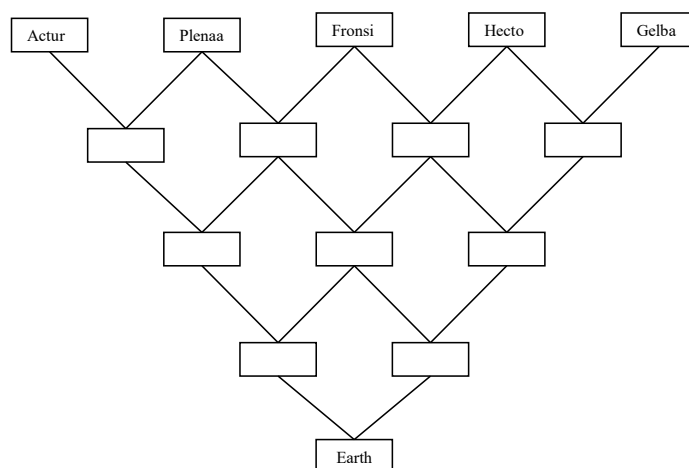
   Question (after many games): What is the most likely planet to reach? Why?

3. **Feud**

   Materials: two dice, two players.

   Rules:

   (a) Players in turn throw dice and add the numbers.

   (b) If the point sum is 2, 3, 4, 10, 11 or 12, player 1 receives one point. If the sum is 5, 6, 7, 8 or 9, player 2 receives one point.

   (c) The first player to 10 points wins.

   Questions (after many games):

   (a) Is the game fair?

   (b) Does it matter whether you are player 1 or 2?

   (c) What extra numbers could we give player 1 to even the contest?

4. **Making Fair**

   Materials: A page of spinners with different areas for yellow, red, blue (see Appendix C2 for spinner examples). Racetrack game board (see Appendix C2) with tracks marked yellow, red, blue.

   Activity:

   (a) Choose which of the spinners would be fair.

   (b) Decide which colour is likely to win with the other (unfair) spinners.

   (c) Choose an unfair spinner and make it fair by having different colours or moving different amounts.

   Notes:

   (a) Make noncontiguous as well as contiguous spinner, e.g. the left spinner is contiguous and the right spinner is not.

   (b) The spinner on the left is not a fair spinner as there is twice the chance for blue as for each of the other colours. However, it would be made fair if red, yellow and green meant move two spaces while blue meant move only one space.

   (c) If the spinners were given numbers instead of colours then you could have two spins and move the sum. Then you can look at what is possible in sums and whether sums can be fair.
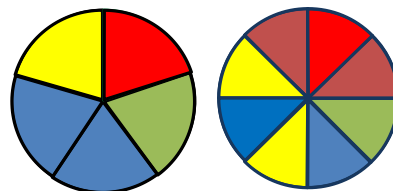
5. **Internet games**

   (a) There are many games of automatic dice rolls, card games, roulette etc. available to use in the classroom.

   (b) Ensure the students have pencil and paper handy to record outcomes as it is possible learning opportunities will be missed if the students are simply playing the games without reflection.

3.6   Experimental probability

Skill 7: The student can estimate the numerical probability of an event using "simulation" methods.

Big idea: Frequentist vs theoretical.

This section follows the theoretical one of section 3.5. However, now the method to obtain the probability as a fraction is frequentist – found from experiments.

3.6.1   Defining Skill 7

Materials: Thumbtack, pen and paper.

Directions:

1.   Toss a simple spinning top 50 times and record the number of times that it lands point up and point down in a frequency table.

Outcome	Tally	Frequency
Point up		
Point down		

2.   Use your results to estimate the probability of a spinning top landing point up, and point down, in a single toss.

3.6.2   Teaching ideas

Some points to consider are as follows.

1.   It is useful to begin with experimental probability by constructing experiments where outcomes are equally likely before moving on to not equally likely examples. In this way, experimental and theoretical can be connected. It also allows the experiments to be validation of the theoretical outcomes.

2.   This approach is also useful because it shows that experiments often give different results to theory. This leads to important discussions regarding why experimental (observed) and theoretical (expected) probabilities may differ. One is chance but another is that the experiments become non-random or there are too few trials.

3.   This leads to spending time on ensuring that: (a) randomness is retained in experiments that do not have the backing of theory, and (b) sufficient trials are completed to remove extreme probabilities. One needs to check that tosses produce the best possible random results (this often involves bouncing the thrown object back off a wall – see activity 1 in subsection 3.6.3).

4.   Once skill is acquired, students should move from one step to two or more step experiments. This will lead to the need to undertake complete sample spaces, and to the use of tree diagrams, two-way tables and Venn diagrams.

5.   As the number of trials is important to accuracy in probabilities, technological simulations become important tools for detailed investigations.

3.6.3 Classroom activities

1. **Tossing coin**

    Materials: One coin.

    Directions:

    (a) A trial consists of seeing a coin and recording how it lands. Perform some preliminary flips to determine the best technique for tossing which obtains random landing positions.

    (b) Perform an experiment of 30 trials. Record results after each trial: tally the result, then record the numbers and percentage cumulatively.

Trial	Results		Cumulative record			
	Head	Tail	No. of heads	No. of tails	% of head	% of tails
1						
2						
and so on						

    (c) Obtain two sheets of graph paper. Draw a line graph using axes as below:
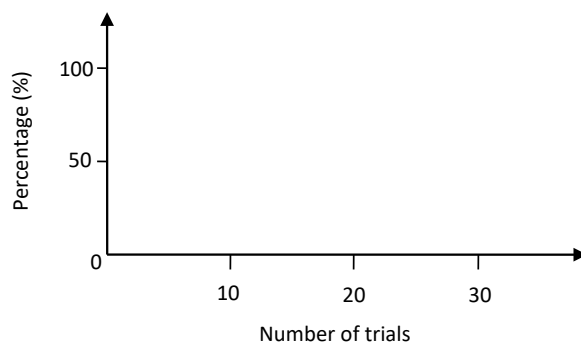
    Questions:

    (a) Which of heads or tails occurred the most in the 30 trials? Would this remain the case if we completed more trials?

    (b) Was there a trend in the graphed percentage? Would this trend give rise to a reasonable estimate of probability? Is this what you expected?

2. **Two-dice difference**

    Materials: Two dice, pad and pencil.

    Experiment: Roll two dice. Calculate the difference between the uppermost faces – take low from high.

    Questions: What difference is likely to occur most frequently? What about least frequently? What differences can occur? What is the probability of these differences?

    Procedure:

    (a) Investigate these questions and record your results and findings on pad.

    (b) Devise an experiment to test your conclusions. What did you find? Were your expectations confirmed? How could you improve your experimental procedure?

    (c) If you were the banker in a gambling game of two-dice differences, what odds would you strike for each difference?

3. **Biased Die**

   Materials: Cubes with sides as follows: 1, 1, 2, 3, 3, 3.

   Directions:

   (a) Create a biased die with numbers e.g. 1, 1, 2, 3, 3, 3. One die for each student or in pairs, but students do not get the opportunity to "study" the arrangement of numbers.

   (b) Repeatedly roll the die and record the numbers that show up. Students keep rolling till they think they know the number arrangement. A pattern may appear after 10 rolls, where students can make a hypothesis as to what numbers are featured on the die.

   (c) After 20 rolls they may be in a position to make a statement as to what the number arrangement is.

   (d) Clear frequencies will be seen with over 50 rolls of tally data.

4. **Cube toss**

   Materials: Cubes with sides as follows: 1, 1, 2, 3, 3, 3.

   Directions:

   (a) Ask students what they might get when they roll the cube. What is likely, what is impossible?

   (b) Have students roll the cube and record results in bar graph as shown below. Stop when a bar is full. Discuss the findings.

1										
2										
3										

   (c) Now introduce the notion of rolling two cubes and recording the sum of the two cubes. Have a short discussion where students predict which bar will fill the fastest or if they will be equal.

   (d) Roll the cubes, stop when a bar is full. Discuss the findings.

1										
2										
3										
4										
5										
6										

5. **Spots on rocks**

   Material: Four flat stones or rocks with 1, 2, 3 and 4 in the form of dots on one side and nothing on the other side (see below).
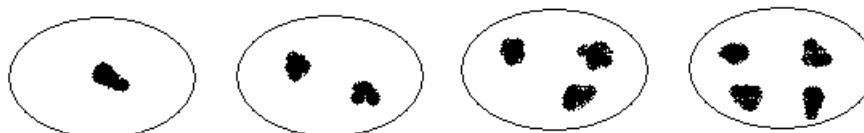
Directions:

(a) The game involves the tossing of four rocks and calculating the resulting score.

(b) Each player tosses the rocks, and keeps a cumulative score (checked by other players if necessary). The first player to reach exactly 50 wins.

(c) In a discussion, ask students what sums are possible? What sums were common? What scores are possible in a single turn?

(d) What are all the outcomes (possible combinations of stones)? What is the probability of each score?

Note: This game is good for exploring the notion of independent events.

(e) As a variation for scoring, as students toss their way to 50, the computation can be varied once they have achieved at least 40 points. The students can use subtraction, multiplication and division to reach exactly 50.

(f) Again discuss what computations are possible? What computations were common? What scores are possible in a single turn?

(g) What are all the outcomes (possible combinations of stones)? What is the probability of each score?

6. **Cylinder experiment**

   Materials: Cylinders of different heights, red and blue coloured marking pens.
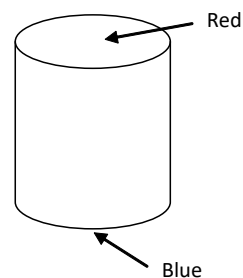
   Directions:

   (a) Mark the ends of the cylinders red and blue, as shown on right, with a marking pen. A trial consists of flipping the cylinder and recording how it lands. Perform some preliminary flips to determine the best technique for flipping which obtains random landing positions. Once you've worked this out, stick to it. (Hint: It is best to throw the cylinder against a wall.)

   (b) Perform an experiment of 30 trials with one of the cylinders. Record results after each group of three trials. First tally the results and then record the number and percentage of each outcome cumulatively after each group of three trials (use table below).

Trials	Result			Cumulative record					
				Red end		Blue end		Curved Surface	
	Red end	Blue end	Curved Surface	Number	%	Number	%	Number	%
3									
6									
and so on									

   (c) Find two other groups using the same size cylinder and copy their results. Record their tallies, then cumulatively add these tallies to your number and percentage columns to gain results of 90 trials.

   (d) Repeat directions (b) to (c) with the second cylinders (of differing height), using another version of the table above.

   (e) Draw line graphs of the cumulative percentages for both cylinders on graph paper using axes as on right.
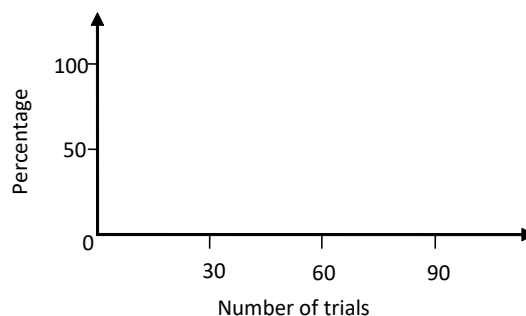
Questions:

(a) Which outcome occurred most frequently for the first cylinder? For the second? Would this be the same if we completed more trials? Is there the same chance of getting a blue as a red end? Should there be?

(b) What were the differences in frequency of outcomes for the two cylinders? Is this the difference you would expect taking into account the different height–diameter ratios? Suggest a relationship between the geometry of the cylinder and the frequency of outcomes. How could you test this?

(c) Is there any trend in the graphed results? Will this trend give rise to a reasonable estimate of probability? Do these probabilities accord with your expectations for the cylinders? Why?

Notes: Using the probabilities worked out above, determine the probability of getting three blue ends in three successive trials. How would you test this? List five other experiments that could be conducted and for which there is no easy theoretical answer.

7. **Match sticks**

Materials: Box of matches, pad and pencil, ruled board as below.

Experiment: Hold the matches vertically about the centre of the board. Drop the matches (shown below).

Lines one matchstick length apart

match

board
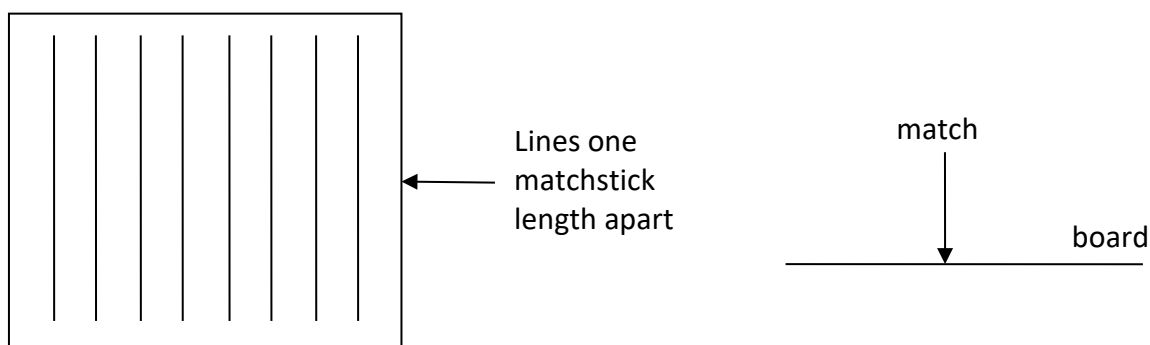
Questions: What is the probability of a match falling across a line?

Procedure:

(a) What would you guess the probability to be?

(b) Design an experiment to test your guess and record your work on your pad.

(c) What factors influence the result of your experiment? Can they be controlled?

(d) What effect would doubling, halving, etc., the distance between the lines have on probability? Test and record your results.

Notes: Here is some "Pi in the Sky". If $\frac{2n}{d \times f} =$ _____, where n = number of matches dropped, d = distance in matches between lines, f = number of matches on line, what is _____? Use your results to find _____.

3.7 Inference from probability

Inference is the last section of this chapter. It is the ability to use probability data to make decisions and predictions regarding probabilistic situations and frequencies of different outcomes.

3.7.1 Teaching ideas

1. The focus in inference is on making decisions from experiments. This means that time is needed for the exploration and thinking. It also means that a wide variety of methods should be used to explore the data (e.g. tables and graphs) and care should be taken in gathering the data (care with ensuring randomness).

2. Ensure that all activity is reversed. In other words, teaching that is the teacher providing materials for the students to find probabilities should be, in part, turned around so that some of the time the teacher provides the probabilities and the students come up with the materials and activity that reflects that probability.

3. Such inferential investigations have the ability to handle large data sets and so this adds in the opportunity for the mathematics to start to involve analysis of a wide range of data, analysis that is necessary to solve today's problems.

4. Inferential activity has a large aspect of problem solving and requires confidence, self-efficacy and resilience.

3.7.2 Classroom activities

1. **Guess what is in the bag**

   Materials: 1 bag, 10 blocks of different colours. Groups of students.

   Directions: Place blocks in bag, draw one out at a time and record. Replace and shake bag. Repeat this. Use record to predict what is in the bag.

   Notes:

   (a) If students find this difficult, have fewer blocks and fewer colours.

   (b) Talk to students about how many times they should repeat the drawing of the block. Why would multiples of 10 be good?

   (c) If student gets answer wrong, ask to do more draws, discuss making draws random.

3.7.3 Inference from experimental probability

Materials: Bag, small counters or pegs of two colours, another bag with 14 counters of three colours (these bags need to be prepared ahead of time and labelled with the total number of counters so that students do not know ahead of time what colours are in the bags), notebook, pen, graph paper, hand calculator.

Directions:

(a) Place one counter of one colour and two counters of a second colour in the bag.

(b) A trial consists of shaking the bag, selecting a counter (without looking) and replacing the counter in the bag (without looking). Ensure the bag is well shaken.

(c) Perform an experiment of 30 trials. Record results cumulatively after each trial.

(d) Complete a table with headings as below.

Trial	Result		Cumulative Record			
	1st colour	2nd colour	1st colour		2nd colour	
			Number	%	Number	%
1						
2						
3						

(e) Perform a second experiment of 20 trials with the second bag containing 14 counters. Record results cumulatively after each group of 5 trials.

(f) Find three other groups who have finished this experiment and record their tallies and then work out your own cumulative results (making 80 trials in all).

(g) Complete a table with headings as below (use your calculator).

Trial	Result			Cumulative Record					
	1st colour	2nd colour	3rd colour	1st colour		2nd colour		3rd colour	
				Number	%	Number	%	Number	%
1									
2									
3									

(h) Draw line graphs for all colours in both experiments using axes similar to below (use graph paper).

Cumulative % of 3 coloured counters

Cumulative % of 14 coloured counters
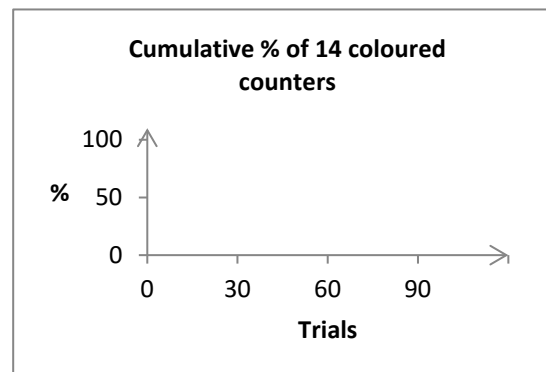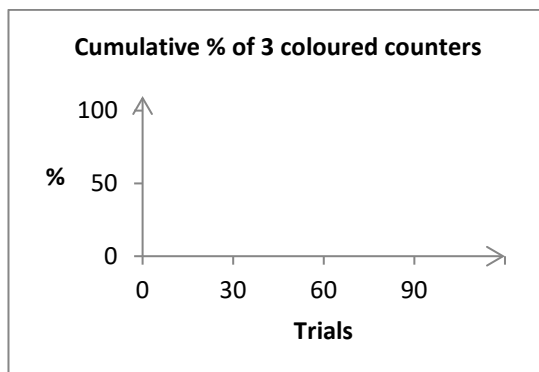
Questions:

(a) From your results what is your estimate of the number of counters of each colour – in the first experiment (3 counters)? In the second experiment (14 counters)? How could you improve this estimate?

(b) Do any trends evident in the graphs enable you to make an even more accurate determination of the number of blue counters? Why?

(c) Open the second bag. Check its contents. Was your estimate right? Why or why not? Calculate the theoretical probabilities for each colour in the first and second bags. Were these close to the experimental probabilities? Why or why not?

2. **Peggie's Pick**

Materials: Bag containing pegs (2 red, 4 blue, 8 green), pad and pencil.

Experiment: Shake the bag well. Select a peg without looking. Record its colour. Replace the peg.

Questions: What colour pegs are in the bag? If there are 14 pegs, how many of each colour are there?

Procedure:

(a) Design an experiment to investigate the above questions and record all work in your pad.

(b) How could you improve your estimate of the number and colour of pegs? Would more trials help?

(c) Open the bag. Check its contents. Were you right or wrong? If you were wrong, what factors could have influenced the results of your experiment?

Activities: List three other experiments to give, as above, experiences of inference.

3. **How many beads in the container?**

Materials: Plastic or glass container full of beads (with lid taped down), pad and pen. Any measuring device you need.

Procedure:

(a) Guess the number of beads in the sealed container. How accurately can you guess? Within 100, 50, 20, 10, ... of the actual number?

(b) Devise a method which will give a better estimation than guessing. Record your work in your pad. The sealed container must not be opened. You may use another container and beads.

(c) What factors influence your calculation? Can you think of other better methods?

Activities: List other situations in which estimation using the above method is required.

4. **Footy kicks or Netball passes**

This activity is a repeat of goal shooting games played earlier, however at the conclusion of the chapter, it is expected that the language and representations relating to the goals would contain accurate probability description.

(a) Discuss the chance of getting a goal from a variety of distances and angles.

(b) Have students keep a record of shooting positions and draw diagrams.

4   Statistical Inference

There is a body of opinion that statistics is a strand different to the other strands of mathematics. This is because the other strands of mathematics are seen to focus on invariant or unchanging properties and processes that generalise across contexts. For example, "turnarounds" or the commutative law for addition (i.e. a+b = b+a) is a law that holds for all numbers – it is a general or generic law. Against this, statistics is seen as understanding variation (or difference) within a particular context, that is, the data that has been gathered on a particular situation. It certainly appears to be a reasonable argument that statistics is applied to particular and often complex contexts (e.g. what is the height of a typical Year 7 student?) which have error and uncertainty at their basis.

However, YDM is unsure about this difference because: (a) statistics has invariant properties (e.g. formula for mean), processes (e.g. box and whisker graphs) and concepts (e.g. the idea of error or uncertainty); (b) other strands of mathematics (e.g. probability and measurement) appear to have uncertainty and error in their applications; and (c) even operations have to take account of error when applied to real-world applications (e.g. bridge building). It seems that the particularity of statistics is in terms of the data and questions that it is designed to handle. However, statistics reflects traditional Aboriginal and Torres Strait Islander thinking in its ability to handle large data sets and complex interactions in particular contexts. In rich tasks that focus on complex but particular situations, Indigenous students have been found to excel (e.g. the tasks in the New Basics program of the 2000s).

It is important to realise that statistics requires students to **infer from data** not just describe data. This difference between description and inference is shown below.

DESCRIPTION	INFERENCE
• Describes data	• Goes beyond data
• Expressed with certainty	• Uncertain (how sure are we)
• Often based on personal experience	• Evidence-based arguments

Because of its nature, statistics and statistical inference are now **very important** in mathematics. This is because they reflect modern life and modern problems in terms of the information revolution that is occurring. Statistics enables complexity to be solved within a context. Statistics is part of normal development – people make inferences everyday (e.g. what film shall we go to?).

This chapter looks at the teaching of statistical inference and its sections cover: sequencing for statistical inference (4.1), early inference (4.2), development of inferential reasoning (4.3), central tendency (4.4), data distribution (4.5), and inferential misrepresentation (4.6).

4.1   Sequencing for statistical inference

This section covers sequences in teaching and doing statistical inference.

4.1.1   Sequencing in teaching approaches

Statistics benefits from active teaching approaches, particularly inquiry-based approaches that give precedence to experiences that enable students to come to terms with variation and uncertainty. Thus, like problem solving, statistics is best learnt by doing because only in this way can students appreciate uncertainty and variation, and the undetermined nature of much of this; that is, appreciate and understand the untypicality of typical.

Statistics is an opportunity for integration with other maths topics (particularly measurement). It benefits from finding contexts and problems that interest students. Its focus, and most important component, is learning to make decisions with data: to make inferences. Also importantly, this decision-making or inferring is part of a process or sequence that has components as below, and can be seen in the cycle on below right:

Posing a question/problem and predicting possible answers

Devising a plan to tackle the question

Collecting and analysing data

Making decisions acknowledging uncertainty (inferring)

Defending the inferences with argument

PROBLEM

CONCLUSION → PLAN
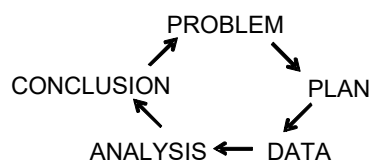
ANALYSIS ← DATA

In the early years, this is an informal process involving understanding variation, prediction, hypothesising, and criticising. In the later years, it involves: (a) analysis and interpretation of data; (b) investigation and comment on different forms/representations of data; (c) relationship of data to questions/issues and evaluation of these issues/questions in terms of data, particularly relationship between purpose and choice of data; (d) box plots and relation to distributions; and (e) discussion of distribution of data, using terms including *skewed*, *symmetric* and *bimodal*.

As an important footnote, teaching statistics also provides the opportunity for enhancing the **proficiencies in the Australian Mathematics Curriculum**: (a) **understanding** – covering terms such as *sample*, *population*, *random*; (b) **fluency** – covering working with calculations; (c) **problem solving** – devising a strategy for analysing data to answer a question; and (d) **reasoning** – generalising from data and analysis to a conclusion.

4.1.2 Sequencing content

The development of statistical inferences goes through **three steps** as below:

Statistical literacy
- Focuses on utility and purpose of tools; interprets, critiques, debates and judges.

Statistical reasoning
- Focuses on reasoning and making sense; utilises data, graphs and statistics information to understand the situation.

Statistical thinking
- Focuses on the why and how of statistical investigations; understands distributions; infers, creates and sees things as big ideas.
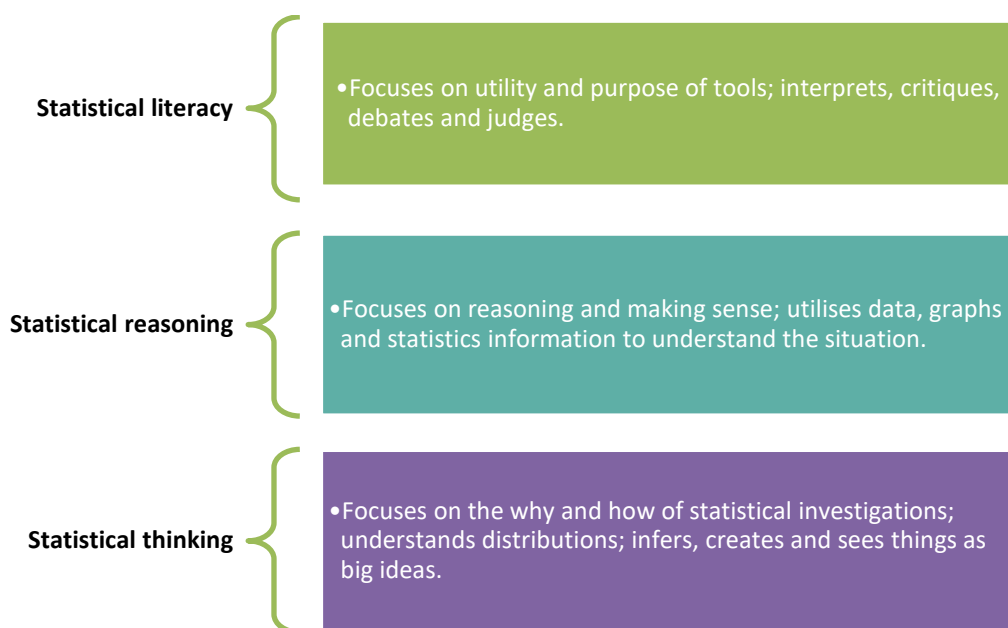
For the purposes of this chapter, we have divided the focus of the inferential statistics into a **five-step sequence** from early years to later years:

Early inference

Development of inferential thinking

Data and central tendency

Data distribution

Inferential misrepresentation

The reason for these five stages is to look at statistical literacy development in the first section, begin the movement to statistical thinking in the second section, and then start to build towards statistical thinking in the next two sections. We end at looking at the role of misrepresentation in statistics (i.e. "how to lie with statistics") that has led to the statement "there are lies, damned lies, and statistics".

This sequence will also enable us to cover the development of the important process of making decisions with respect to data, and also the development of meaning and formulae for a series of concepts that assist with describing data and its distribution, for example, mean, mode, median, range, deviation, quartiles, and outliers. As these concepts with respect to statistics are late in mathematics development, it means that early learning should focus on the development of statistical literacy leading to reasoning, inferential thinking and the strategies and approaches that go with this. Then, later, the new ideas will be added for a more sophisticated language for inference which will build onto the early thinking.

Finally under sequencing, we look at the types of questions, tasks or projects that can be set across the years of this chapter (Year 3 to 9). This we break into four levels of **problem types**:

> **Level A: Simple** – one uncertainty, e.g. *Do most students have brown eyes?*
>
> **Level B: Multiple** – two or more uncertainties, e.g. *Do tall children run faster?*
>
> **Level C: Extended** – two or more uncertainties plus need for other maths/science knowledge, e.g. *What year level has the healthiest lunch?, What is the best design for a loopy aeroplane?*
>
> **Level D: Complex** – all of Level C plus differences between types, e.g. *Do typical Year 7 students eat healthy cereals?*

We will look at Level A problem types in section 4.2 and then move on to Level B problem types in section 4.3. Sections 4.4 and 4.5 will cover Levels C and D problem types.

4.2 Early inference

This section covers early activities in statistical inference – it looks at the middle primary years. The focus is on understanding statistical literacy as the first step in inference. This means a focus on utility and purpose of tools, and on building abilities to interpret, critique, debate and judge.

4.2.1 Overview of section

As stated above, this beginning section focuses on statistical literacy as a meeting point of probability and statistics in the everyday world. It begins the development of statistical tools in relation to general contextual knowledge and critical literacy. The tools are primarily those relating to the collection and organisation of data and using that data to make decisions. The activities tend to include: (a) posing questions and collecting categorical/numerical data; (b) describing and interpreting data in context; and (c) carrying out surveys and recording data accurately. They begin to build an appreciation of informal inferences and some informal understanding of variation and uncertainty.

The outcomes aimed for include:

(a) building proficiencies – understanding, fluency, problem solving and reasoning;

(b) building critical and creative thinking – developing inquiry (identifying, exploring, organising), generating ideas, reflecting on thinking and process, and analysing, synthesising and evaluating; and

(c) building the heart of statistics – formulating and testing of hypotheses, justifying conjecture with evidence, and inferring with convincing argument.

In particular, the aim is to integrate measurement with statistics and challenge students' idea of certainty; for example, how certain are they that their measure is correct?

The practicalities for this include shifting focus from data points (e.g. "Kym watches 10 hours of TV") to holism or characterising groups (e.g. "most of my class watch between 10 and 15 hours of TV a week"). This has been characterised as having the ability to "distinguish signal from noise" (e.g. tuning a small radio). It is best undertaken through investigating questions characterised as Level A (see section 4.1.2) – by allowing students to draw inferences from data they have gathered.

It is important to remember that students' capabilities in drawing informal inferences need to be recognised, with increased exposure to a range of statistical representations that require interpretation and explanation beyond basic descriptions. If students are not exposed to informal inference in the primary school, the introduction of formal statistical tests in the late secondary school can become a meaningless experience because students will not have developed an intuition about the story conveyed by data.

4.2.2 Activities

Introductory activity
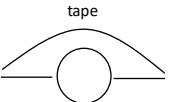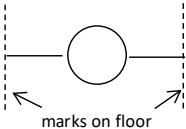
Materials: Measuring devices, methods to represent bar graphs (rectangular sheets of paper, Maths Mat, graph paper, pen and paper, and so on).
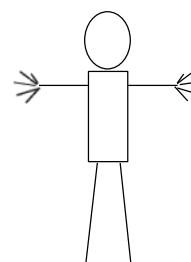
Instructions:

- Choose a student to stand with arms outstretched and organise all students to measure the student's arm span from fingertip to fingertip.
- Record all the students' data on the board to nearest cm. Organise the students to graph this data in terms of individual lines or frequencies.
- Use the data and the graphs to discuss the following informally: range, centre, outliers, certainty, and typical.
- Ask the students to make decisions from the data as to: What is John's arm span? What is the variation and why is it occurring? What is uncertain?
- In particular, ask what the students would do if all they had was the data and no way to measure the student's arm span. What do they think is the "correct answer" or the "best answer" and why?

Points for discussion: In most examples of this activity, there is a wide variation in measures. This gives an opportunity to discuss errors in measurement and the way in which different ways of measuring may lead to different measures. For example, the following can lead to error:

(a) tape

Measure too long because of bend in measuring tape.

(b)

marks on floor

This can lead to discussion of different ways of measuring such as laying the students on the ground and marking lines on ground and measuring between these. Would this be more accurate?

(c) Students not accurate when measuring with a tape – particularly if have a 1 m tape and it has to be moved for the overall measure.

(d) The student moves between different students measuring – the arms could move backwards and forwards, after a time tiredness may make the arms sag so not at right angles to the body, the fingers may curl, and so on.

This activity can lead to good discussion on data such as what is the middle, what is the average and are there outliers, what are these and why would they occur? Finally, asking the students to come to a conclusion or consensus just from the data can lead to great discussion justifying different outcomes. This then is an opportunity to discuss the idea of error but, even more importantly, the idea of uncertainty.

4.2.3 Foci of activities

There are four foci here. Teachers need to choose which of these is/are appropriate for their students:

1. **Ensuring basics**. It is important to run activities that build the abilities that underlie inference. The first of these includes gathering data, recording data, graphing data and describing data. For example, it is possible to gather all students' shoe sizes by using shoes to make the graph. This allows for recording (tallies and tables), graphing (bar graphs and frequency bar graphs) and informally discussing what "most students' size is" (e.g. centre, average or typical) and what are unusual sizes (e.g. range, outliers). The second of these is to return to measuring. This leads to discussion on errors and how they can be made and how to take account of them.

2. **Making decisions**. It is important, after or during basics, that activities like measuring arm span above are undertaken to (a) extend discussion to **error and uncertainty**; (b) build ability to **make decisions** from the data; and (c) **defend decisions** from the data in relation to the specifics of the measuring.

3. **Posing problems and devising data gathering**. As well as (2) above, it is important to build the students' ability to work from the problem only. So we need to build ability to pose questions and devise ways to gather data for their answers. We need to reduce our support for the students – just ask a question like "how far do we jump?" and allow the students to work out ways to gather data for this. Then, justification for the inference is not just in terms of data but in terms of relevance of the data.

4. **Building complexity**. The above introductory activity on measuring arm span is so specific yet filled with uncertainty. The next step that could be undertaken is to begin to add in extra uncertainty by asking not for a specific arm span but for a "typical" arm span. This starts to extend the ideas in earlier sections.

4.2.4 Investigations

Investigation 1

1. Choose an investigation like one of those in the Level A examples below. Try to make it relevant and motivating for your students.

2. Let the students work out their own way to tackle the question – discuss and reflect.

3. Use every opportunity to direct attention to and reinforce the outcomes for this section and the four foci in the activities as appropriate to the students.

Level A investigations	How tall is John?
	How far do we jump?
	What is the best recipe for play dough?
	What kind of books do we like?
	What is the best design for an obstacle course?
	What makes a toy car go further – a steep or a low ramp?
	How long does it take to tie a shoelace?
	Do most kids in class have brown eyes?
	Are we getting better at skipping? or Can we get better at skipping?

Investigation 2

Choose something that has more than one way of getting an answer, for example:

1. How many adverts do they have on TV? Does it change for different programs?

2. What is the most popular car colour?

3. Who is the best player on a football team? [Class chooses team]

4. Which is the best class from the maths test?

Always predict to start and then follow Polya's four stages (SEE, PLAN, DO, CHECK).

1. To see, discuss the question so everyone is clear what has to be done and what has to be known or assumed to be able to tackle the question.

2. To make a plan, discuss what should be done, and what you have to collect and what you have to look up to follow the plan. Work out the sequence/order in which you will do things. Check if you've missed something, or that there is not another way.

3. To do, follow the plan and write a report, giving inferences.

4. To check, go back over what you have done, see if there is another way to solve it or another solution, try to highlight what you have learnt, and try to generalise what you have done to an extension of the question.

4.3 Development of inferential reasoning

This section covers the move from statistical literacy to statistical reasoning – it looks at the upper primary years. Statistical reasoning focuses on reasoning and making sense of data in context – it utilises data, graphs and statistics information to understand problems and situations.

4.3.1 Overview of section

As stated above, this section covers the move from literacy to reasoning. Its aim is to:

(a) reinforce students' understanding of statistical literacy and begin their movement to statistical reasoning;

(b) enable students to experience statistical processes before they learn more formal rules and formulae (this is considered crucial to lead to better statistical understandings in secondary school);

(c) facilitate students to begin to focus more on inference ("beginning inference") – covering variation, prediction, hypothesising and criticising; generalising beyond data, using data as evidence, and continuing to acknowledge uncertainty; and

(d) prepare students to move their understanding of variation and uncertainty from informal/intuitive to formal.

This section also aims to ensure students understand and can use a variety of representations of data. This is because statistical reasoning benefits from seeing connections between different representations of data, particularly when students move to new representations to better infer findings. In particular, understanding of inference improves as students move through the graphical forms of data used in inference as below:

unordered value plots

ordered tallies

ordered bar graphs (frequencies)
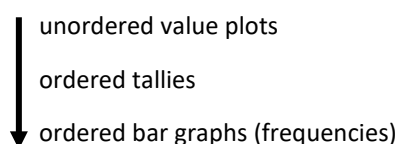
This section also covers building appropriate language, more formally introducing terms such as: outliers, error, most likely, centre, and so on; and introducing sampling and the relation of samples to population. In terms of practicalities, it is important to provide experiences such as: (a) carrying out surveys and recording data accurately; (b) working with frequency data and converting it into percentages; (c) appreciating and employing the process of making "informal inferences"; (d) learning and using the language of the process (pose, predict, sample, random, population, infer, conclude, decide, certainty); and (e) using random samples in collecting data and using the results from the samples to make decisions about a statistical question. The Australian Bureau of Statistics has websites with many useful data sets that students can use, sample and contribute to.

In particular, this section advocates the following.

1. **Challenge that randomness is determined**. Teachers need to **challenge** the common misconception that randomness is determined (e.g. "I never throw sixes"), especially with respect to life. Students need to be challenged to understand that throwing a die can give random numbers from 1 to 6; but they also need a stronger challenge to ensure that random events in life give all possibilities.

2. **Integrate contextual knowledge**. Give a variety of experiences of data and making inferences in particular contexts and show how the context affects decision-making.

3. **Give intuitive experiences first**. Teachers should always give experiences that build intuitive understanding before moving to formulae and to procedures (e.g. look at data intuitively for average before teaching the formula for mean).

4. **Teach purpose and aggregation**. Teach that graphs are a purposeful tool before stressing the features of good graphs, and aggregate data in the mind; that is, stress seeing data globally as a distribution not as a collection of points.

It is important to go beyond routine questions that relate directly to data and get the students to interpret information from data sets and graphs. Inference includes taking account of variation, predicting, hypothesising, and criticising. It has three components: generalising beyond the data, using data as evidence, and acknowledging uncertainty in the conclusion. There are three types of questions:

(a) from the data (answers can be read directly from the graph);

(b) between the data (involves comparing categories on the graph); and

(c) beyond the data (students infer reasons why or predict from the graph).

As a reversing activity students can be asked to suggest what questions may have been asked to generate the data in the graph. Sharing questions and responses can lead to significant engagement with the data and may suggest further avenues that students can explore in generating their own data collections.

4.3.2 Activities

1. **Softball throwing**

    (a) A class had to choose a representative for a softball throwing contest. Three students volunteered. Each volunteer was asked to make five throws which were measured with a trundle wheel to the nearest metre. The results were as shown on the table below.

Volunteers	Their 5 throws (to the nearest metre)
Rachel	28, 23, 22, 24, 27
Betty	24, 23, 27, 24, 27
Tony	23, 27, 29, 18, 26

    (b) Who would be the best representative? Who is the most consistent? Who has the longest throw?

    (c) What should our criteria be for selecting the best representative? Who has the best typical throw? How do we define typical? Is consistency important? Should we have measured more or less than five throws? Should bad throws be excluded? Is anything important lost in rounding to the nearest metre?

    (d) Develop an argument for your choice.

    Note 1: Students can be allowed to put information on tables, or to tally throws into sections, say 15–19, 20–24, 25–29, etc. if this helps them. Students can also graph the results and work out averages if this also helps. Encourage students to take into account the context (one big throw wins) and their analysis of data in their arguments for their representative.

Note 2: If students refine the way to represent the data in arriving at their inference (e.g. from a simple plot of points to a frequency bar graph), this is said to be an example of "transnumeration" (Wild & Pfannkuch, 1999) – "changing representations to engender understanding" (p. 227). This shows strong knowledge growth and increased understanding. Observe students to see if this happens.

2. **Canned food**

   (a) There are 23 brands of baked sausage in plum sauce. Sixteen use microwaves to cook the sausages and sauce, and seven use steam. The cans always display how they are cooked but stores always seem to have different brands or unbranded cans. To enable consumers to determine which type of can to buy, a consumer group tests all brands and marks them out of 20 on quality. The marks for each of the 23 brands are given in the table below.

Type of can	Marks for all brands of that type
Microwaved	7 4 18 5 4 12 9 3 4 3 16 9 5 13 7 18
Steamed	10 14 5 11 15 12 9

   (b) Which type, microwaved or steamed, is best to buy when there is no brand? Which type has the best mark? Which type is the most consistent?

   (c) Determine criteria for making a judgement. Develop an argument for your choice.

   Note: Again let students use any form of table, graph or determination of average that they think will help them. Check for the misconception that the type with the highest score is best even if other scores are low – in this context, it could be argued that one needs consistency as do not know what is being bought.

4.3.3   Investigations

Investigation 1

1. Choose an investigation like one of those in the Level B examples below. Try to make it relevant and motivating for your students.

2. Let the students work out their own way to tackle the question – discuss and reflect.

3. Use every opportunity to direct attention to and reinforce the movement from statistical literacy to statistical reasoning, and take every opportunity to meet the four challenges and use the three types of questions.

Level B investigations	What year level has the healthiest lunch?
	What is the shortest shadow?
	How much do we spend at the fete?
	What is a typical hand span?
	Is rolling your tongue hereditary?
	How many commercials do we watch?

Investigation 2

1. Choose a topic that has rich online data; for example, something from statistics that can be found on the Internet such as crime statistics, road safety statistics, weather statistics, sport statistics, TV ratings.

2. Pose a question or, better still, have a discussion and see what the students come up with (problem posing).

3. Put students into groups to investigate the question and infer and defend a conclusion.

Use the process from subsection 4.2.4 – PREDICT, SEE, PLAN, DO, CHECK – the plan now must focus on data to select.

4.4 Central tendency

This section continues to focus on statistical reasoning and introduces the mean, median and mode as another tool to facilitate the comparison of data sets and the process of inferring and predicting from data. This period is the time when students' understandings should be changed from informal to formal and from intuitive to using formulae. However, this move should follow understanding. This means spending time building intuitive understandings for mean as average, median as the middle one, and mode as the most common one. YDC would recommend allowing students to develop their own formulae before giving them the precise statistics formulae (this is the creative component of Abstraction in the RAMR cycle).

4.4.1 Overview of section

It is recommended that: (a) students be encouraged to predict before finding mean, median or mode and to reverse from mean-median-mode to data (as well as data to mean-median-mode); (b) teaching integrates statistical inference with probability (e.g. using experimental probability investigations as a basis for inferences); and (c) questioning keeps attempting to move students' understandings from literacy to reasoning to thinking. [*Note*: An activity that can be used here is the Monte Carlo method to find area of irregular shapes. To do this: (a) copy the shape onto a rectangular grid of Cartesian coordinates; (b) use random numbers to generate points and plot them; (c) count points in and out of the shape; and (d) use the percentage in the shape × total area of grid to give area of the shape.]

The formal definitions of mean, median and mode are introduced along with formulae and examples. Consider the following data set containing results in a mathematics test out of 20 for two groups of students (the Angels and the Aces).

Angels	7	8	10	9	5	7	11	5	10	7	9
Aces	8	7	14	10	8	9	7	16	8	15	8

1. **Mean**. The mean is the average of the scores. To calculate it, the scores are added and the result divided by the number of scores (i.e. mean = Σ score/number of scores). In the Angels and Aces example, the following means are evident:

    Angels   7+8+10+9+5+7+11+5+10+7+9 = 88   88÷11 = 8

    Aces   8+7+14+10+8+9+7+16+8+15+8 = 110   110÷11 = 10

2. **Median**. The median is the middle number when numbers are placed in order from lowest to highest. For an odd number of scores like here with 11 scores, the median is the 6th score. For an even number of scores (say 14), the median is the average of the middle two scores (for 14 numbers, this would be the 7th and 8th scores).

    Angels   5, 5, 7, 7, 7, 8, 9, 9, 10, 10, 11 – the 6th term is 8 – the median

    Aces   7, 7, 8, 8, 8, 8, 9, 10, 14, 15, 16 – the 6th term 8 – the median

3. **Mode**. The mode is the most commonly occurring score. In the example above, the mode for the Angels is 7, while the mode for the Aces is 8.

Once calculated, the measures of central tendency (mean-median-mode) and the data sets are applied back to reality. The questions are:

*   What do these measures tell us? How might they be useful?

*   What can be inferred from these measures? What relationships can be seen?

*   How representative of the class are the measures of central tendency?

4.4.2 Intuitive and process activities

Intuitive introductory activities

The best introductory activities are those that enable the students to intuitively understand mean, median and mode. Here are some examples. Choose one or more that are appropriate to your students. It is not necessary to do them all – only those that are needed.

1. **Straws or strips**. Cut 6–10 straws or strips of paper in various lengths. Ensure that there are more than one of some lengths. Give strips to another group. They put the strips side by side (shortest to longest) and:

    (a) find median (middle one) and mode (most common);

    (b) predict where they think mean will be, and work out an informal way to find mean through making lengths the same (cutting bits off the longest and adding to the shortest).

2. **Unifix**. Repeat the above using coloured unfix cubes – have sets of 5–7 colours and students make bar graphs by putting colours together. Then they move cubes around for mean (best to prepare Unifix so mean is a whole number).

3. **Packets of M&Ms or Smarties**. Give out packets, students sort into colours and make bar graph shortest to longest. Students find mode and median – then predict and move M&Ms/Smarties to get all same height bars for mean.

    Can move bars so in a line with a marker between bars – this shows that the mean is found by adding and dividing by number of colours (leads to formulae) – see below.

4. **Birthdays**. The above can be repeated with A4 sheets with names of students placed on a bar graph under months of their birthdays. This allows the paper sheets to be cut into fractions to get common length for the mean.

5. **Picture**. A picture of a row of people of different heights can give a visual image of average – it is the line in which tall can be added to short to get common height. Heights made with streamers is another introductory activity for mean.

Process activities

This section more formally introduces the three measures of central tendency, moving from data to formulae. They also look at the importance of allowing students to make up their own formulae before they are given the formal ones of mathematics.

Investigation 1: Data and centres

1. Pick a topic to gather data on, e.g. shoe sizes. Construct a physical graph from everybody's shoe sizes like the one below (each 0 represents a student).

					0				
					0				
				0	0				
				0	0	0			
			0	0	0	0			
			0	0	0	0			
			0	0	0	0		0	
	0		0	0	0	0	0	0	0
Shoe size:	3	4	5	6	7	8	9	10	11

2. List the shoe sizes above in order, e.g. 3, 5, 5, 5, 5, 6, 6, ... and so on. Calculate mean, median and mode. Repeat this for your data.

3. Record data on a frequency table as below for the data in (1). Add cumulative frequencies. Repeat this for your data.

Cumulative frequency (Cf)	1	1	5	11	19	24	25	27	28
Frequency (F)	1	0	4	6	8	5	1	2	1
Number (n)	3	4	5	6	7	8	9	10	11

4. Graph the data from the frequency table as a bar graph and the cumulative frequency as a line graph. Repeat this for your data.

5. Compare your data with data from (1). What is the difference in the mean, median and mode? If there is a difference, why? If there is not, why not?

6. Describe and justify from your class's data only (as the other is made up) what you would think a typical shoe size for a class at the same year level as yours would be. Do you need more data for this? Why?

Investigation 2: Building formulae

1. Repeat 1 to 4 from Investigation 1 for another set of data, e.g. hand length to nearest centimetre.

2. If you had to calculate the mean, median and mode from the frequency and cumulative frequency table, how would you do it? Try some ways to see if you arrive at the same mean, median and mode.

3. Here are some data in a table:

Cf	2	6	11	12	18	20	23	24
F	2	4	5	1	6	2	3	1
n	3	4	5	6	7	8	9	10

Calculate mean, median and mode from the table.

4. **Challenge**: In your group construct a formula for finding the following measures (if a formula is possible): (a) mean, (b) mode, and (c) median.

5. Compare your formulae with the formal formulae from your teacher. What is different? What is the same?

4.4.3 RAMR lesson: Measures of central tendency
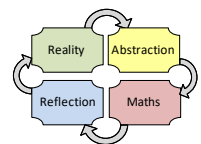
This is an example of a RAMR lesson for mean, mode and median.

Reality

*Determine from students their initial understanding of the term **average**. Discuss populations (Australian Bureau of Statistics website has much useful statistical data) as a basis of statistical information or find a relevant topic with statistical information for students to start with. Perhaps cricket run rates, bowling statistics of players, swimming times if near Olympic or Commonwealth Games. Discuss the uses of these statistics – how are they used, what information do they convey? This leads into need for calculations and interpretation of measures. Gather own personal statistics – height, mass, foot length, arm length … make three paper streamer replicas of these measures (not crepe streamers as these will stretch). Ensure students have their names on their streamers and a label for what measure it represents.*

Abstraction

Body

Students line up in order of height. Discuss what the average height might be. Have students consider what the middle value is. Find middle value by counting in from the ends. Create a chart of students' heights by securing

*streamers to a large flat surface (wall, floor, Maths Mat). Clearly label the middle value as **median**. (If there are even numbers of students the median will be halfway between the two central students' heights.)*

*Looking at the chart, have students identify any sections where students are the same height. Find the height that occurs most often. Discuss this measure as the **mode**. Label clearly.*

*Consider the tallest and shortest values, run a line of string across the chart from left to right across all streamers at the shortest and tallest values, discuss the difference between the two measures as the **range**.*

*Ask students what could be done to make everyone the same height. How could this be achieved? Some suggestions may be to take height off all taller people or add height to all shorter people. Have students consider how to make all the streamers the same length without having any streamer left over. This is time consuming but worthwhile to build a kinaesthetic understanding of the process behind calculating the **mean.** Students shorten longer streamers and add to shorter streamers to make them all the same height.*

*Make a new chart with an additional set of streamers. Mark across the mean with string or a drawn line. Discuss the differences between the mean and the actual measurements of height. This is the **mean deviation.***

Note: This activity can also be linked to graphing activities to discuss scaling and truncation of axes. Students can be challenged in groups to make their graph of heights fit on an A3 page while still representing their heights. Students usually tend to fold their heights in half or quarters – scaling. It is also useful to discuss the possibility of choosing a base height to work from and remove the section of streamer that is this long. What this shows is that scaling on a graph indicates the overall relationship between values in proportion; truncation displays the difference or the range more effectively but loses the overall relationship.

Hand

List students' heights on board. Link heights to scale on graph. Students make own graphs to represent heights. Mark in mean, median, mode as discovered. Record median and mode as numeric values. Discuss the actions taken in determining the mean. Explore ways of working with the values to make them all the same.

Mind

Look for strategies to simply calculate the mean. Consider patterns in students' explorations which lead to adding all values and divide by the number of values to determine the mean.

Mathematics

Discuss measures of central tendency discovered. Where might they be most useful and/or most appropriate? Explore other measures taken to further practice calculation of mean, median, mode, range, mean deviation. Explore relationships between mean and median. Investigate what happens to the mean and median if further values are added. This can be done by working through a second abstraction cycle using foot lengths and calculating the mean and median as lengths are added to the list (i.e. start with two values and calculate the mean and median [will be the same], add a value and recalculate, record changes, …). Discuss the effects of adding values to the mean and median.

Reflection

Generalise the effects of changes in the data set on the mean and median.

Explore ways of making the mean and median of a set of figures the same and different.

Explore the effect of skewed data on mean values. Reverse from a given set of values to indicate what the data set might look like.

4.4.4 Further mean, mode and median activities

Once again, it is not necessary to do all these activities. They give a variety of ideas – choose the activities that your students need. The first four reinforce a particular central tendency – the last one practises all three measures of central tendency (mean, mode and median).

Mean activity – Drawing instruments

Materials: Box or tray of writing instruments (students can use their own); strips of scrap paper, or students can tear these themselves; sticky tape.

Instructions:

1. Discuss **Mean** – simply the average of all the items in a sample. To compute a mean add up all the values and divide by the total number of items in the data set.

2. Do a hands-on activity with drawing instruments. Each student selects out two things that can be used to draw with (pens, pencils, coloured pencils, crayons, charcoal, etc.). Have the students be creative as to what can be used to draw.

3. Students work with a partner to cut a strip of paper to the length of each drawing instrument. Then, they tape the strips together and fold into four equal pieces.

4. Discuss why four? [four drawing instruments used so divide by four].

5. Students measure the length of these pieces and that is their mean.

6. Discuss how this is the same as what we do mathematically. Finally, apply it by finding the mean of the same data used in previous lessons.

Mode activity – Measuring smiles

Materials: String, tape measures, paper strips or streamers, calculators.

Instructions:

1. Discuss **Mode** – is the most frequently occurring value in the data set.

2. Working with a partner, students use a piece of string and a measuring tape to measure their own smile – rounded to the nearest centimetre.

3. Each student then cuts out a strip of paper streamer the length of their smile and writes their name on their strip as well as the length of their smile.

4. Create a column graph of smile lengths; the column with the most smiles in it is the mode.

5. As a class manipulate the smiles to discuss mean and median.

6. Introduce the notion of outlier data if you have any worthy smiles.

7. You can investigate metric conversions to millimetres and metres.

8. Students can then work together in small groups then as a class to tape their smiles together. Does your class smile stretch across the room?

Median activity – Finding what is typical

Materials: Unifix cubes.

Instructions:

For finding median, use Unifix cubes (but you could use stacks of non-interlocking as well) to show how many of something we have.

1. Discuss **Median** – is the middle number in a series of numbers, stated in order from least to greatest. If there is an even number of items in the data set, the median is the average of the two middle values.

2. Have class make a stack of cubes to represent:

    (a) how many people in your family

    (b) how many pencils in your desk

    (c) how many pets you have.

3. To find the median, line up in order from least to greatest (left to right) and find the exact middle person.

4. Discuss with the class how the middle person gives us information about what is typical for whichever numerical data we are investigating. Be sure to demonstrate examples using an even number of pieces of data and an odd number of pieces of data so students can see how that works.

5. After doing several of these types of lining up, counting to the middle, discussing that median shows us what's typical, then you can move to a paper and pencil method for finding median, always emphasising the importance of putting the data in order from least to greatest before finding the exact middle.

6. As far as connecting median to the real-world reality, think of some things that we make decisions about based on what we have come to believe is typical for certain situations. For example:

    (a) holiday plans may be based on typical weather patterns, temperature or rainfall;

    (b) expectations for sporting event outcomes may determine whether fans turn up to watch;

    (c) a coach looks at what is typical about an athlete, and then makes decisions on whether to use the player, when to send a player in or take one out.

One thing to remember is that median is just one kind of average – just one way to look at what's typical.

Median activity – Hat sizes

Materials: Tape measures, pencil, post-it notes or scrap paper.

Instructions:

1. Discuss **Median** – is the middle number in a series of numbers, stated in order from least to greatest. If there is an even number of items in the data set, the median is the average of the two middle values.

2. To create a **reality link**, use hat sizes. Pose a problem about a shop owner not knowing how many hats to order in each size. The students, in pairs, estimate and then measure each other's head size with tape measures.

3. The students agree on their head measures, and each student writes theirs on a post-it note or piece of paper.

4. Have the students line up from greatest to least. One from each side of the line can sit down until you reach the median.

5. You can then use the data to make a variety of graphs to show the information. Mean and mode can be calculated from the data.

6. As an independent activity, have students repeat hat activity but have the retailer selling sneakers. The students will need to understand the need to look for shoe size, order these from largest to smallest, eliminate from each end until median is found, graph etc. to find mean and mode.

Mean, mode and median card games

Materials: Deck of cards (1 [Aces] to 9 cards only), scrap paper, pencil, calculator (optional).

Number of players: 5

Instructions:

Review the definitions of these key terms with the class:

- **Mean** is simply the average of all the items in a sample. To compute a mean add up all the values and divide by the total number of items in the data set.

- **Median** is the middle number in a series of numbers, stated in order from least to greatest. If there is an even number of items in the data set, the median is the average of the two middle values.

- **Mode** is the most frequently occurring value in the data set.

Deal out 7 cards to each player. Ask each player to arrange their cards in sequential order. Aces count as the number 1. Then, depending upon which game you want to play, follow the directions below:

1. **Finding the Mean game.** Each player finds the total value of the digits on their cards, then divides the total by 7 (the total number of cards) to find the mean. For example, if the cards in your hand are Ace, 2, 4, 6, 8, 8, 9, then the sum of those digits is 38. Dividing the sum by 7 yields 5 (rounding to the nearest whole number). If this was your hand, you would have scored 5 points in this round. Because computation can be tricky without paper at this age, feel free to give your students a pencil and paper to find the mean. Or, to keep the game moving at a faster pace, you may allow use of a calculator.

2. **Finding the Median game.** Each player finds the median card in their hand and that number is their point value for that round. Thus, using the hand above, the median of the cards is 6, since it is the value of the middle card.

3. **Finding the Mode game.** Each player finds the mode in their hand of cards, which represents their point value for that round. If there is no mode, then they don't score any points in that round. However, if there are two modes (two numbers occur the same number of times), then the player snags the point values for both modes! In the example above, the mode would be 8, since it occurs most often.

The winner of each game is the first person who scores 21 points.

4.4.5 Investigations

Any group of people vary in their data, and many distributions are possible. What does this say for the numbers (mean, mode, and median) that we use to describe them? The following investigations involve constructing different distributions so we can look at this. They look at the question – what does mean, mode, and median tell us about a distribution?

It is not necessary to do all these investigations – simply do the one or the ones that are appropriate to your students.

Investigation 1: Answering a question

1. Choose an investigation like one of those in the Level C/D examples below. Try to make it relevant and motivating for your students. But make sure that the investigations move onto two or more uncertainties and other knowledges required.

2. Let the students work out their own way to tackle the question – discuss and reflect.

3. Use every opportunity to direct attention to and reinforce the outcomes for this section and utilise central tendencies as appropriate for your students.

- Do we eat healthy cereal?
- What is the best design for a loopy aeroplane?
- How long is 10 000 steps?
- Is it better to buy or make Chinese food?
- Does Barbie have human dimensions?
- What is reaction time?
- How far does an origami frog jump?

Investigation 2: Constructing distributions

1. Suppose a class had small and large students and no in-between sizes.

    (a) Construct a frequency table for shoe size for this class which gives the same mean as in section 4.4.2 *Process activities 1: Data and centres* but with no 6, 7, or 8 shoe sizes.

    (b) What happens to mean, median and mode?

    (c) Draw a frequency graph. How is it different to your original graph from section 4.4.2 *Process activities 1: Data and centres*?

    (d) Can you keep the median the same?

2. Suppose two giants joined the class with shoe sizes of 25 and 27.

    (a) Redo mean, median and mode for your data. What is the difference?

    (b) Draw a frequency graph. How is it different to your original graph?

    (c) How could you change the shoe sizes for the rest of the class so that the mean was reduced to the same number as in section 4.4.2 *Process activities 1: Data and centres*?

    (d) Can you make the median the same as in the process activity?

3. What if we had a lot of students with size 2 shoes?

    (a) Let's add five size 2 students and remove two from each of 6, 7, and 8. What happens to the mean, median and mode?

    (b) Draw a frequency graph. How is it different to your original graph from section 4.4.2 *Process activities 1: Data and centres*?

    (c) Can you increase the sizes of some of the other students so you get the same mean? What happens to mode and median?

4. The temperatures for a week were:

S	M	Tu	W	Th	F	S
23	24	23	22	23	24	22

    (a) What was mean, median and mode? Did you notice that all three are the same?

    (b) Can you get the mean to differentiate from the median and mode by changing some temperatures? Which ones and by how much? Can it be done with a simple change?

    (c) Modify the above to make a distribution with a low mean.

    (d) Modify the above to make a distribution with a high mean.

    (e) Draw the frequency graphs for (b), (c) and (d). How are they different?

Investigation 3: Understanding what mean-median-mode mean for a distribution

1. Prepare/find data of the following types:

(a)   symmetrical data (like normal curve) with low and high numbers matched and most numbers in the centre;

(b)   symmetrical data but with a hole in the middle – very little data in middle;

(c)   skewed low data – most data in low end;

(d)   skewed high – most data in high end; and

(e)   data with a few high outliers.

2.   Find mean, median and mode. Compare and contrast for different data sets. Draw conclusions regarding the effect of different distributions on mean-median-mode.

3.   What kind of data would have mean, median and mode the same? What would have mean and median the same but two modes? What would have mean higher than median, lower than median?

4.5   Data distribution

This section continues the move from literacy → reasoning → thinking in statistical inference. It does this by looking at range, quartiles, mean deviation and standard deviation. It also looks at two new graphing techniques of complex bar graphs and box plots which use range, mean/median and quartiles in their design. It continues discussion of distribution of data, using terms including *skewed*, *symmetric* and *bimodal*. Finally, it looks at sampling.

4.5.1   Overview of section

The first focus of this section is distributions, particularly in frequency graphs. It continues to look at the relationships of mean, median and mode with data but also introduces new distribution concepts of range, quartiles, mean and standard deviation, and their relationships to data. It also focuses on reversing, that is, constructing data to achieve given ranges, centres and deviations. Finally, it introduces new graphs based on distributions, that is, complex bar graphs showing range, and box and whisker graphs, introducing new language such as skewed and normal.

The second focus of this section is to begin to introduce sampling. To do this we look at gathering data from large populations and discuss informally how it would be done. We also introduce terms such as *sample* and *represents population*. Thus we discuss ways to have a sample more closely represent the population, introducing such ideas as random sampling, stratified random sampling, cluster sampling and multi-stage sampling (strata and clusters).

Note: This is where we need to go back and look at the earlier sections on challenging misconceptions and building understanding.

The new ideas raised in this section include the following.

1.   **Range**. In a list of data this is the difference between the greatest and least value. Consider the data for the Angels and Aces mathematics test shown in subsection 4.4.1:

   The range for the Angels is 11−5 = 6

   The range for the Aces is 16−7 = 9

2.   **Quartiles**. The four quartiles show the scores at which data is divided into four parts. For the Angels and Aces data, this is as follows (put data from smallest to largest):

   Angels   5   5 ⑦ 7   7 ⑧ 9   9 ⑩ 10   11 – quartiles are at 7, 8 and 10

   Aces   7   7 ⑧ 8   8 ⑧ 9   10 ⑭ 15   16 – quartiles are at 8, 8 and 14

Note: For continuous data, median is found by finding/calculating the score that is halfway (halving the number of scores) and the first and third quartiles are calculated by finding/calculating the scores that are ¼ and ¾ of the number of scores.

3. **Deviation**. This is the measure of difference between the scores and the mean. Early deviation looks at the average of the differences between score and mean (where differences are always positive) – the mean deviation. Deviation is a concept that is tied to the idea of centre, it measures the extent to which your data deviates from the centre or mean. As an example, the Angels and Aces data gives the following:

| **Angels** | 1, 0, 2, 1, 3, 1, 3, 3, 2, 1, 1 | sum is 18 | Mean deviation is 1.6 |
| **Aces** | 2, 3, 4, 0, 2, 1, 3, 6, 2, 5, 2 | sum is 30 | Mean deviation is 2.7 |

There are two matters that should be especially noted. The first is that statistical measures are only possible if data can be added, subtracted and divided. Scores from counting (interval data) are acceptable but scores denoting only the order (ordinal data) or simply the membership (category data) of categories are not acceptable. The second is the reason for calculating these statistical measures in the first place. This is that they enable a mass of data to be succinctly described. Reducing data to sets of representative figures enables comparison, inference and prediction to be more easily deduced from the data set.

Standard deviation is a measure of difference between score and mean which gives more weighting to large differences. This is because it averages the square root of the square of the differences between mean and scores. In the Angels and Aces example it is as follows:

| **Angels** | squares of differences: | 1, 0, 4, 1, 9, 1, 9, 9, 4, 1, 1 |
| | sum = 40 | standard deviation = $\sqrt{40}$ = 6.3 |

| **Aces** | squares of differences: | 4, 9, 16, 0, 4, 1, 9, 36, 4, 25, 4 |
| | sum = 112 | standard deviation = $\sqrt{112}$ = 10.6 |

Compare the standard deviation to the mean deviation. What are the differences? Why?

Now we have a complete set of centres and deviations, they can be used to compare data. Central tendency may appear to be a straightforward way of comparing two data sets to make inferences and predictions. However, the incidences of misrepresentation and misconceptions with statistics (see section 4.6) mean that it is important that students are encouraged to explore the use of misinformation in statistics for robust understanding and critical literacy in statistical situations in the real world. Rather than just learning these as a set of ideas and features to watch for it is more useful to challenge students to understand misrepresentation sufficiently that they can produce deliberately misleading statistics. This is much more engaging and will result in a stronger learning experience.

In the Angels/Aces example we can tabulate the measures as below for comparison.

	Range	**Mean**	**Median**	**Mode**	**Mean Deviation**	**Standard Deviation**
Angels	6	8	8	7	1.6	6.3
Aces	9	10	8	8	2.7	10.6

As can be seen, the scores of the Aces are more spread out than the scores of the Angels (this can be seen in both the range and the deviation) and are higher (this can be seen in the mean and mode), yet this is due to a few high scores as the median shows there is little difference in the bulk of the scores.

Thus, it is important to build intuitive understanding of measures such as mean or deviation so they can be applied back to the original question to analyse the trends shown in the data. This is applying the statistics back to reality and is the culminating activity for all data handling and analysis activities. Inferring from the measures

and predicting outcomes are important skills for decision-making and should involve discussion and debate. Written justifications for decisions using the measures calculated are also appropriate activities here.

4.5.2 Activity

Are you a deviant? Comparing hand spans

Materials: Pencil, tape measures or rulers, paper.

Instructions: Have students work in groups of four.

1. Record the hand span for every person in your group. Let the students decide if it matters if the left or right hand is used.

2. Students find the mean hand span for your group.

3. Make a dot plot of the results for your group. Write names or initials above the dots to identify each case. Mark the mean with a wedge (▲) below the number line.
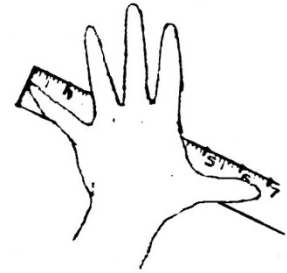
    For example:
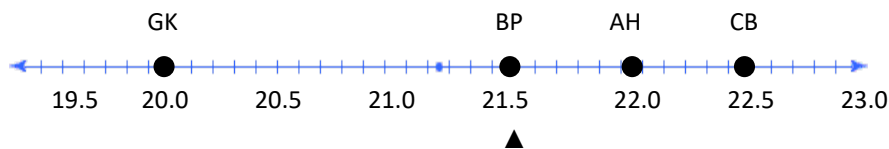
4. Students to discuss two reasons why the measurements are not all the same.

5. Students are to measure (either with ruler or calculate numerically) how far their hand span is from the mean of their group? How far from the mean are the hand spans of the others in your group?

6. Make a second dot plot. This time, plot the differences (deviations) from the mean. (Compute these by subtracting the mean from each observation, e.g. GK has hand span of 20.0, mean = 21.5. So 20.0 − 21.5 = −1.5). Again, label each dot with names or initials.

    For example:
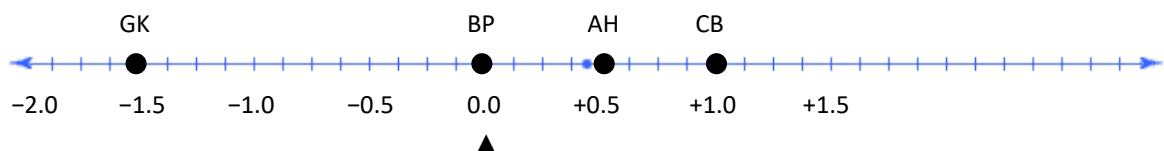
7. Explain how to get the second plot from the first without computing any differences.

    We can just shift the number line (think of the line being able to slide to the right) so that the wedge now indicates the zero point on the number line.

8. Using the idea of deviations from the mean, can you come up with a "typical" distance (deviation) from the mean? Explain.

    A "typical" distance from the mean seems to be around 0.75. We can get this by taking the average distances: (1.5 + 0.0 + 0.5 + 1.0) / 4 = 0.75. We do this by adding everything as if it is positive – this is the mean deviation.

Extension activity: Create some data in your class that can be plotted in a dot plot where multiple occurrences are found for a given value (e.g. number of hours of TV watched on Saturday). Find the mean and plot as per graph on right. By finding the mean distances of the horizontal lines from the mean, an approximate standard deviation can be found.
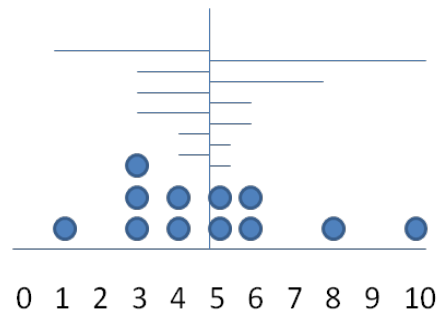
4.5.3 Distribution graphs

There are two types of graphs.

Complex bar graph showing range

Take data on temperatures for a week. One way to look at these would be to graph the maximum temperature on a bar graph. However, if we graph bars from minimum to maximum temperature (this will look like the graph on the right below) this shows maximum and minimum temperatures and the range of the temperature across the day.
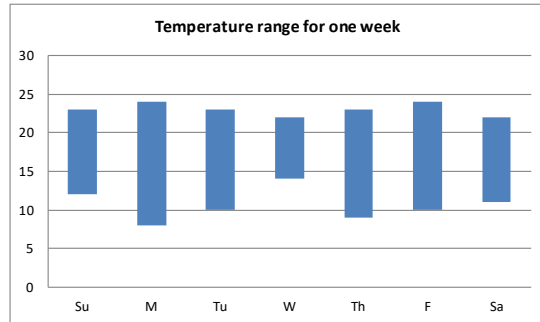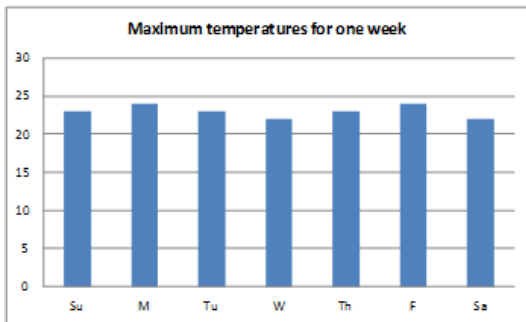
Activity

1. Collect shoe size data for four classes.

2. Average the data for each class and calculate low/high and range for each class.

3. Represent the average shoe size data on a bar graph for the four classes.

4. Represent the data with bars showing low to high and a line for average on a graph. The two graphs should be different as follows:
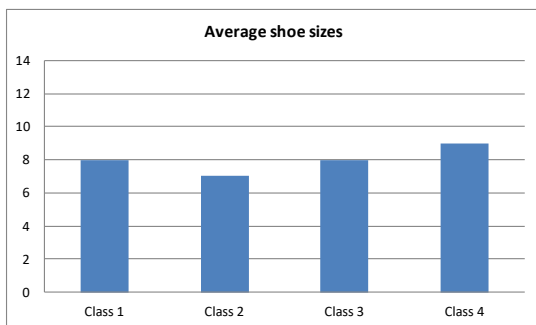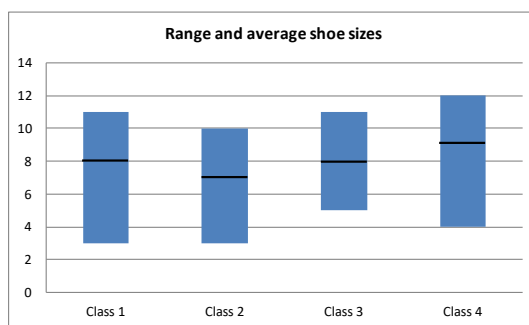
Simple bar graph   Complex bar graph showing top, bottom and mean

5. Which gives the best results in terms of simplicity? Which is best for inferences?

Box and whisker

Take data on shoe sizes for a class. Find median and divide into quartiles. Graph this as on the right:

Activity

1. Find data on temperatures for four months (find data from the Internet).

2. Calculate lowest and highest temperature and median and quartiles for each month.

3. Draw a box and whisker graph for each month, side by side (called stacked) – you should get a graph that looks like that on the right.

4. Draw a box and whisker graph for the three months combined.

5. Which is better? The stacked or the combined? Which is easier to make inferences from?
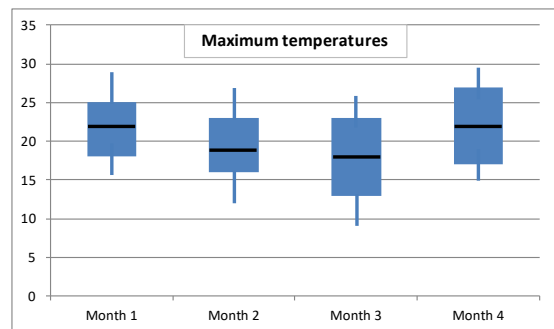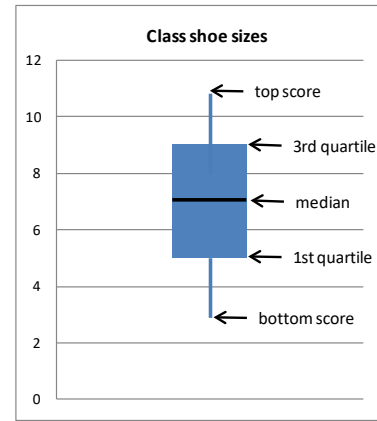
4.5.4 Sampling

In the real world, and sometimes in class, there are too many sources of data. Therefore, to get an idea of the data distribution, we take a sample. The important thing for accuracy is that the sample reflects the population. Thus, the sample that is considered most likely to do this is the random – or formed by chance. However, as it is chance, it still may not represent the population – but it is considered to have more chance of doing this than other ways.

Activity

1. Gather data from all the students in the class – say height.

2. Put names of students on pieces of paper, put pieces of paper in a bag, and select 10 at random.

3. Compare mean, mode, median, range and standard deviation of the 10 with the same measures for the whole class – were the 10 representative of the class?

There are four types of sampling:

1. Random sampling – sample chosen by chance.

2. Stratified random sampling – population broken into different characteristics which are given a percent representation, then random within each strata (e.g. may wish men and women to be the same percent – 50% for each – so randomly sample the same number from each strata).

3. Cluster sampling – population divided into clusters that are the same in some way, clusters are chosen randomly, but all people in the clusters chosen are surveyed.

4. Multi-stage sampling (strata and clusters) – combination of 2 and 3.

Activity

1. Choose something on which to survey all the students in the school.

2. Choose a sampling technique and implement it.

3. Analyse the data (mean, mode, median, range, deviation, and so on).

4. Infer findings from the population from your sample data. Do you think that the sample was accurate?

5. In elections, the whole population votes but to predict the election, polls use samples. Find out how they do this. Are they accurate?

4.5.5   Investigations

As stated before, any group of people vary in their data and many distributions are possible. What does this say for the numbers (mean, median, mode, range and deviation) that we use to describe them? The following tasks involve constructing different distributions so we can look at range and deviation. It looks at the question – What do mean deviation and standard deviation tell us about a distribution?

Investigation 1: Answering a question

1. Choose an investigation like one of those in the Level C/D examples below. Try to make it relevant and motivating for your students. But make sure that the investigations move onto two or more uncertainties, other knowledges required, and differences between types.

2. Let the students work out their own way to tackle the question – discuss and reflect.

3. Use every opportunity to direct attention to and reinforce the outcomes for this section (moving from reasoning to thinking, using measures of deviation, using complex bar or box and whisker graphs, and using sampling) as appropriate for your students.

Levels C and D Investigations	Do we eat healthy cereal?
	What is the best design for a loopy aeroplane?
	How long is 10,000 steps?
	Is it better to buy or make Chinese food?
	Does Barbie have human dimensions?
	What is reaction time?
	How far does an origami frog jump?

Investigation 2: Why we need standard deviation

1. Take all the data sets from subsection 4.4.2 *Process activities 1: Data and centres* and 4.4.5 *Investigation 2 Directions 1-3*. This is five sets of data. For all five distributions, calculate:

   (a) range;

   (b) mean deviation; and

   (c) standard deviation.

2. What do you notice about the mean deviation and the standard deviation? Was one distribution's deviation higher than the rest? Which distribution had this higher deviation? Why? Any interesting results in the other deviations? How did the mean and standard deviations relate to each other?

3. In a new study, you gathered data on shoe size and found that the mean was 13.

   (a) What does the distribution look like if the standard deviation is high?

   (b) What does the distribution look like if the standard deviation is low?

   (c) What does the distribution look like if the standard deviation is high and the mean deviation is low?

Investigation 3: Reversing

Provide students with means, medians, modes, ranges, mean and standard deviations and see if they can develop data for such measures.

4.6   Inferential misrepresentation

This section is different to the others in that it looks at how statistics can be used to misrepresent or "lie". This is a powerful way of reinforcing statistical inferential thinking, and also a powerful way to understand how persuasion may be given the credibility of mathematical correctness. The section looks at methods for misrepresentation and then at activities and investigations.

4.6.1   Overview of section

Ways of misrepresentation

Darrell Huff in his book *How to lie with statistics* (Gollancz, London, 1969 – now out in Penguin, 1991) describes a series of ways in which statistics can misrepresent.

1.   **Sample bias**

Statistics showing central tendency for a large population are nearly always based on a small sample. Such samples may not represent the population as a whole and so bias the statistic. For example, Oxendorf University may trumpet that its graduates of 10 years standing earn on average $145 165 per year. Is this really correct? A closer look at how this statistic was arrived at may provide insight.

It is likely that the information on salaries was collected by replies to an emailed survey. In this case, only those graduates whose email addresses were known and who bothered to reply were included in the result. Furthermore, the statistic was calculated not on their actual salary but on what they said their salary was. These factors mean that the statistics are open to problems of lying and bias towards more successful graduates, whose addresses are known and who have support staff to reply to the email (and who may inflate their salaries).

The statistics of average salary being $145 165 may also conceal large differences. What about deviation? The statement seems to imply that such a salary is what **every** graduate can expect!

We should always realise that all samples have a bias – towards people with more money, more education, more information and alertness, better clothing and more conventional and settled appearance – because these are the people who most interviewers feel more at ease with.

So, when faced with a survey result, say "how was the information collected?".

2.   **Wrong average**

In a factory or enterprise, there may be, for example:

- a manager/owner earning $900 000 per year;
- a partner earning $300 000 per year;
- two assistant managers earning $200 000 per year;
- a sales manager earning $114 000 per year;
- three sales people earning $100 000 per year;
- four information technology staff earning $74 000 per year;
- a foreperson earning $60 000 per year; and
- 12 workers/clerical staff earning $40 000 per year.

In this case, the **mode** (the wage/salary occurring most frequently) is $40 000 (the workers at the bottom of the range). The **median** (the wage/salary in the middle – 12 people earn more, 12 people earn less) is $60 000 per year (the foreperson), while the **mean** (the average) is $114 000 per year, but only 4 of the 25 people earn more than this. Depending on the data, mean, median and mode may be the same or differ widely. Where there is a large range of values which contains a few very large values and many close together low values (which is typical of income statistics), the mean is high and the mode low. The median is the best measure of centre.

So, when faced with an average, say "which average?".

3. **Missing information**

Statistically inadequate samples (small ones) can produce just about any result. Therefore if we ignore unfavourable samples, we can end up with an "independent laboratory test" certified by a "public accountant" proving just about anything. Four out of five people liking "Exo teeth liquorice" can be just that – groups of five people were asked if they liked "Exo" until one group was found where four out of five did. The 30 previous groups in which less than four liked "Exo" need not be considered.

The average alone can be misleading. A town with cold nights and hot days can end up with a delightful average temperature. We need information on range and deviation as well as average.

Words have different meanings to different people. What do statisticians mean when they say that "Tuffo cleans twice as bright"? Is this twice as bright as other cleaners or twice as bright as before cleaning?

4. **Irrelevant statistics**

Darrell Huff gives the old adage: "if you cannot prove what you want, demonstrate something else and pretend it is the same". Statistics about related matters are often used to support arguments for which there is no direct support. The statistic quoted may well be true but not for the situation to which it is directed.

For example, "laboratory controlled tests" may indeed show that "Basho" destroys 9 out of 10 germs when used in high concentrations in a test tube, but will it do anything in your mouth in dilute concentrations? Young people from 16 to 21 may indeed have more car accidents than the 50 to 55 age range, but this may be due to driving more. Accidents per person **per kilometres driven** may show that it is safer, for a 100 kilometre drive, to be with the young person!

5. **Direct misrepresentation**

Statistical data can be directly misrepresented. For example, juvenile delinquency figures can take a large jump when the courts change their recording procedures to count charges for group activities to each individual. Five youths stealing from a house can change from one offence to five break and enter offences, five being unlawfully on premises and five stealing offences (15 offences in all).

Percentages can make increases smaller or larger, depending on what you want, by choosing the appropriate base. Percentage increases can also look different to absolute increases. For example, someone taking a 50% pay cut from $800 to $400 per week would not be happy if they were told that the 50% would be returned but then that return was based on the $400, i.e. to $600. We would not think it right if our 50% rabbit burger was made by mixing one rabbit with one bullock.

4.6.2   Talking back to statistics

Darrell Huff recommends that we form a general impression of the argument and then ask the following five questions. Make these the basis of teaching.

1. **Who says so?** Look for bias, missing information, wrong measures, ambiguous statements and value-laden names (the prestigious university). What are the interests of the claimants?

2. **How do you know?** Look at how the statistic was calculated (the gathering of the data, sample size, type of data, calculations etc.).

3. **Is there anything missing?** Is there range and deviation as well as average? What average? Try to look past percentages to raw scores. Look to see if key words are properly defined. Be wary of value-laden labels. Look at the groupings and categories. How were these selected?

4. **Did someone change the subject?** Watch for the switch from data to conclusion. More reports of cases do not mean more cases! What people say they do may not be what they do. Watch comparisons. Are they between different things?

5. **Does it make sense?** Many a statistic is false on its face; the magic of numbers suspends belief. In particular, for example, be wary of the unnecessary decimal. Often for examples like the poverty line for a family of four, a number will be given to two decimal places. However, such a poverty line is near impossible to directly measure; it has to be calculated from estimates. But the use of the decimal at the end gives the impression that it is exactly calculated.

4.6.3   Activities

Introductory activities

Complete the following activities. Discuss Huff's misrepresentation types after the questions have been attempted and discussed.

1. Suppose a university wanted to show that its students earned more by going to university.

    (a) What data would they need to gather?

    (b) How would this gathering be done?

    (c) Does this lead to bias? How?

2. We have the following data:

    *   a manager/owner earning $900 000 per year;
    *   a partner earning $300 000 per year;
    *   two assistant managers earning $200 000 per year;
    *   a sales manager earning $114 000 per year;
    *   three sales people earning $100 000 per year;
    *   four information technology staff earning $74 000 per year;
    *   a foreperson earning $60 000 per year; and
    *   12 workers/clerical staff earning $40 000 per year.

    (a) Calculate the mean, median and mode.

    (b) What do you notice?

    (c) How does this come about?

    (d) What is the best of the three measures (mean, mode, median) for giving the centre of the data?

    (e) What is wrong with the average or mean as the number giving the middle?

3. Answer the following statistical questions.

    (a) What does an advertisement mean when it says there is evidence that Tuffo cleans twice as bright? [Twice as bright as what?]

    (b) What does the Tuffo advertisement mean when it says four out of five housewives recommend Tuffo? [Does it mean 80% of all the housewives?]

    (c) The disinfectant ingredient in Basho cleaner has been shown in laboratories to kill 99% of all bacteria. What does this mean for Basho coming out of the spray can onto the kitchen table? [Does spray can Basho have the same concentrations as Basho's ingredients in the laboratory?]

    (d) Do statistics showing drivers aged 22–30 have more accidents mean that they are worse drivers than drivers aged 52–60? Why or why not?

(e) Is it a 50% rabbit burger if you mix the meat of one rabbit with the meat of one cow? Is there any way this could be looked at as true?

4.6.4 Recognising misrepresentations

Refer to the five misrepresentations of Huff. Classify each of the following as one of the misrepresentation types. Then look at them in detail and answer the questions supplied.

1. Johnny threw a coin five times. He got a head only once. "That convinces me," said Johnny, "from now on, I'm always going to call tails!"

    (a) What is wrong with Johnny's conclusion?

    (b) What could a teacher do to show Johnny the incorrectness of his conclusion?

    (c) What does this say about the teacher who says that MAB are useless because when she tried them on some students last year, they did not work?

2. Bread rose from 80 cents to $1.20, milk dropped from $1.00 to 60 cents. "This is straightforward," said June, "bread has increased 50% and milk has decreased 40%, we have had a rise in the cost of living!" "No!" said Janette, "bread has increased 33 $\frac{1}{3}$ % and milk dropped 66 $\frac{2}{3}$ %, we have had a reduction in the cost of living."

    (a) Both are right in their percentages but how?

    (b) What has really happened to the cost of living, based on the cost of these two items?

    (c) How is this possible?

3. "The average family of four requires $673.86 per week to survive", states the report.

    (a) How is such a statistic calculated – from what information?

    (b) Is the decimal justified?

    (c) Why is it nearly always used?

    (d) Does this mean that all families of four can survive on this amount?

4. The principal says that the class has a mathematics average of 6 out of 10.

    (a) What could this mean?

    (b) Is it useful information on its own?

    (c) Does it mean that over half the students passed?

5. Before she used "Mucho" hair lotion, the lady is pictured dowdy, lank and sad. Afterwards, she is pictured smiling with bouncy and glistening hair.

    (a) What do these pictures tell us about "Mucho"? Anything of value?

    (b) Why do such pictures continue to be used?

6. The factory has an increased wages bill of 10%, an increased advertising bill of 10% and increased material costs of 10%.

    (a) Are they justified in raising their prices by 30%?

    (b) Why/why not?

4.6.5 Investigations

1. **"Against the assertion" poster**. The world is full of assertions and sayings. For example, "there is more crime now than in the past"; "a stitch in time saves nine"; "the unemployed are dole bludgers"; "many hands make light work"; "people are less friendly than in the past".

   (a) Choose an assertion that most people agree with. Gather data regarding the assertion, represent the data and draw inferences from the data, to make a poster that disagrees with the assertion.

   (b) Look at Huff's five ways of misrepresenting. For example – gather data from a sample which is biased by choice of sample or choice of interviewer or statements in the interview; reinterpret data so that there is a different result (e.g. look at accidents per km driven not total accidents, or look at crimes per person not crimes on their own); gather data on something that looks like it is the same but is not, but can act as if it is; or use percentages of centres differently.

   (c) Construct the poster. Try to be convincing in data and arguments. Make it really attractive and appealing.

   (d) Did you find that there is some data that cannot be misrepresented easily? Why?

2. **"Both sides" posters**. Prepare a double poster display for an assertion as below.

   (a) Choose an assertion (e.g. Young drivers are more dangerous than old drivers) that is suitable for (b) below.

   (b) Gather data and prepare two side-by-side posters, one presenting data and drawing inferences in a way that supports the assertion, and the other presenting data and drawing inferences in a way that rejects the assertion (make the posters attractive and appealing on both sides, with graphs and headings and so on).

   (c) Think of opposing ideas and how you could be able to find opposing data – remember that there may need to be a different way of looking at the data. Think of different ways your data could be biased and your graphs look better in their support of bias. Try not to simply repeat the idea in 1 above.

   (d) Display your posters side by side.

3. **"Paradox" poster**. There is a paradox in percent in that two sets of data can show a reduction yet the combined data show an increase. For example, there could be two groups – in the first, most prefer A and in the second most also prefer A; however, when put together, the combined data shows most prefer B.

   (a) Find this paradox. Describe it.

   (b) Why is it possible? Represent it on a poster.

5   Teaching Framework for Statistics & Probability

The teaching framework organises the content for statistics and probability into a framework of three topics: tables and graphs, probability, and statistical inference. Each of these topics is partitioned into sub-topics, chosen so as to represent ideas that recur across all year levels. The resulting framework is given in Table 1. This overall framework can be compared to the Australian Curriculum to produce year-level frameworks.

Table 1. *Framework for teaching statistics and probability*

TOPIC	SUB-TOPIC	DESCRIPTION AND CONCEPTS/STRATEGIES/WAYS	BIG IDEAS
Tables and graphs	Data gathering and organisation	Tables, charts and maps	Interpretation vs construction; Matrix structure; Notion of unit
	Data comparison	Venn and two-way diagrams; picture graphs; bar graphs; complex graphs	Accuracy vs exactness; Interpretation vs construction; Matrix structure; Notion of unit; Scale; Multiplicative structure; Complex/Inferential thinking
	Frequencies	Histograms; two-sided graphs; stem and leaf graphs	
	Trends and relationships	Line graphs; circle graphs; scattergrams	
	Misrepresentation	Using graphs to misrepresent data ("how to lie with graphs")	Interpretation vs const.; Scale; Efficacy of models and simulations; Complex/Inferential thinking
Probability	Perceiving and identifying attribute	Classifying an event as impossible, possible or certain	Chance vs certainty; Interpretation vs construction; Notion of unit; Scale; Variation and uncertainty; Formulae; Complex thinking
	Comparing and ordering likelihood of outcomes	Listing all outcomes; listing all outcomes in consecutive trials; stating which outcome is most likely	
	Comparing and ordering situation for desired outcome	Correctly choosing in what situation an event is more/most likely	Chance vs certainty; Accuracy vs exactness; Interpretation vs construction; Multiplicative structure; Centrality of context; Integration of information; Relation between sample and population; Formulae; Complex thinking
	Measuring probabilities	Assigning a numerical probability to an event; probability as a fraction; estimating numerical probability using simulation	
	Applying probabilities in inferences	Using probability to make informed decisions and predictions	Interpretation vs construction; Notion of unit; Mult. structure; Variation and uncertainty; Centrality of context; Relation between sample and population; Formulae; Complex/Inferential thinking
Statistical inference	Early inference Early statistical literacy	Making decisions and ensuring basics; posing problems and devising data; building complexity	Chance vs certainty; Variation and uncertainty; Centrality of context; Beginning complex and inferential thinking
	Development of inferential reasoning	Utility and purpose of tools; learning to interpret, critique, debate and judge	
	Central tendency (mean, mode, median and using formulae)	Mean, mode and median; reasoning and making sense of data; using graphs and other models; experiencing statistical processes; beginning inference; informal → formal	Interpretation vs construction; Variation and uncertainty; Centrality of context; Integration of information; Efficacy of models and simulations; Formulae
	Data distribution (move to statistical thinking and formulae)	Range and quartiles; mean deviation and standard deviation; statistical thinking and central tendency; samples and populations	Inter. vs const.; Mult. structure; Variation/uncertainty; Integration of information; Relation between sample and population; Efficacy of models/simulations; Formulae; Data driven/Complex thinking; Evidence-based/Inferential thinking
	Inferential misrepresentation (reinforcing thinking)	Sample bias; wrong average; misusing information; irrelevant statistics; direct misrepresentation; talking back to statistics	

Appendix A: Types of Tables and Graphs

Tables

The simple table (list)

This is represented by the table of contents on a cereal packet or the list of prices at a shop. It consists of words and figures in two columns. An example is:

Table 2. *Ruffy's Corn Cereal*

Average contents per serving:	
Vitamin C	25 mg
Iron	17 mg
Niacin	11 mg
Riboflavin	38 mg

Another simple table which is useful to link to probability activities is the frequency table. Frequency tables list the sample space (possible outcomes) of an event in a table with the number of combinations that lead to the outcome listed underneath. Frequency tables can be converted to a single continuous strip of boxes with each outcome coloured suitably, similar to a stacked bar graph. This can be curved in a circle and used to create early circle graphs or combined with hundreds beads to convert to percentages.

The regular table

This is the matrix style table where there are two or more columns of data. For example:

Table 3. *Materials collected by the students in 3Z*

	Dan	Joe	Fred	Sue	Anne
Milk bottle tops	5	7	4	11	2
Cotton reels	2	9	12	4	3
Orange juice bottles	5	2	6	2	11
Egg cartons	15	5	4	9	10

The irregular table

This is similar to the matrix table above but not all rows are the same (e.g. a class seating plan).

Table 4. *Seating plan of our classroom*

	Table 1	Table 2	Table 3	Table 4	Table 5	Table 6
A	Joe	Fred	Sue	Joan	Bill	Alex
B	Abel	____	Jill	Judy	Bob	____
C	Anita	____	Greg	Tom	Betty	Brigit
D	Derek	Ted	John	____	____	____
E	Nick	Chris	Peter	____	____	____
F	Frank	Fran	Greta	____	____	____

Charts

Charts attempt to display information more visually, to relate the display to what actually occurs (e.g. road maps, bus routes, timelines of history).

The strip map

This could be the bus route of an area, the major highway route of a journey or the timeline of a history topic. A line is drawn and on this line are marked references to major features (e.g. bridges, towns or happenings). Two examples are:

Bus route

My Saturday

| ------------------ | ------------------- | ------------------ | ------------------ | --------- |

| *Breakfast* | *Shopping with Mum* | *Lunch* | *Play with friends* | *Dinner* |
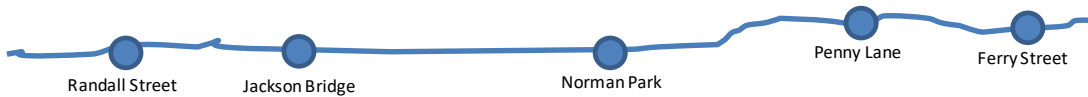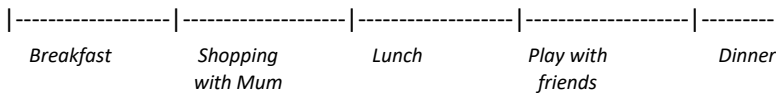
The branch map

This is a combination of strip maps, involving branching as in a tree. The most straightforward examples are genealogy diagrams (family tree of parents, grandparents, etc.). An example of a branch map is:

Mary Watson's family tree
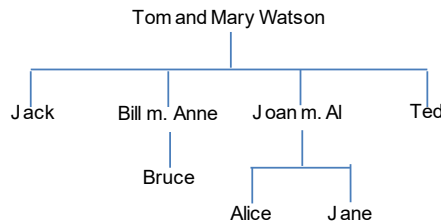
Venn and Carroll or Two-way diagrams

These are visual ways to represent membership in different sets and subsets. An example is:
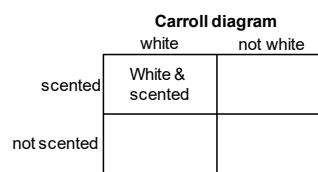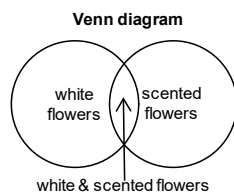
Graphs

Picture graphs (pictographs)

Picture graphs facilitate comparisons of quantities. They can represent the data with one-to-one or one-to-many correspondence. It is important to ensure that students understand the need for picture graphs to use the same size images to represent each piece of data and that pictures must be consistently spaced. These are the beginning understandings for scale and proportion and are most easily accomplished by giving students same-sized pieces of paper to create their initial graphs. Picture graphs are easily converted to bar graphs.
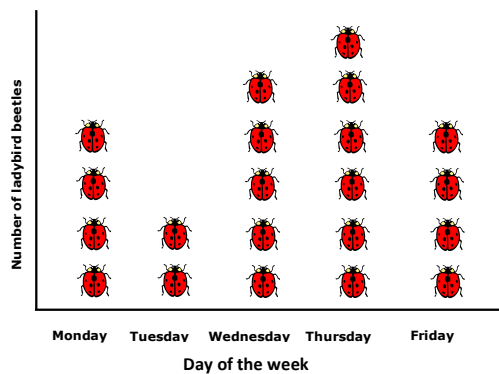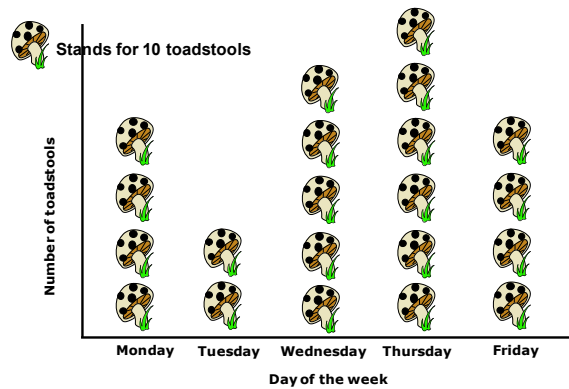
Picture graphs showing one-to-one and one-to-many correspondence

Bar graphs

Bar graphs are easily created initially with blocks and sticky notes which can then be transferred to squared paper before working with abstractly ruled scales. Bar graphs facilitate comparisons of quantities and can be displayed with vertical bars (also called columns), horizontal bars, or bar lines. Below is an example of each.
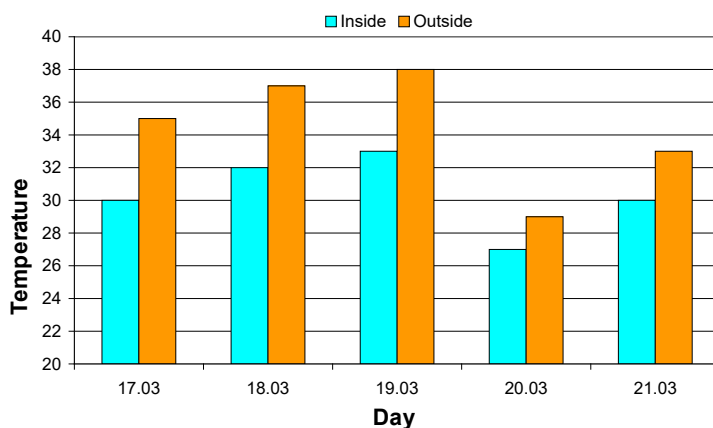
Inside and outside temperatures collected in 1 school week.

Example 1: Vertical bar or column graph
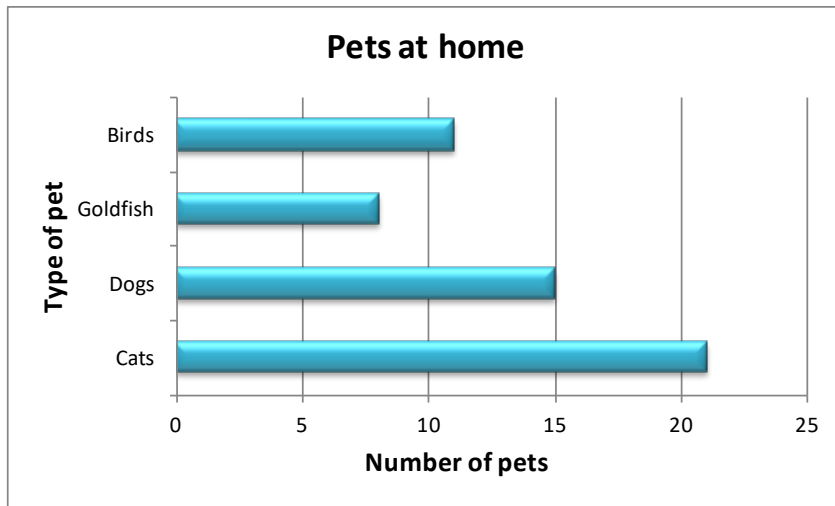
Example 2: Horizontal bar graph
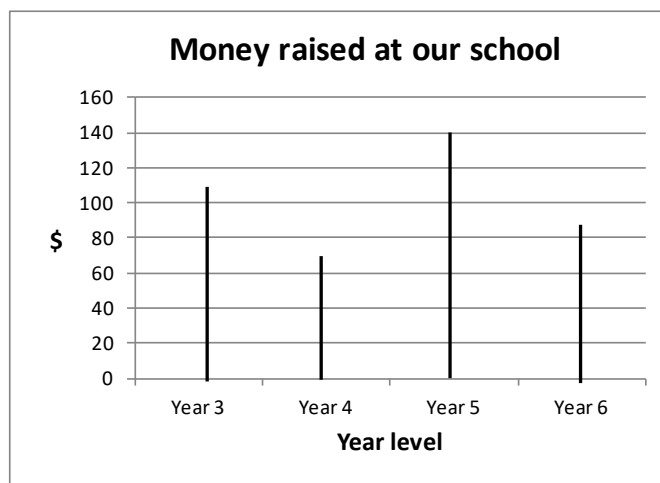
Example 3: Bar line graph

Line graphs

Line graphs can be used for comparison and for expressing allocations of resources, but they are particularly useful for communicating trends. Line graphs are also useful for representing continuous data.
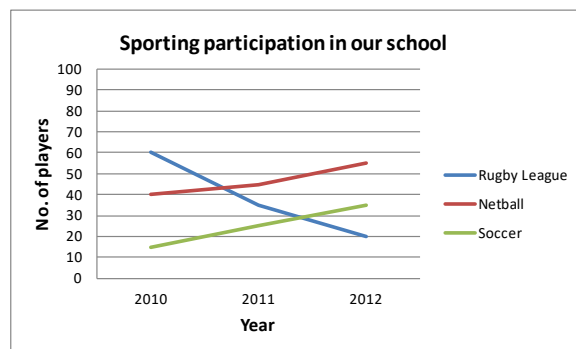
Circle graphs (pie charts)

Circle graphs are used to picture the totality of a quantity and to indicate how portions of that totality are allocated. The example on right is a circle graph indicating how one college student spent his budget.
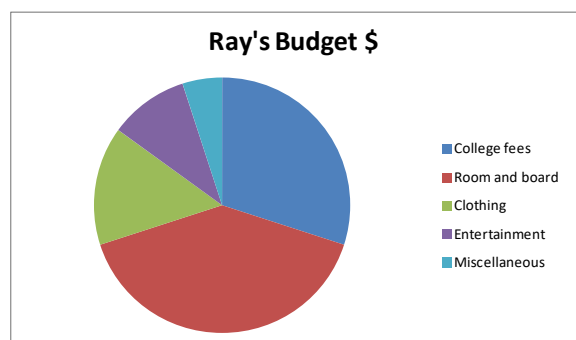
Circle graphs are in some instances made on shapes other than circles. For example:

Where the egg money goes

Where the pork money goes
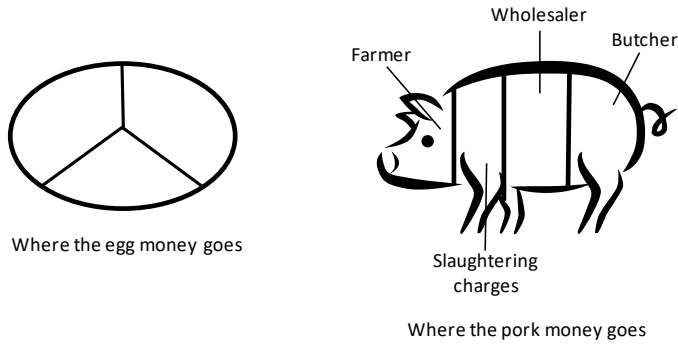
To successfully engage with circle graphs students need to understand Part-Part-Whole relationships and that the complete shape represents the whole while the segments represent the parts of the whole. Circle graphs link very closely with fraction and percent understandings.

Scattergrams

Scattergrams show relationships between two different sets of data. The scattergram is made for data which is not in sequence (in terms of the horizontal axis) and is unsuitable for a line graph. Here is a scattergram which shows that mass is related to height.
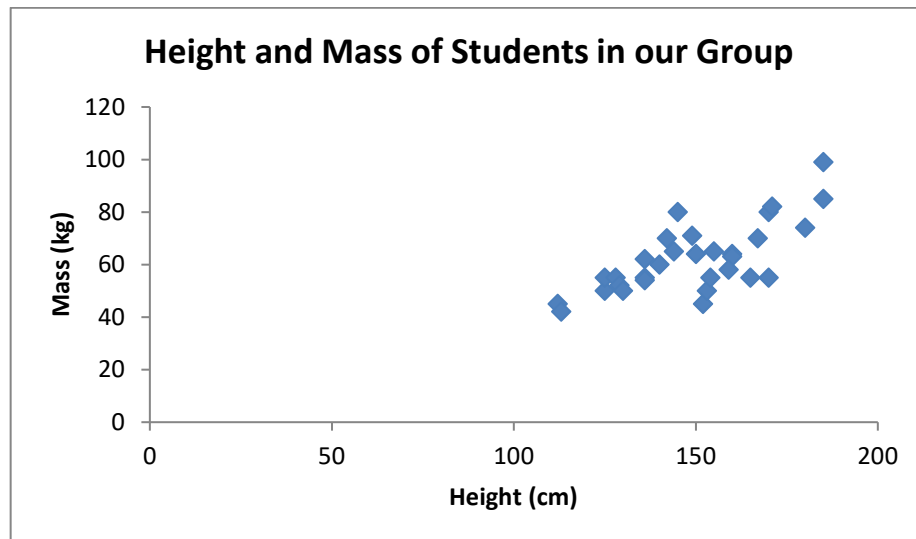
Overall features of graphs

A graph should have the following components:

1. Title

2. Vertical
   (a) Axis
   (b) Scale
   (c) Title
   (d) Units

3. Horizontal
   (a) Axis
   (b) Scale
   (c) Title
   (d) Units (if relevant)
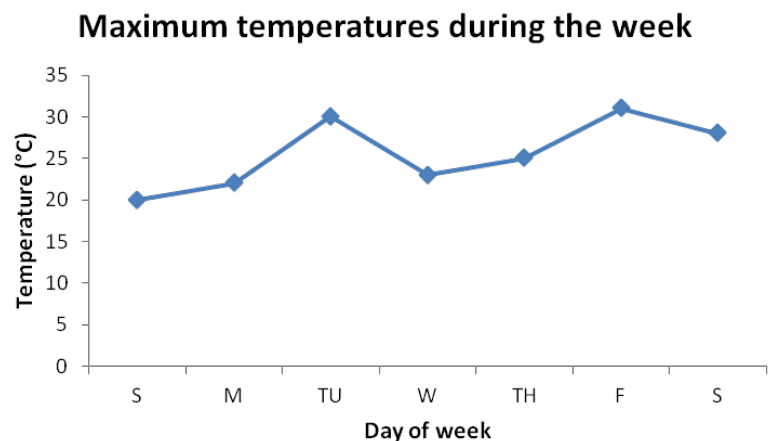
Appendix B: Extra Material for Tables and Graphs

B1   Data grouping activity

Data for activity:

Person 1 Gender: F Eye colour: Blue L/R handed: R	Person 2 Gender: F Eye colour: Brown L/R handed: R	Person 3 Gender: M Eye colour: Brown L/R handed: R	Person 4 Gender: F Eye colour: Green L/R handed: R
Person 5 Gender: M Eye colour: Blue L/R handed: R	Person 6 Gender: F Eye colour: Brown L/R handed: R	Person 7 Gender: F Eye colour: Green L/R handed: R	Person 8 Gender: F Eye colour: Blue L/R handed: R
Person 9 Gender: F Eye colour: Brown L/R handed: R	Person 10 Gender: M Eye colour: Brown L/R handed: R	Person 11 Gender: F Eye colour: Green L/R handed: R	Person 12 Gender: M Eye colour: Green L/R handed: R
Person 13 Gender: F Eye colour: Blue L/R handed: L	Person 14 Gender: F Eye colour: Brown L/R handed: R	Person 15 Gender: F Eye colour: Green L/R handed: R	Person 16 Gender: F Eye colour: Blue L/R handed: L
Person 17 Gender: F Eye colour: Brown L/R handed: L	Person 18 Gender: F Eye colour: Blue L/R handed: R	Person 19 Gender: M Eye colour: Blue L/R handed: R	Person 20 Gender: M Eye colour: Brown L/R handed: L

ACTIVITY

- Cut out the squares. Keep the frame.
- Group them any way you like (be prepared to explain your grouping).
- Count the number in each group.
- Repeat above, gathering your own data using blanks on next page – compare the two sets of data.

Blank cards for data gathering

Person 1	Person 2	Person 3	Person 4
Gender:	Gender:	Gender:	Gender:
Eye colour:	Eye colour:	Eye colour:	Eye colour:
L/R handed:	L/R handed:	L/R handed:	L/R handed:
Person 5	Person 6	Person 7	Person 8
Gender:	Gender:	Gender:	Gender:
Eye colour:	Eye colour:	Eye colour:	Eye colour:
L/R handed:	L/R handed:	L/R handed:	L/R handed:
Person 9	Person 10	Person 11	Person 12
Gender:	Gender:	Gender:	Gender:
Eye colour:	Eye colour:	Eye colour:	Eye colour:
L/R handed:	L/R handed:	L/R handed:	L/R handed:
Person 13	Person 14	Person 15	Person 16
Gender:	Gender:	Gender:	Gender:
Eye colour:	Eye colour:	Eye colour:	Eye colour:
L/R handed:	L/R handed:	L/R handed:	L/R handed:
Person 17	Person 18	Person 19	Person 20
Gender:	Gender:	Gender:	Gender:
Eye colour:	Eye colour:	Eye colour:	Eye colour:
L/R handed:	L/R handed:	L/R handed:	L/R handed:
Person 21	Person 22	Person 23	Person 24
Gender:	Gender:	Gender:	Gender:
Eye colour:	Eye colour:	Eye colour:	Eye colour:
L/R handed:	L/R handed:	L/R handed:	L/R handed:

B2   Timetables

Constructing a timetable

The best way to understand timetables is to construct one. For this, you have to consider:

1. **Activities** – this is the focus of the timetable, for example, trains, buses, classes in a school, appointments. These are usually down the side of the table.

2. **Places** – this is where the activities occur or may stop, for example, trains stop at stations; appointments occur in numbered surgery rooms. These are usually across the top of the table (but may not be, sometimes timetables mix things up).

3. **Times** – these are what are in the squares or cells of the table – the times at which the activities occur at a certain place.

Thus a timetable is as follows:

To teach how to construct a timetable, think of a timetable situation. Try to choose something different. For example, when bands are playing at different venues, get students to create their own personal weekly or monthly "gig guide".

Four ways to read a timetable

Consider the following timetable for trains – columns are stations, rows are trains, and cells are times.

Train	Start	Jackin	Karlin	Mont	Nanty	Ooptan
1	9:16 am	9:47 am	10:23 am	11:26 am	11:58 am	12:22 pm
2	9:45 am		10:43 am		12:06 pm	12:42 pm
3	10:07 am	10:39 am		12:09 pm		1:35 pm
4	10:54 am	11:28 am	12:03 pm	1:01 pm	1:42 pm	2:06 pm

1. **Way 1**: Finding information in a cell (here, finding a time) – to do this, look across → and down ↓ . For example: to find when train 2 gets to Karlin, we start at train 2 and move across to Karlin's column. Then circle what time appears in that box – 10:43 am.

Train	Start	Jackin	Karlin	Mont	Nanty	Ooptan
1						
2			10:43 am			
3						
4						

2. **Way 2**: Finding which row to use (here, finding a train) – to do this, look for the station and then go down for the time and across for the train. For example: if you need to get to Nanty just before 12:30 pm, you start at Nanty, and look down until you get to 12:06 pm, then across to the left for the train. Then you can circle which train number is needed.

Train	Start	Jackin	Karlin	Mont	Nanty	Ooptan
1						
2					12:06 pm	
3						
4						

3. **Way 3**: Finding which column to use (here, finding a station) – to do this, find the train required, go across to the time and look up for the station. For example, to find which station you would have to get off from train 3 that was just after 12.00 pm, we start at train 3 and move across to a train near 12:00 pm, then move up to the station. Then circle the station you need to get off at.

Train	Start	Jackin	Karlin	Mont	Nanty	Ooptan
1						
2						
3				12:09 pm		
4						

4. **Way 4**: Finding things that have no answers: (here some cells are blank) – to do this, you have to understand that sometimes tables have blank spaces because, here, the train does not travel to a certain station. For example, train 2 does not stop at Jackin. To find these things out, look for gaps and empty spaces.

B3   Teaching principles of good picture graphs

1. Construct a stage 2 / stage 3 bar graph as described below.

Type of Pet	Number of students with that pet
Dog	9
Cat	7
Fish	3
Bird	5

2. Construct a poster framework as below:

PETS WE HAVE	
Dogs	
Cats	
Fish	
Birds	

3. Cut out 24 rectangles of paper of the same height (a height to fit in the rows of your poster) but of the length so that 17 are squares and 7 are rectangles where the length is 1.5 times this height.

4. Represent the data on the pets using these squares/rectangles of paper (by placing a "D" on 9 squares for the dogs, an "F" on 3 squares for the fish, a "B" on 5 squares for the birds and a "C" on the 7 rectangles for the cats).

5. Paste the squares/rectangles appropriately on the poster as follows:

    (a) Place the dogs end to end neatly, starting on the left of the poster, e.g.
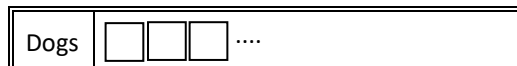
    (b) Place the cats also end to end neatly, starting on the left of the poster, e.g.
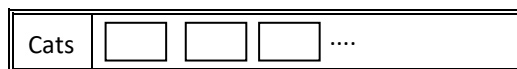
    (c) Place the birds end to end with a gap between (still starting from the left) so that they extend to the right further than the dogs, e.g.
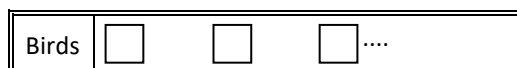
    (d) Place the fish end to end neatly but starting sufficiently to the right that it extends longer than the dogs, e.g.
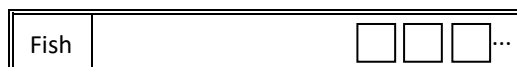
B4   Transition from picture graph to abstract bar graph

Simple picture graphs where one symbol represents one item:
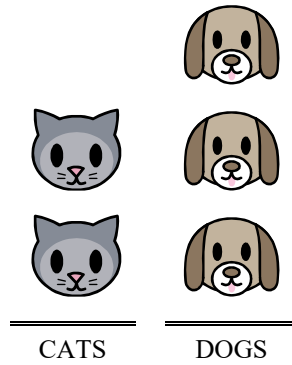
Can have their symbols replaced by squares:
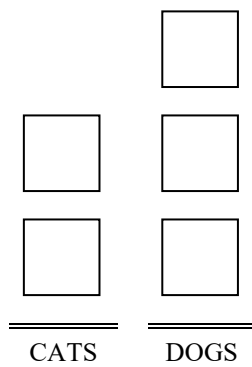
Then we can replace constructing rows or columns of such squares with the shading in of squares on graph paper:
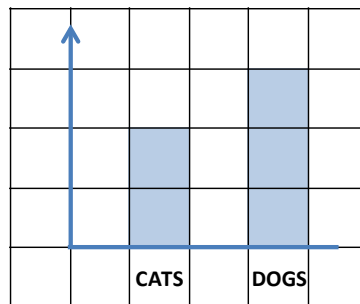
And, finally, remove the graph paper and just use bars:
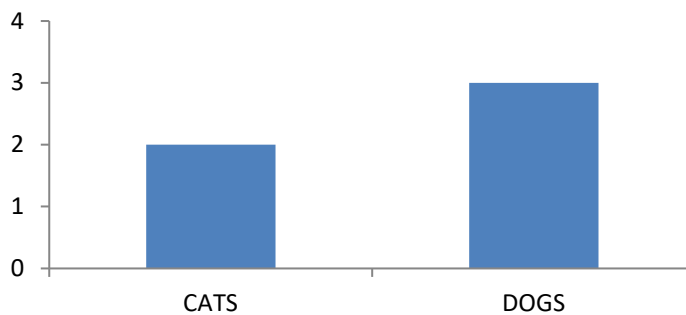
B5   Improving picture and bar graph drawings

Children

Children who live in Boondall and Nundah

our shoe sizes
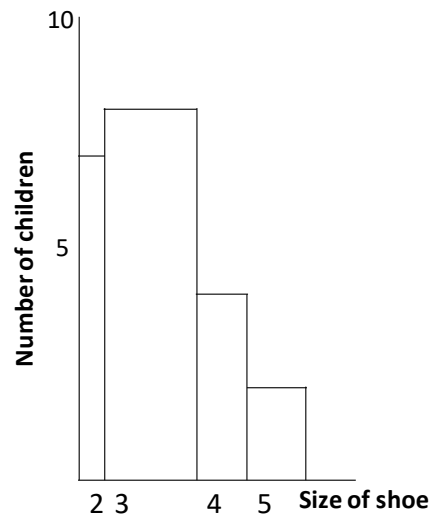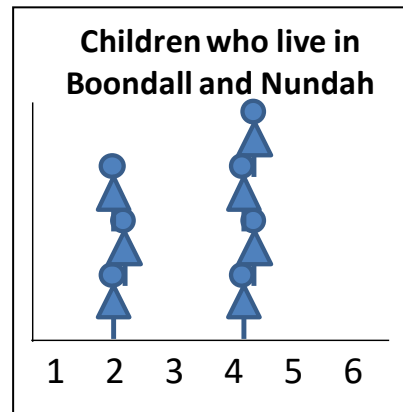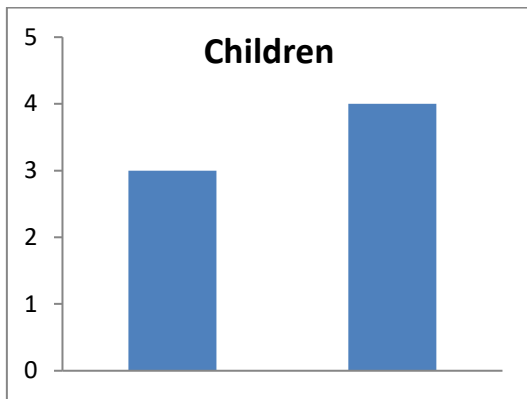
Colours of cars that passed our school

= 5 cars

RED   BLUE
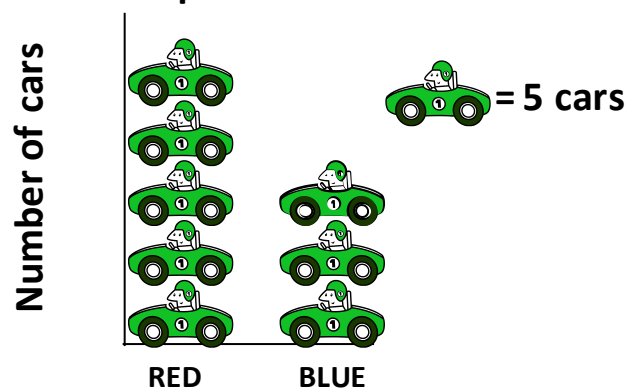
B6 Reading and interpreting graphs

For each of the following graphs, answer the questions listed below them.

1. **Bar Graph.** Students in a Year 4 class constructed the following rainfall graph for one school year.
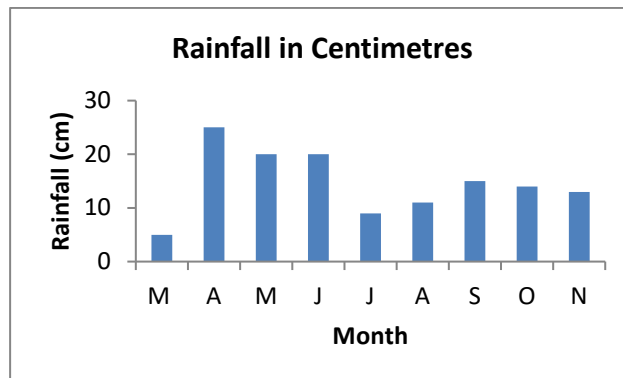
(a) Which month had the most rain?

(b) What were the approximate amounts of rainfall in March, April and May?

(c) What part of the school year would you call the rainy season?

(d) About what percentage of the total rainfall fell in November?

2. **Line Graph.** As part of a program to improve her serve in tennis (where she had a tendency to double fault), Mary decided to make a graph of her daily serving success. Below are the results for 10 days.
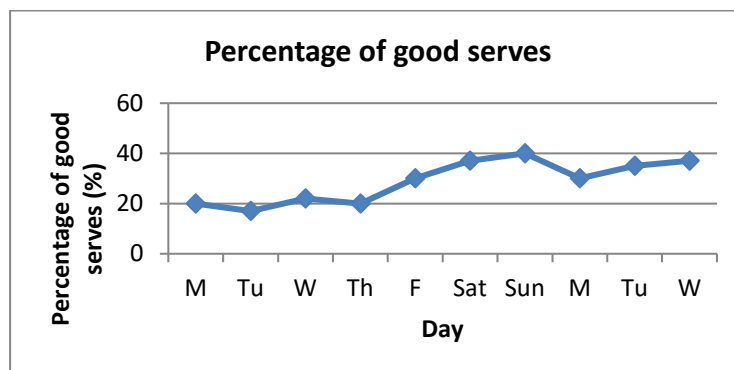
(a) Is Mary improving?

(b) Which was Mary's best serving day? Her worst?

(c) Approximately what percentage of her total serves over the 10 days were good? (be careful!)

(d) Complete:

Day	M	T	W	Th	F	Sat	Sun	M	Tu	W
% of good serves										

3. **Circle graph.** A Year 8 class had lessons in science, social studies, mathematics and English (split by the teacher into literature and grammar). The students were upset about the length of the mathematics assignments and presented the following graph to their mathematics teacher to convince her she was using an unfair share of their study time.

Allocation of study time

(a) The alert mathematics teacher told the class that they were complaining to the wrong teacher. Who did she have in mind?

(b) About what percentage of study time did the class feel they spent on each of their four subjects?

(c) What special skills are involved in reading a circle graph that are not required for the previous types of graphs?

4. **Picture graph.** A Year 2 class wanted to keep track of the number of dry and rainy days during a week, so they used the following method of representing the data, using cut out faces.

Rainy Days

RAINY | ☹ ☹ ☹ ☹

DRY | ☺ ☺ ☺
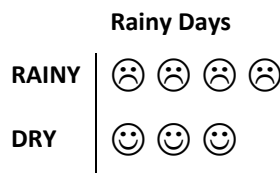
(a) What advantages can you see in using a picture graph in this situation instead of a bar graph?

(b) An upper primary class may use a picture graph for rainy and dry days over half a year as follows:

Rainy Days

RAINY | ✗ ✗ ✗ ✗ ✗ ✗ ✗ ✗

DRY | ✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓

(each ✗ and ✓ represents 5 days)
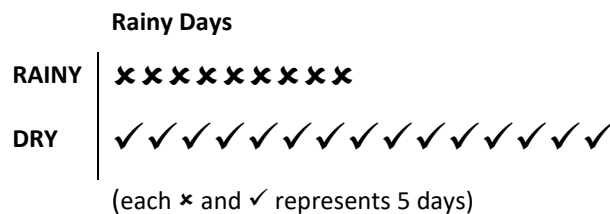
What is the extra difficulty here for students reading the graph? For what questions concerning the graph will errors mostly occur?

(c) Which of the following kinds of data lend themselves to picture graphs?

i) Maximum temperature on each day of the week.

ii) The number of students who ride bikes, walk or are driven to school.

iii) The relative numbers of science, mathematics and social studies books in the library.

iv) The number of recyclable cans collected by each child in the class.

5. **Scattergram**. To show the relation between time they spent on homework and time they spent watching TV, a class made a scattergram as follows:

Time spent during school week on homework and TV
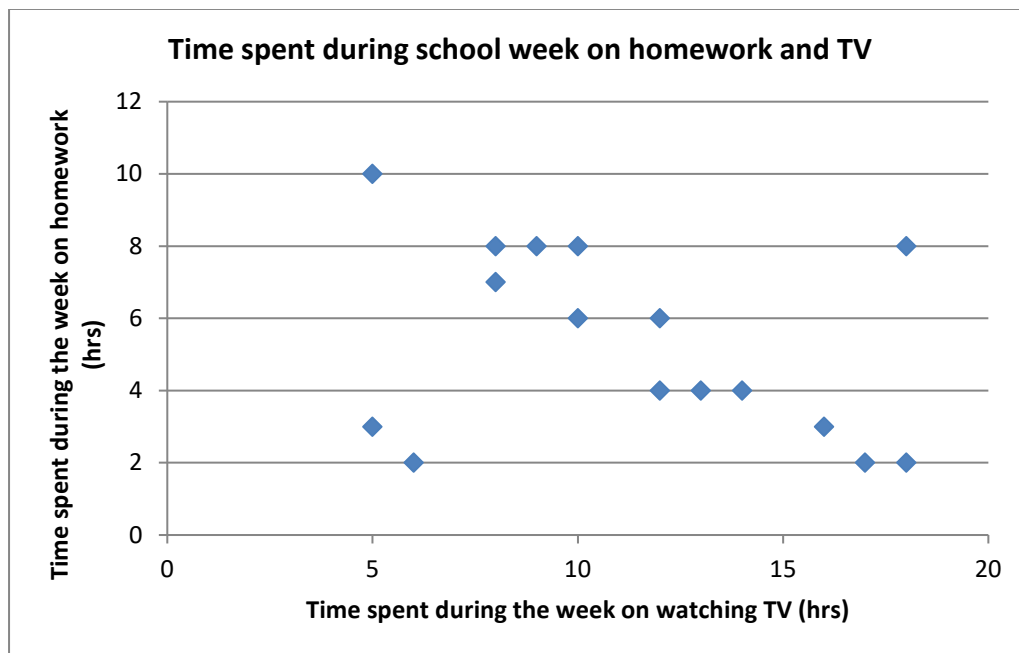
(a) Is it true that students who watch a lot of TV do little homework (and vice versa)? That is, is TV watching inversely proportional to homework time or is there no relation?

(b) Are there any students who do not fit the pattern? Describe them.

(c) Which of the following kinds of data lend themselves to scattergrams?

   i)   Length of bean plants for each day of the week

   ii)   Length of foot and circumference of head

   iii) The sunset hours of each day of the week

   iv)   Circumference and diameter of circles

What is particular about data that is amenable to scattergrams?

B7   Using tables to work out costs

Setting up tables

To show how lists can become tables, we will look at an example, *What would be the cost of clothing a man for a 4-day trip?* There are four steps as follows.

1. **Develop a list**. One way to do this is to consider the clothing needed in terms of parts of the body and then different activities; for example, a list could be:

    Shoes, Socks, Underwear, Pants, Shirts, Coat/jumper, Hat, Pyjamas

2. **Consider all needed for table**. One way to do this is to think of one item and what is needed to work out cost (e.g. the number of items, where to buy it, the cost and so on). Because of the nature of the task of buying clothes, there is also a need to consider any discounts.

    Item, Place to buy it (shop), Number of items, Cost of each item, Any reductions, Totals

3. **Translate to a table**. An effective way to do this is to put the needs (Step 2) across the top and the list of all items (Step 1) down the left-hand side of the table.

Shop	Item	Number	Cost/item	Any reduction	Totals
Athlete's Foot	Shoes				
Target	Socks				
Target	Underwear				
Jeans West	Pants				
Chaps Menswear	Shirts				
Chaps Menswear	Coat/jumper				
Target	Hat				
Target	Pyjamas				
OVERALL TOTAL					

4. **Determine numbers**. The next step is to complete the table by filling in the gaps – determining the number of items, the shop, and the cost/item (plus any reductions).

Shop	Item	Number	Cost/item	Any reduction	Totals
Athlete's Foot	Shoes	1			
Target	Socks	4			
Target	Underwear	4			
Jeans West	Pants	2			
Chaps Menswear	Shirts	4			
Chaps Menswear	Coat/jumper	1			
Target	Hat	1			
Target	Pyjamas	1			
OVERALL TOTAL					

Calculating with tables (Budgets)

Once the table has been set up, then operations can be used to determine totals (budgets) as follows.

1. **Select appropriate operations**. Selecting the right operations within a table is easier than in real-life situations, but still needs to be accurate. In general, the rules are: (a) addition for totals and overall totals, (b) subtraction for any reductions (discounts), and (c) multiplication of number × cost or number × reduced cost for total in each row. Of course, if we have to work backwards (e.g. find the cost per item when we know the cost of four items), the operations can invert (e.g. from multiplication to division).

2. **Determine form of totalling**. In a table, you can find the overall total at the end or you can total cumulatively – keep a running total that advances at each row.

3. **Complete the operations**. Here are examples for overall total and cumulative total based on the cost of clothing for a 4-day trip for a man.

Overall total:

Shop	Item	Number	Cost/item	Any reduction	Total
Athlete's foot	Shoes	1 pair	$160.00	$32.00	$128.00
Target	Socks	4 pairs	$6.90		$27.60
Target	Underwear	4	$3.50	$0.30	$12.80
OVERALL TOTAL					$168.40

Subtraction $160.00–$32.00

Multiplication 4 × $6.90

Subtraction $3.50–$0.30=$3.20 Multiplication 4 × $3.20

Addition of the column

Cumulative total:

Shop	Item	Number	Cost/item	Any reduction	Cumulative total
Athlete's foot	Shoes	1 pair	$160.00	$32.00	$128.00
Target	Socks	4 pairs	$6.90		$155.60
Target	Underwear	4	$3.50	$0.30	$168.40

Adding $128 to total of row ($27.60)

Adding $155.60 (the amount in the line above) to the total of the row ($12.80)
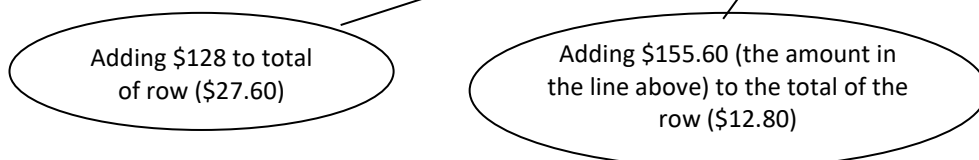
B8   Histograms

1. Obtain the height information from measuring heights to make a bar graph. List the highest and lowest height. Subtract these two values to get the **range** of the heights.

2. Using this range, determine an interval size that divides the heights into approximately seven sections or intervals. For example, a list of heights from 121 cm to 165 cm (a range of 44 cm) could be divided into seven 7 cm intervals (it is important to have the interval length an **odd number** so that there is a central number to define each bar), as follows:

    120 – 126
    127 – 133
    134 – 140
    141 – 147
    148 – 154
    155 – 161
    162 – 168

3. Add the previous and following intervals to this list. For example 113 – 119 and 169 – 175 would be added to the above list. This is so that the ends of the range of data can be shown by the zero values.

4. Make up a table as below and tally into each section the heights that are present in that interval. For example, see the possible tally below:

INTERVAL	TALLY	FREQUENCY
113 – 119		0
120 – 126	II	2
127 – 133	IIII II	7
134 – 140	IIII	5
141 – 147	IIII	4
148 – 154	IIII III	8
155 – 161	IIII II	7
162 – 168	III	3
169 – 175		0

5. Use this data to draw a bar graph, as the example below shows.
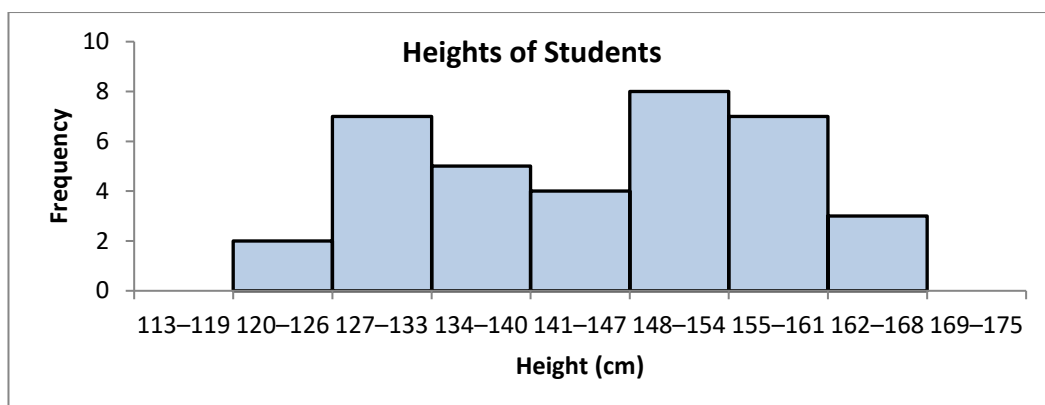
6. Convert the bar graph to a line graph (remember the zeros at the end) as the example shows. This graph is called a polygon.

Heights of Students
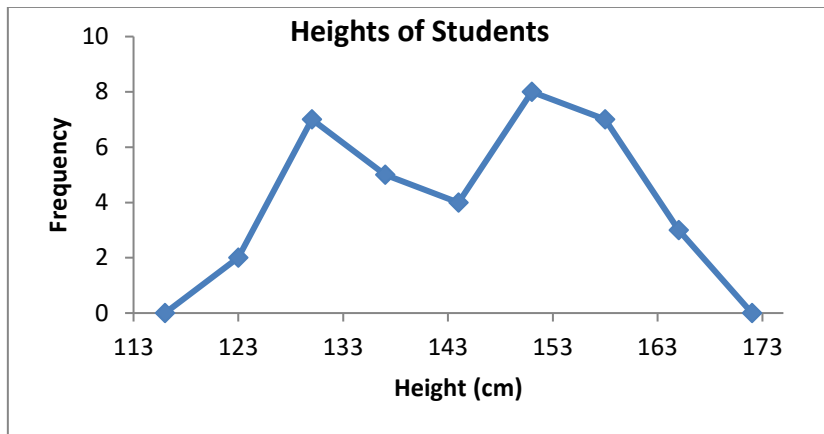
7. The example above is a common bar graph for coeducational classes. A bar graph of heights for boys only or girls only is as below.

Heights of Boys
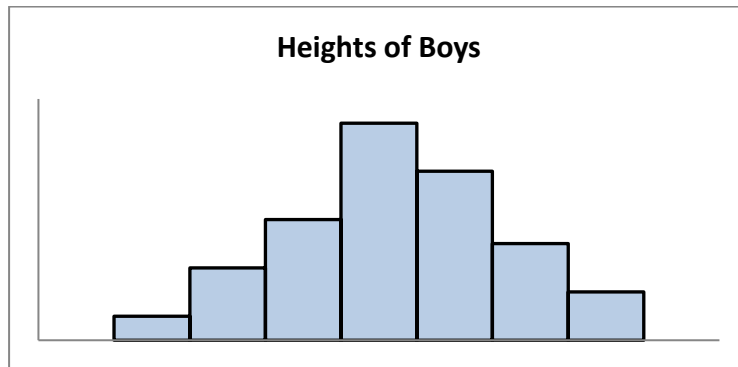
Why is there a difference?

B9   Bar to line graphs

For bar graphs with an appropriate horizontal axis, the bars may be replaced by dots on the top centre of each bar:
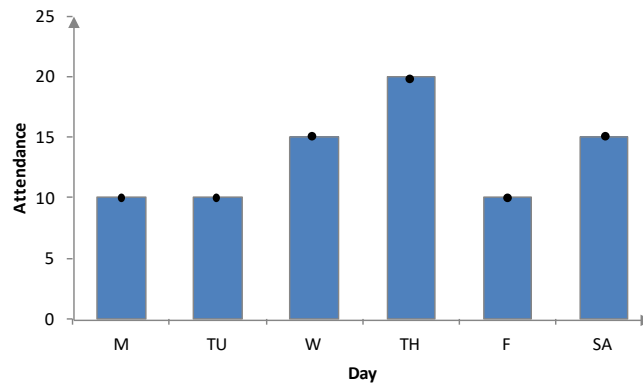
And the dots can then be joined by a line to make a line graph:
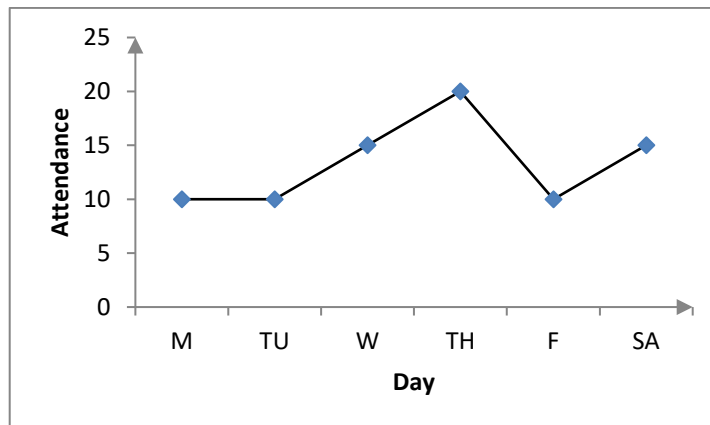
Once ability in plotting points has been developed for line graphs, it can be transferred to scattergrams.

B10   Line graphs, circle graphs and scattergrams

1. Complete the following:

    Step 1 – using graph paper make a bar graph out of the following data:

Basic facts drill	Number correct
Monday	29
Tuesday	27
Wednesday	31
Thursday	38
Friday	49

    Step 2 – place a dot at the top centre of each bar and join with a line.

    Step 3 – repeat step 2 on a new piece of graph paper without drawing the bar graph.

2. Construct a series of questions that you would ask students to achieve steps 2 and 3 above.

3. Construct a circle graph or pie chart for the following data:

Economic sector	Percentage of export
Farming	43
Mining	35
Manufacturing	22

    Draw a circle. Convert the percentages to an angle out of 360 – multiply by 360 and divide by 100. Use a protractor to mark in the sectors.

4. Complete the following table:

Percentage	Fraction (common)	Sector angle
25	1/4	90
45		
	3/8	
		120
	2/5	
62		
		80

Draw each sector angle (except the 90°) on the circles below:
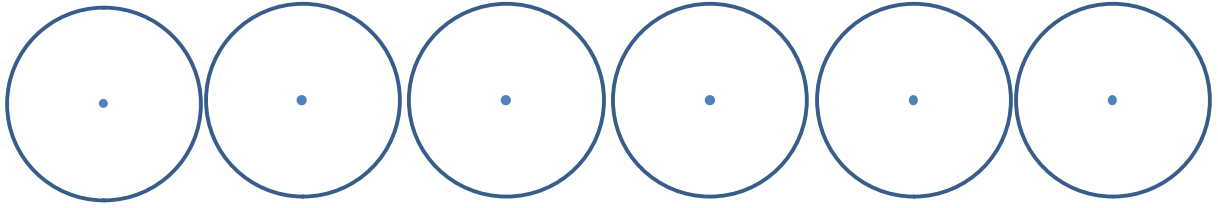
5.   List the prerequisite knowledge of mathematical concepts and processes needed by a student before they could undertake the construction of a circle graph.

6.   Measure the height and mass of each student in the class.

7.   Draw up axes on graph paper.
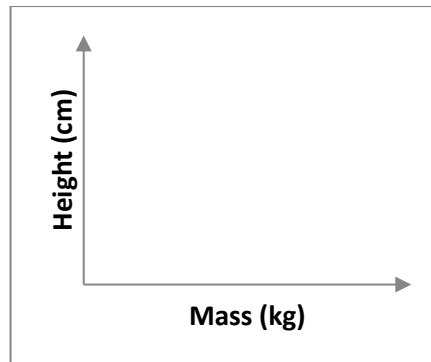
8.   Take the mass of the first student and draw a pencil line with a ruler vertically through the point where this mass is on the horizontal axis. Take the height of this first student and draw a pencil line horizontally through the point where this height is on the vertical axis. Mark where the two lines cross with a bold point.

9.   Mark a point, using the technique in (8) above for the mass and height of every student.

10. Study the resulting scattergram. Is it true that, as a general rule, taller students tend to be heavier and vice versa?

B11 Five data sets

Below are five sets of data. For each set of data construct a bar graph, line graph, circle graph, picture graph or scattergram to represent the data. Choose a different type of graph for each set of data (basing your choice on the appropriateness of the type of graph for the data).

1.

Student	Books read so far this year
Sally	1
Mary	7
Bill	4
Don	3
Joe	2

2.

Place of residence	Fraction of year spent
Home	2/3
Hotels	1/2
Cottage	1/6
Grandmother's	1/12

3.

Age in months	Height in cm
26	71
47	93
71	92
78	120
34	79
54	97
63	105

4.

Spelling test	Johnny's score
1	50
2	90
3	33
4	79
5	97

5.

Diameter of circle (cm)	Circumference of circle (cm)
1	3.1
1.5	4.7
3.3	10.4
4	12.6

B12 Graphing task

1. **Choose a question or position**

   Think of a question or position and devise a data collection experiment to shed light on the question or to support or refute the position. Use your imagination: almost any data collection experiment would be fine – check with your teacher.

   Examples:

   - Take a poll of attitudes towards a political issue
   - Time a particular activity
   - Analyse writing or speech for frequency of use of various parts.

2. **Collect data to answer the question or support the position**

   Set up experiments, surveys, etc. from which you can collect your data (you may have to devise a convenient table or chart for collecting it).

3. **Decide on a message contained in the data**

   Organise your data and look for patterns and trends that support a point of view, position or message.

4. **Choose a type of graph and a scale that will easily and clearly communicate the message**

   Decide what types of graphs (bar, line, circle, picture graph or scattergram) are most suitable to present your data. You may use several types of graphs depending on the type of data and the message you wish to convey. For bar, line and picture graphs, choose a scale that will accurately communicate your required message.

5. **Construct your graph(s)**

   Draw up your graph(s) and prepare a report consisting of a description of your experiment, how you collected your data, the data, your graph(s), and a description of any answers, messages or conclusions that are warranted by the data.

6. **Present the graph to others for evaluation**

   Place this report on a poster and display it for discussion by other members of your class.

   In this discussion, you should first focus on how the data was collected, how it was organised and why particular graphs were chosen in preference to others. Then the class should address itself to the questions:

   - What was the message?
   - Did the data support the message?
   - Could another kind of graph or a different scale have been used to better convey the message?

   The class should pay particular attention to the general question of which type of graph seems most appropriate for which kind of data.

B13   How to lie with graphs

One of the best ways to "lie" or "misrepresent" with data is with graphs. The vertical axis scale can be reduced to increase the slope of the graph. Graphs can be truncated (this suppression of the zero is a powerful tool in misinformation). Scales need not be given. The graph can only show the top of the scale to accentuate differences which are small in comparison to the numbers being considered.

"Trend" lines can be drawn through the scattered sets of points in the scattergram. Over-confident extrapolation which extends the graph well beyond factual content can be full of errors and very misleading if no indication (e.g. a dotted line) is given that it is being done.

Here is an example of how to make a 10% increase in the gross national product more impressive – in three steps (idea from Darrell Huff, *How to Lie with Statistics,* Penguin).
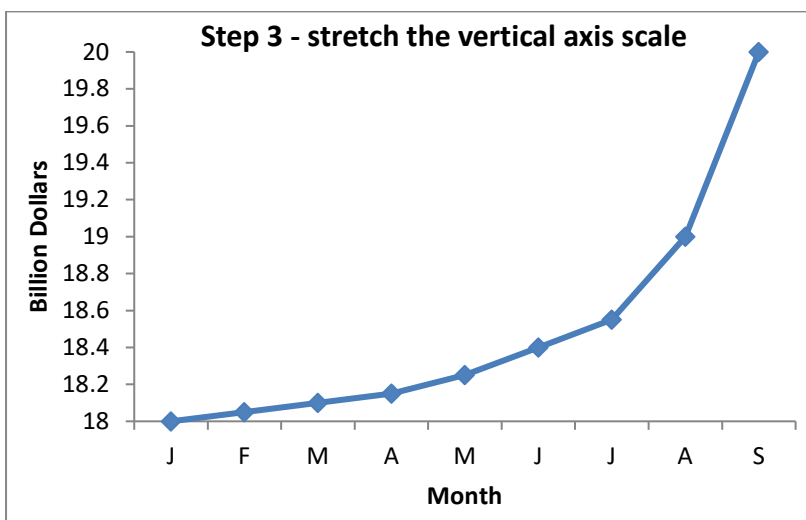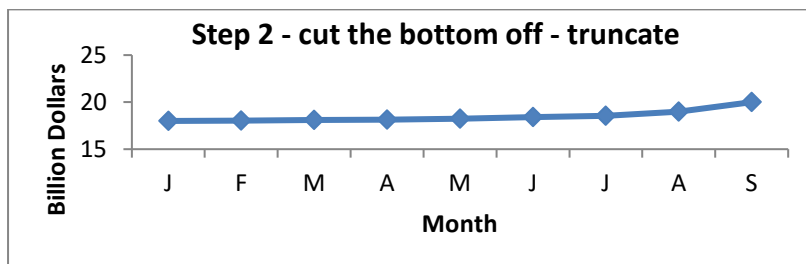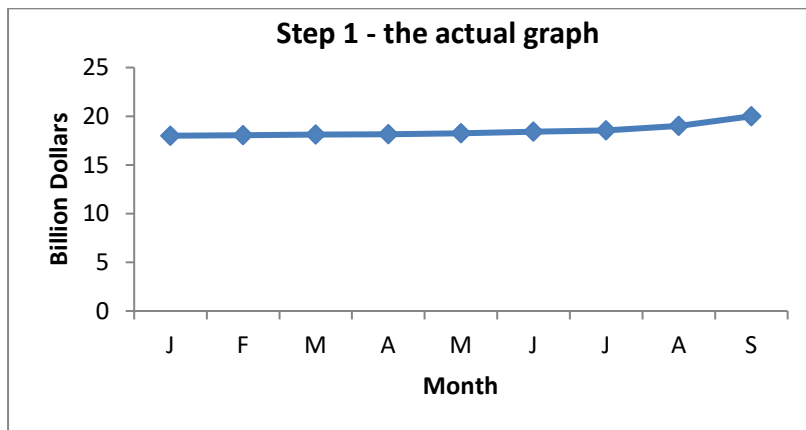
1. Look at these two graphs representing the same data. Do they convey the same message?
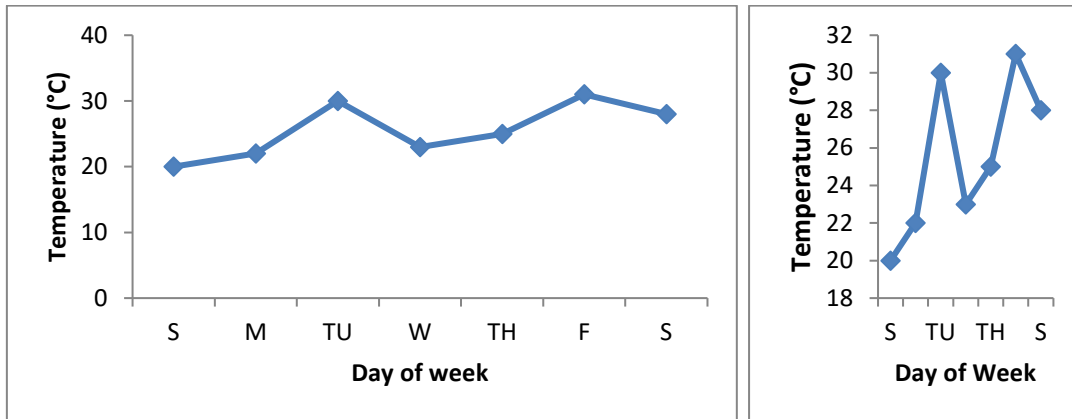
2. If you wanted to emphasise the variability of the weather, which graph would you use? What does the other graph emphasise?

3. What changes have occurred in the vertical and horizontal axes and scales to bring about these changes in impression?

4. Use the following data on government spending in the second half of the year to draw line graphs which emphasis that:

    (a) Government spending has increased; and

    (b) Government spending has remained steady.

Month	Spending ($m)
July	195
August	193
September	196
October	194
November	197
December	202

5. Picture graphs and drawing-type bar graphs (pictograms) can be very misleading. For example, Darrell Huff *How to Lie with Statistics* (Penguin) has the following three ways to show that the workers of Rotundia earn half as much as the workers of USA.
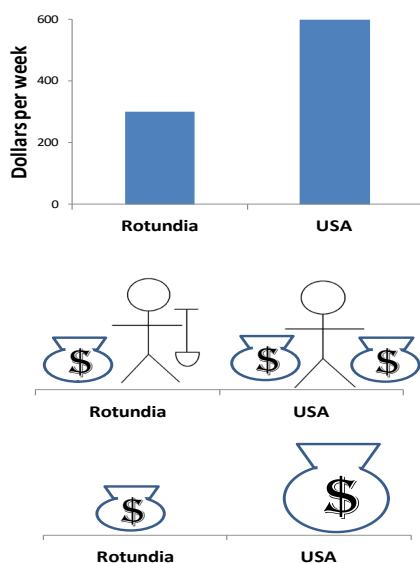
The drawings in the last picture have the USA bag of money being twice as high as the Rotundia bag of money. But the USA bag is also twice as wide and appears twice as thick. Hence the impression is left with the reader that the USA wages are **eight** times more than the Rotundia wages.

B14 Finding better presentations

The following graph, purporting to show students who eat lunch at home and at school, has many inaccuracies.

Lunch
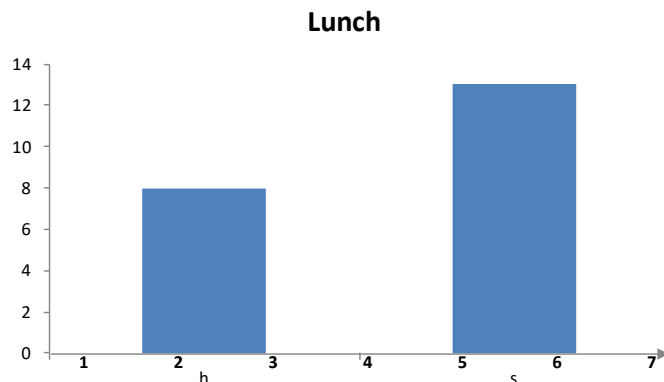
(a) There are no titles giving meaning to the horizontal and vertical axes. There is no indication of what the "h" and "s" stand for.

(b) There should be an arrow on the vertical axis (showing that the numbers continue) but not on the horizontal axis.

(c) The bars should be evenly spaced.

This would be a better presentation:
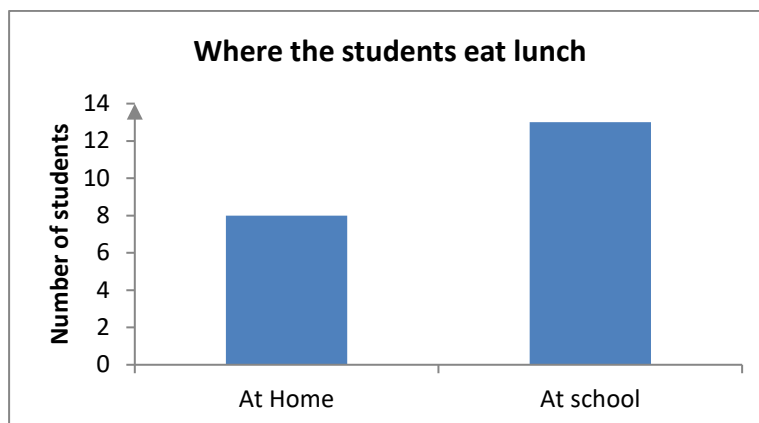
For the following graphs:

(a) list the inaccuracies; and

(b) redraw the graph "correctly" or in a better presentation.

1. Students going to camp

BOYS   GIRLS
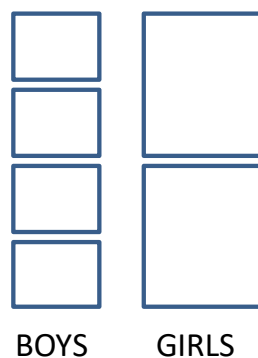
2. The shoe size students have

3.

= 10 people
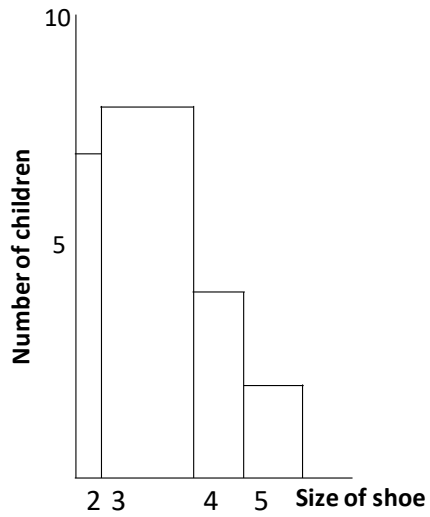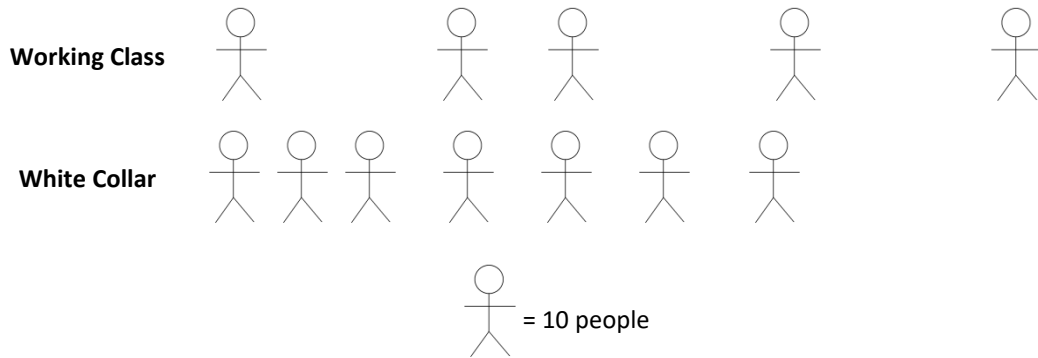
4. School revenue
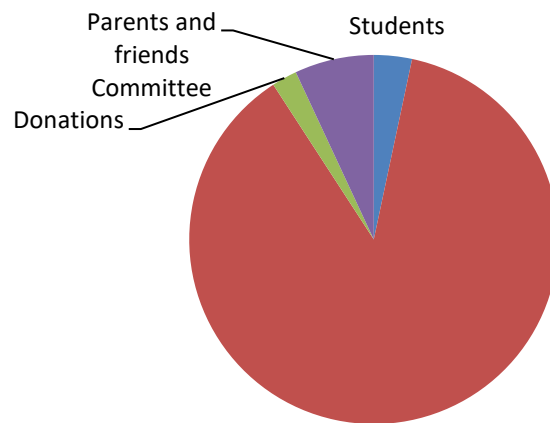
5. Students' favourite TV programs

Favourite TV Program
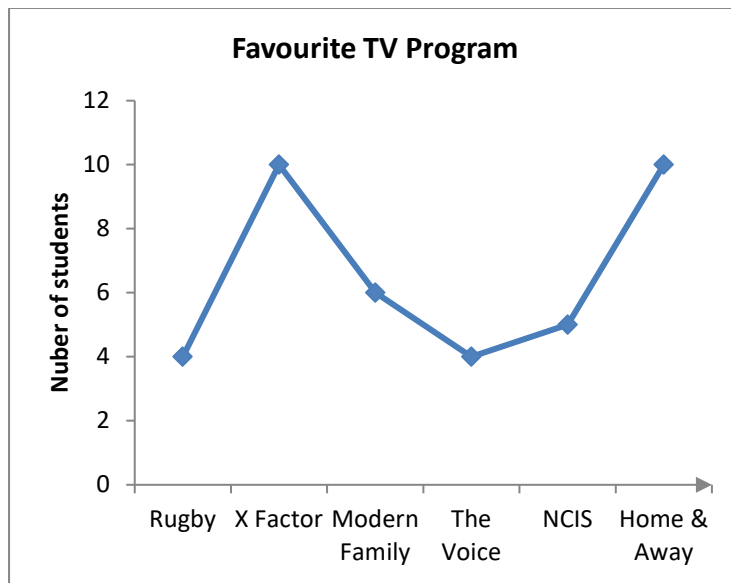
6. Comparing size of schools
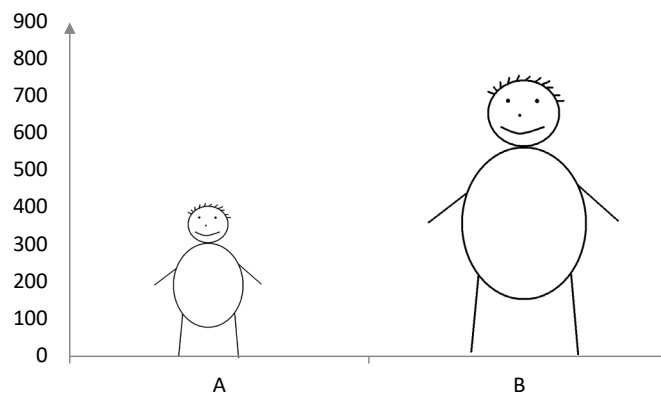
7. Salary changes
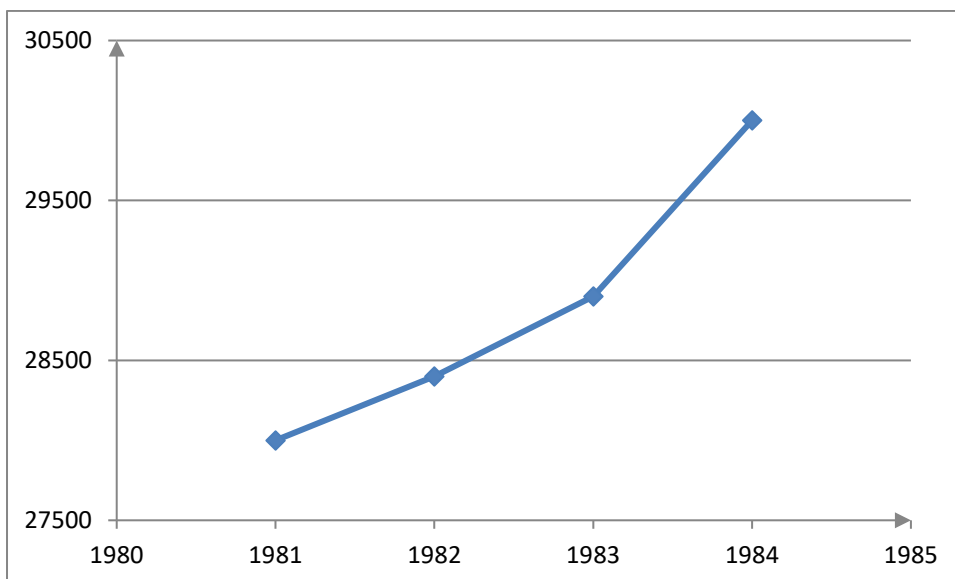
	Features of Graphs	
	Particularly good for:	**Not particularly good for:**
Bar graph		
Line graph		
Picture graph		
Circle graph		
Scattergram		

Appendix C: Extra Material for Probability

C1   Connecting probability to fraction

The major connection is between **probability and fraction** (common, decimal, percent). Because probability and fractions share similar concepts and because probability answers are recorded as fractions, teaching probability requires the teacher to help students make the connection between fractions and probability. The following example shows how these two mathematical domains can be connected pedagogically as well as mathematically.

Aim: To lead the students to discover the probability of an event occurring (informal language and recording) using an area model and a set model.
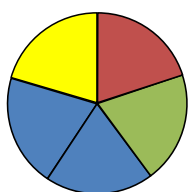
What are the chances of spinning blue on this spinner?

[*2 chances out of 5 chances*]

What are the chances of getting, without looking, a blue marble from this bag?

[*2 chances out of 5 chances*]

Stages/Questions that can be asked:

Identify the whole (i.e. the sample space)
- What colours *could* you spin on this spinner? Would it be *possible* to spin red? purple?
- What colours *could* you get from this bag of marbles? Would it be *possible* to get a pink marble?

Examine the parts for equality
- Has the spinner been divided into equal parts? Would the pointer be *just as likely* to stop on one part as on any other part? (OR: Would you have the *same chance* of stopping on any of the parts?)
- Are all the marbles equal, that is, the same size and shape? Would you be *just as likely* to get one marble as any other marble? (OR: Would you have the *same chance* of getting any of the marbles?)

Name the parts (establish the total number of chances, that is, the denominator)
- How many equal parts does this spinner have? How many chances do you have altogether of spinning a colour?
- How many marbles in this bag? How many chances do you have altogether of getting a colour?

Determine the parts to be considered (the outcome preferred, that is, the numerator)
- How many blue parts are there? How many chances do you have of spinning blue?
- How many blue marbles are there? How many chances do you have of getting a blue marble?

Associate the two parts with the fraction name (the probability)
- What chance do you have of spinning blue? (2 chances out of 5 equal chances)
- What chance do you have of getting a blue marble? (2 chances out of 5 equal chances)

Record the probability: 2 fifths (informal); $^2/_5$ (formal), 0.4, 40%

C2   Probability games and activities

Games as learning experiences in probability

Games of chance provide valid real-world experiences for the learning of probability. However, because of the imperatives of the game, students may retain little of the information on probabilities that could be garnered from the game. It is imperative to keep the focus on the chance events and what they expect to happen throughout the game by asking students to articulate and record predictions and focus on why the outcome matched or did not match their prediction.

Another way to assist students to crystallise knowledge of probability from games is by playing unfair games. In these games spinners or dice are used where some outcomes are more likely to occur (because of appearing more often or because of having a larger area). The method of teaching used is to allow the students to choose the colour they wish to use, to discuss why they chose what they did and then to play the game. After the game is finished the fairness of it all can be discussed. Unfair outcomes can be generated by using colour stickers on a die with, for example, one blue sticker, two red stickers and three green stickers and asking students to choose their colour. Their piece may only move through the game of chance if their colour is rolled. First to finish wins.

Students can also be engaged in exploration of randomness and fairness if challenged to investigate generation of random outcomes in the creation of their own games of chance. This provides a real-world context for exploring probability and motivates students to explore chance in a creative context.

The following pages provide resources and ideas for probability games and activities.

Spinners
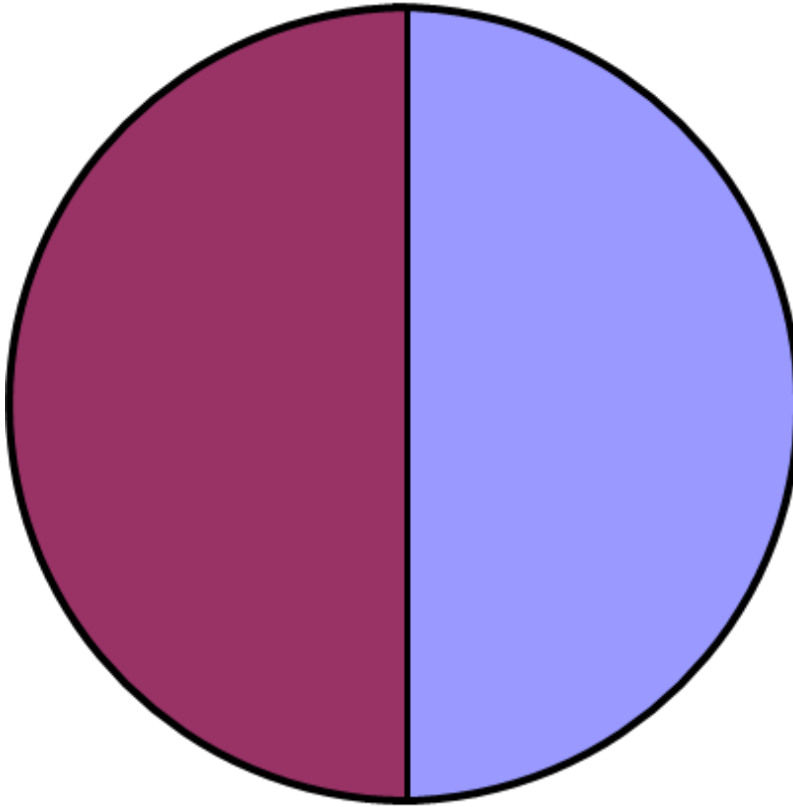
Spinner A
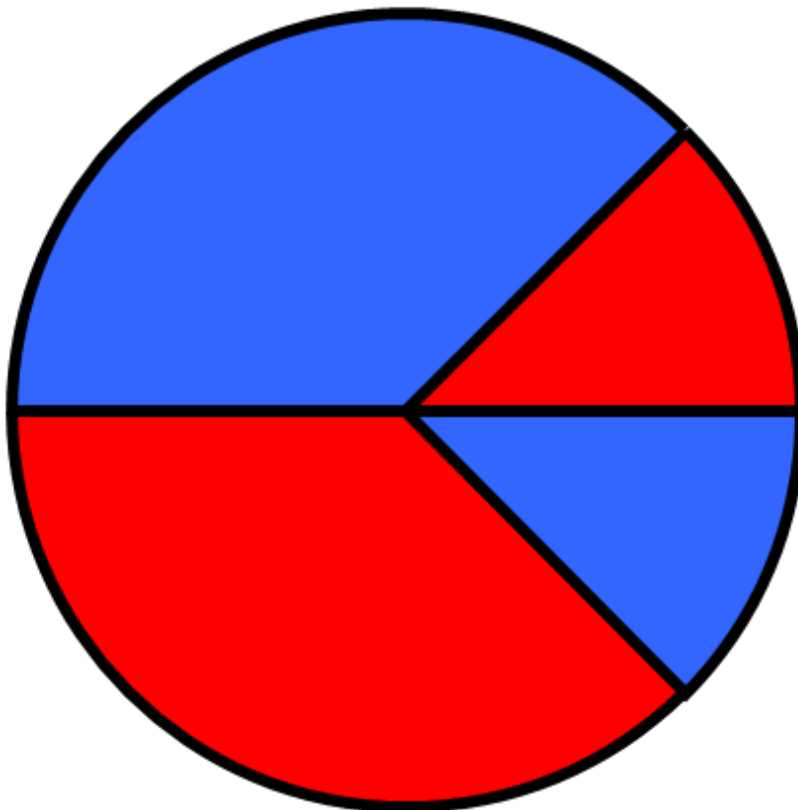
Spinner B
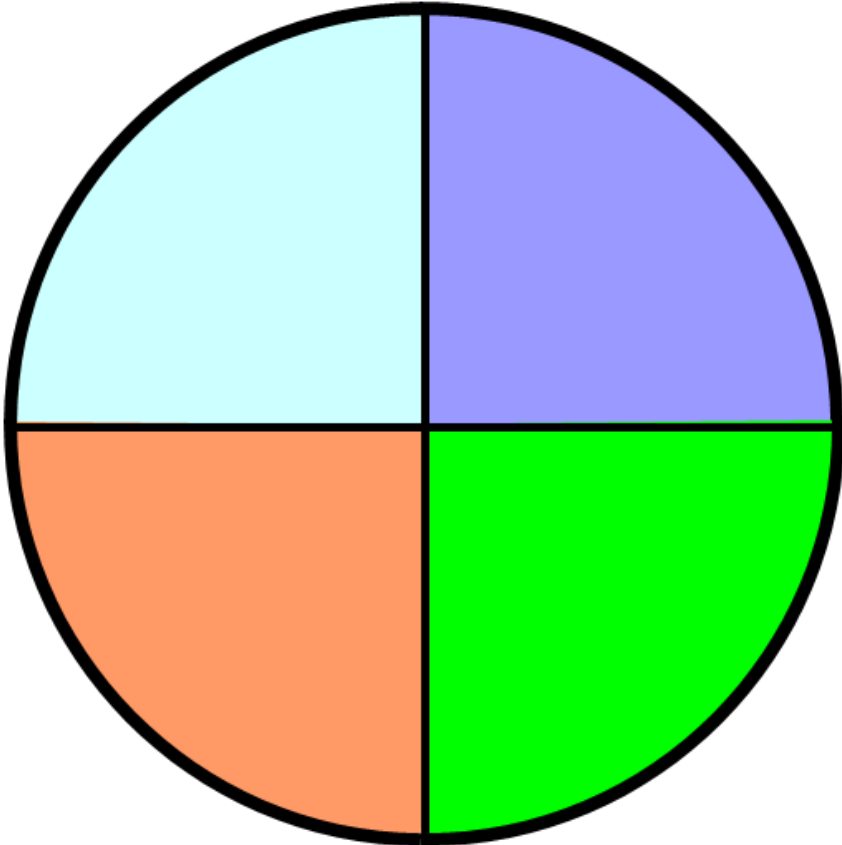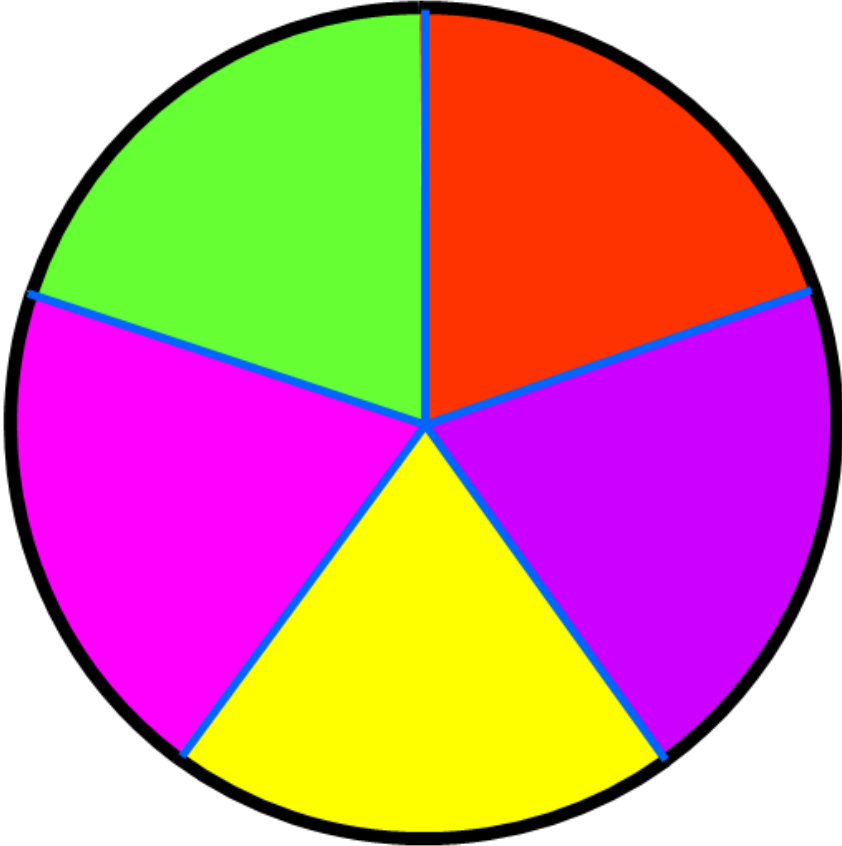
Spinner C

Spinner D

Spinner E
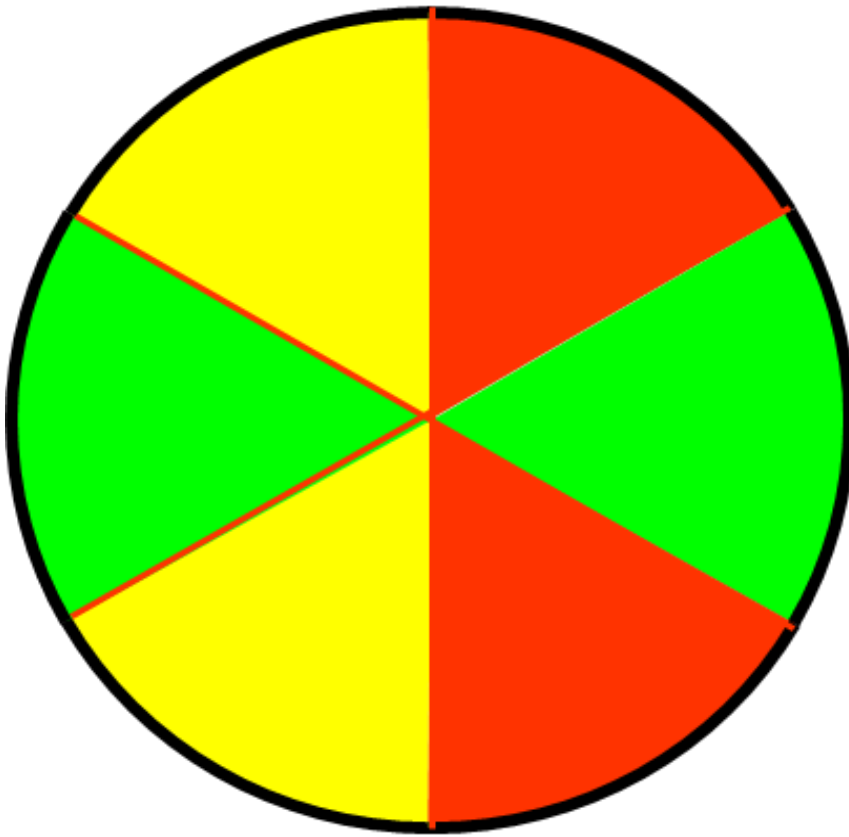
Spinner F
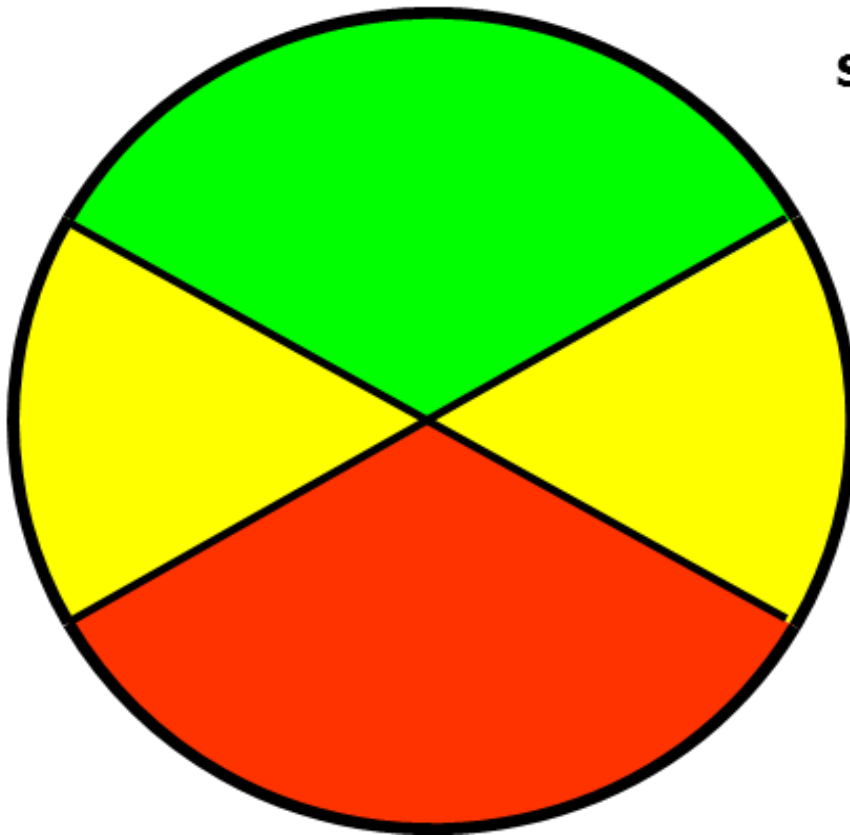
Spinner G
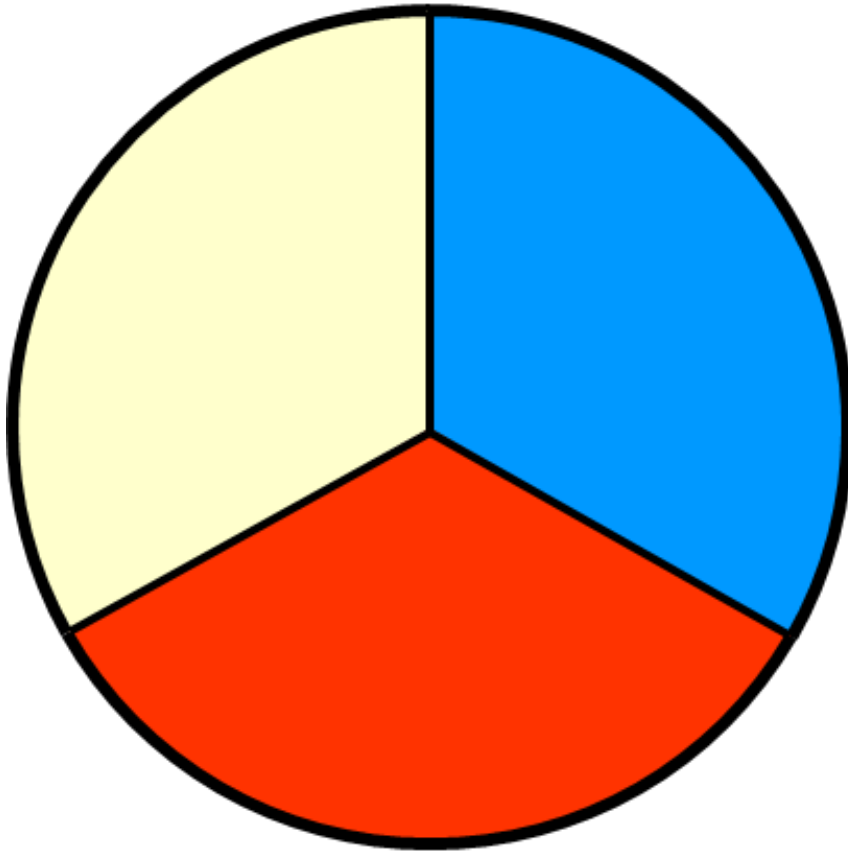
Spinner H
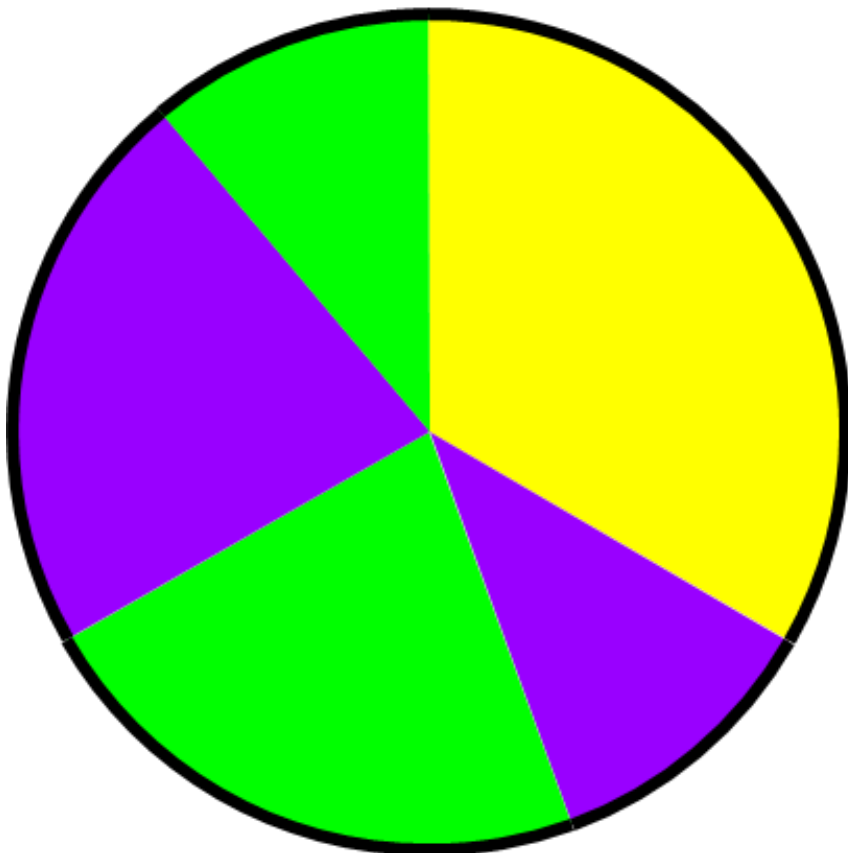
Likely Language
Activity 1
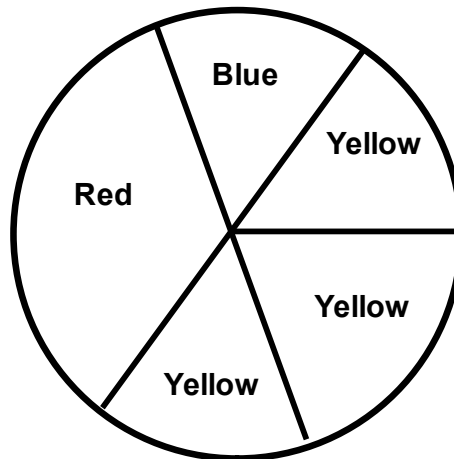
1.   On this spinner, what different colours **could** you spin?

2.   What colour would you be **most likely** to spin?

3.   What colour would you be **least likely** to spin?

4.   If you were playing a game, which colour would give you the **best chance** of winning?

5.   **Play a game with your classmates**. You will need red, blue and yellow Unifix cubes or counters.

Each person takes turns in spinning the pointer.

If your pointer stops on yellow, put out a yellow Unifix cube. (Do the same for red and blue.)

After each player has had 5 turns, stop and count how many yellow, blue and red Unfix cubes each person has. Now put all the reds ones together, all the blue ones together and all the yellow ones together.

Which colour did your group spin most?

What colour did the other groups spin most?

Put all the Unifix cubes together in groups of yellow, blue and red. What colour did the class spin most? Least?

Likely Language
Activity 2
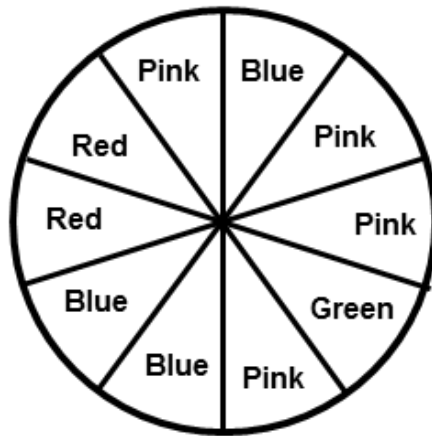
1. On this spinner, what colours **could** you spin? _____

2. What colour would you be **most likely** to spin? _____

3. What colour would you be **least likely** to spin? _____

4. Which colours are you equally likely to spin?
    _____

5. What colour would you be just as likely to spin as red? _____

6. Use 3 colours to colour the spinner to show that:
    •getting green is most likely; and
    •getting blue is just as likely as getting yellow.

Likely Language
Activity 3
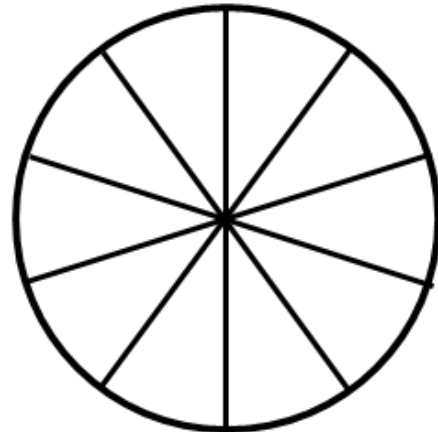
Pr obabil it y Lan guage

Spinner A   Spinner B   Spinner C   Spinner D
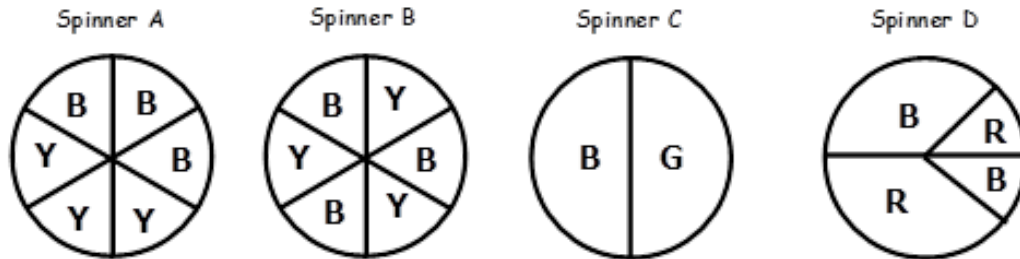
B = blue; Y = yellow; G = green; R = red

Write the colours it is possible to spin on:

Spinner A _____

Spinner B _____

Spinner C _____

Spinner D _____

On which spinner would it be impossible to spin:

Purple: _____   Yellow: _____

On which spinner would you:

More likely to spin blue than yellow?   _____

Less likely to spin green than blue?   _____

Just as likely to spin red as blue?   _____

Is Spinner A a fair spinner?   _____   Is Spinner B a fair spinner? _____

Is Spinner C a fair spinner?   _____   Is Spinner D a fair spinner? _____

Name: _____

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Fair or not fair activities

Fair or not fair?
Activity 1

- Colour the spinner;
- Spin the pointer then tick the colour in the correct box;
- Do this for 20 spins altogether.

On this spinner, is it possible to spin red? _____

On this spinner, can you be certain of getting green? _____

On this spinner, are you just as likely to get purple as orange? _____

On this spinner, are you more likely to get purple than green? _____

On this spinner, are you less likely to get orange than green? _____

If you were to make 300 spins on this spinner, how many times would you expect to get:
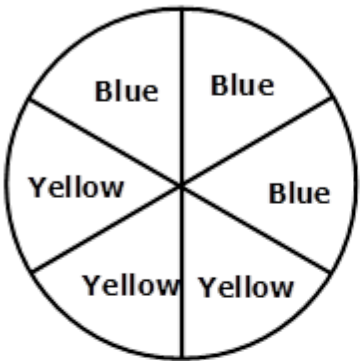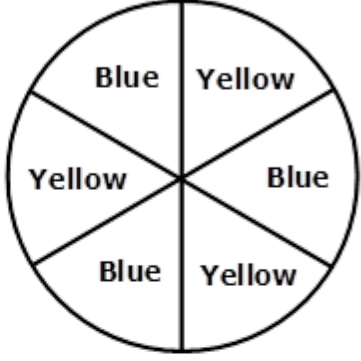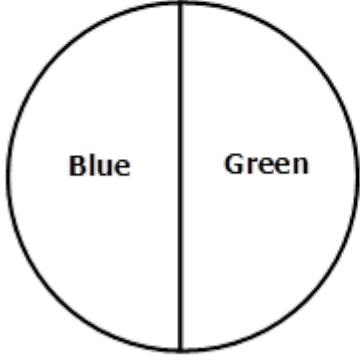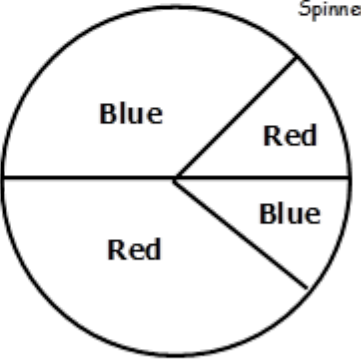
Green?   0   10   50   70   100   200   (Circle one number.)

Orange? 0   10   50   70   100   200   (Circle one number.)

Blue?   0   10   50   70   100   200   (Circle one number.)

Purple? 0   10   50   70   100   200   (Circle one number.)

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Fair or not fair?
Activity 2

Let's find out if the spinners are fair

* Put coloured dot stickers on each spinner;
* Spin the pointer then tick the colour in the correct box;
* Do this for 10 spins altogether.

Spinner A

Blue | Blue | Yellow | Blue | Yellow | Yellow

Blue ☐☐☐☐☐☐☐☐☐☐

Yellow ☐☐☐☐☐☐☐☐☐☐

Spinner B

Blue | Yellow | Yellow | Blue | Blue | Yellow

Blue ☐☐☐☐☐☐☐☐☐☐

Yellow ☐☐☐☐☐☐☐☐☐☐

Spinner C

Blue | Green

Blue ☐☐☐☐☐☐☐☐☐☐

Yellow ☐☐☐☐☐☐☐☐☐☐

Spinner D

Blue | Red | Red | Blue

Blue ☐☐☐☐☐☐☐☐☐☐

Yellow ☐☐☐☐☐☐☐☐☐☐

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Fair or not fair?
Activity 3

Use 3 different colours to make these spinners **fair**.
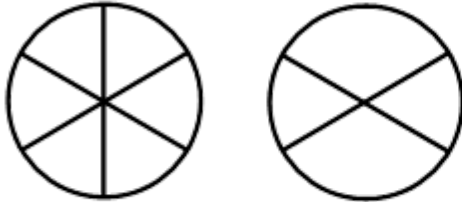
Use the same colours to make these spinners **unfair**.
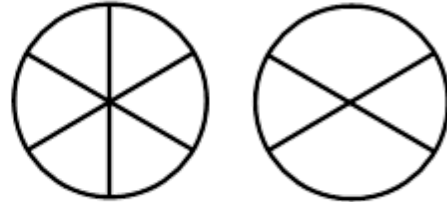
Use the numbers 1, 2, 3, 4 to make these spinners **fair**.
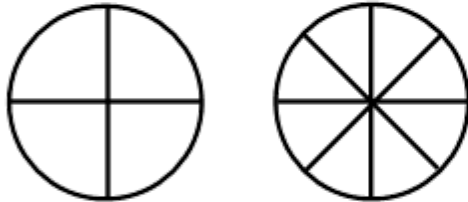
Use the same numbers to make these spinners **unfair**.
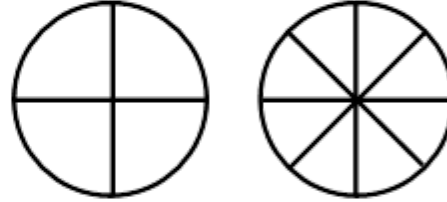
Use the letters A, B, C to make this spinner **fair**.
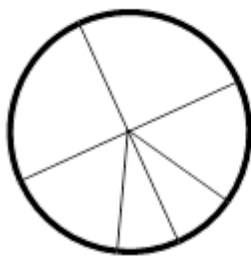
Use the letters A, B, C to make this spinner **unfair**.

© A. R. Baturo, 2002
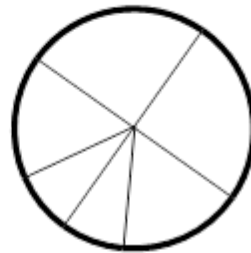
Fair or not fair?
Activity 4

Find out if the spinner below is fair (give each player the same chance of winning)

- Play with a partner. Decide who will be Blue and who will be Yellow.

- Take turns in spinning the pointer at least 10 times each.

- If blue is spun by either player, the Blue player draws a bow on the blue kite's tail.

- If yellow is spun by either player, the Yellow player draws a bow on the yellow kite's tail.
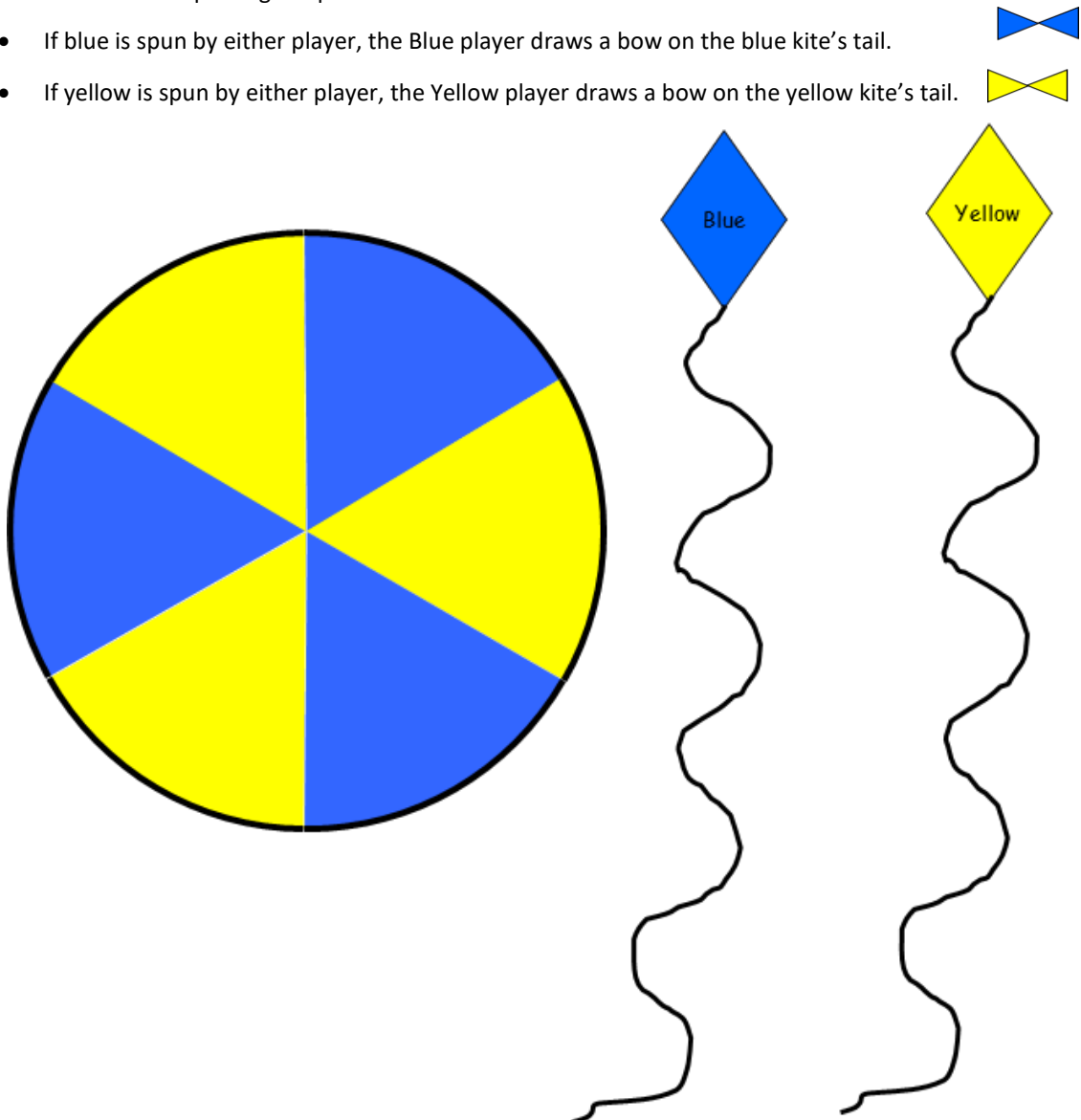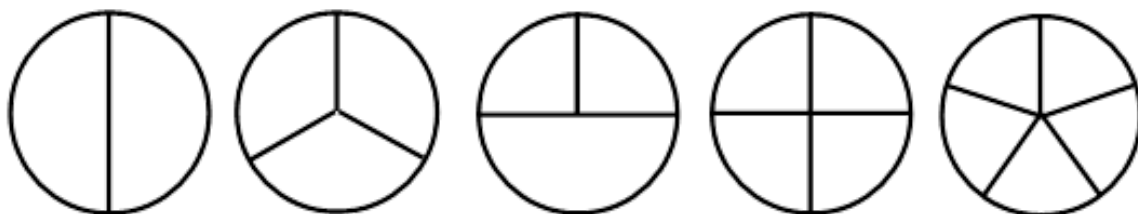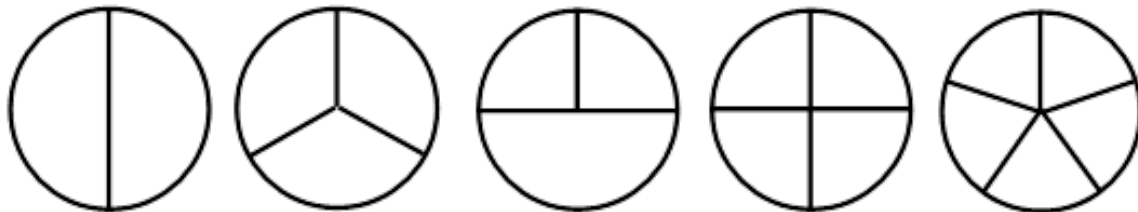
How many blue bows? _____

How many yellow bows? _____

Impossible, Possible, Certain
Activity 1

Colour these spinners so that it would be **impossible** to get blue.
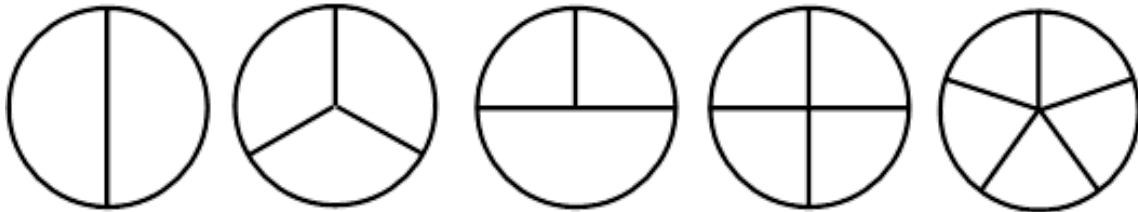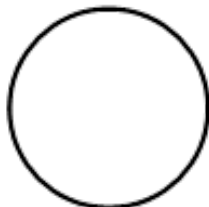
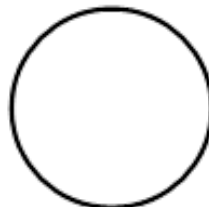Colour these spinners so that you would be **certain** to get blue.

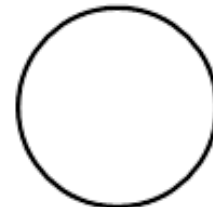Make your own spinner so that getting pink would be:

impossible   **possible but not certain**   **certain**

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Impossible, Possible, Certain
Activity 2

Colour the marbles so that it would be **impossible** to get green.

Colour the marbles so that you would be **certain** to get green.

Make your own marble bag so that getting a purple marble would be:

**impossible**   **possible but not certain**   **certain**

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Decorate the clown game

Number of players: 2, 3, 4

**Material**: Playing sheet for each player, counters, die.

Rules:

- Each player takes a playing sheet showing a clown and small counters.
- Players take turns to roll the die and then cover the matching numeral on the clown with a counter. For example, if a 6 is rolled you may cover a balloon, one spot on the pants, one spot on the arm, or the pompom on the shoe.
- The winner is the first person to decorate the clown completely.
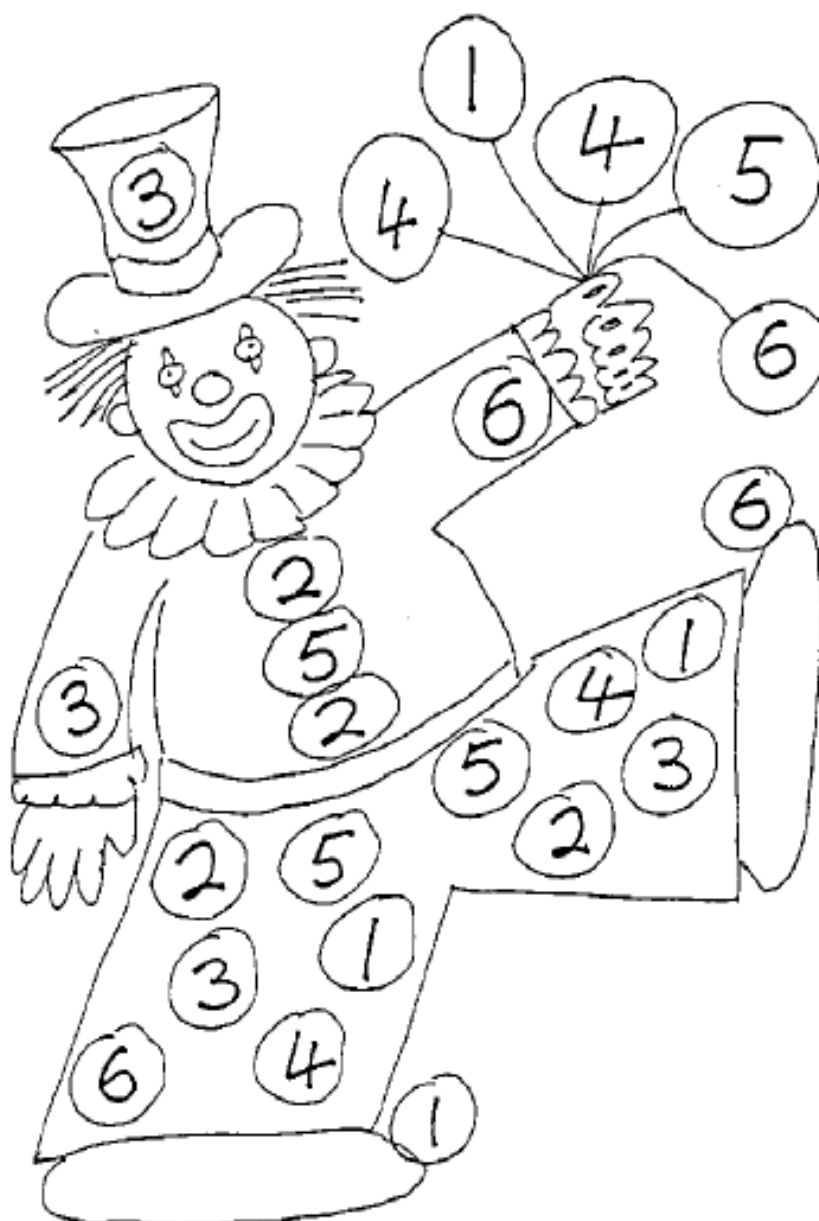
Race game boards

Can be used with fair spinner/die or unfair spinner/die.

| FINISH	FINISH	FINISH
START	START	START
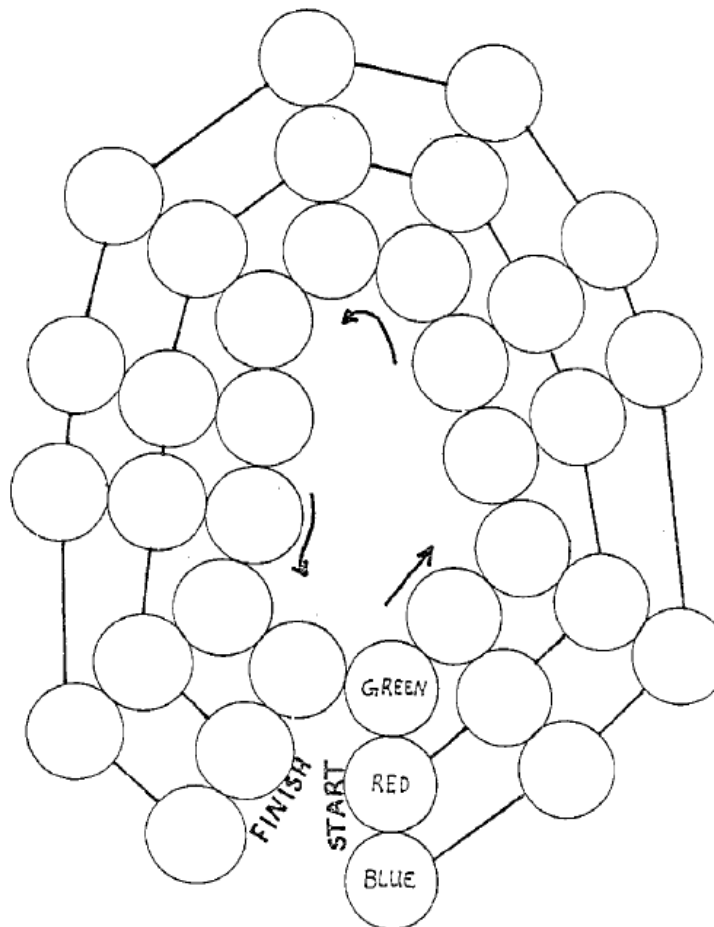
C3   Probability rich tasks

Rich task 1: Fair game

Question: Is this game fair? If it is not fair, how would you change it to make it fair?

Directions: Play this game with a friend. You need a coin.

Rules:

  (a) To decide who goes first, each player tosses the coin. The first player to toss two heads in a row goes first.

  (b) To take a turn, toss the coin. From start, move one block north for heads, one block east for tails. Toss and move two more times.

  (c) Player 1 gets one point by ending up on either 4th Avenue or D Street. Player 2 gets one point by ending up on only C Street.

  (d) The player who gets more points wins the game.

  (e) Write a report. Give proof to support your conclusion about the fairness of the game.
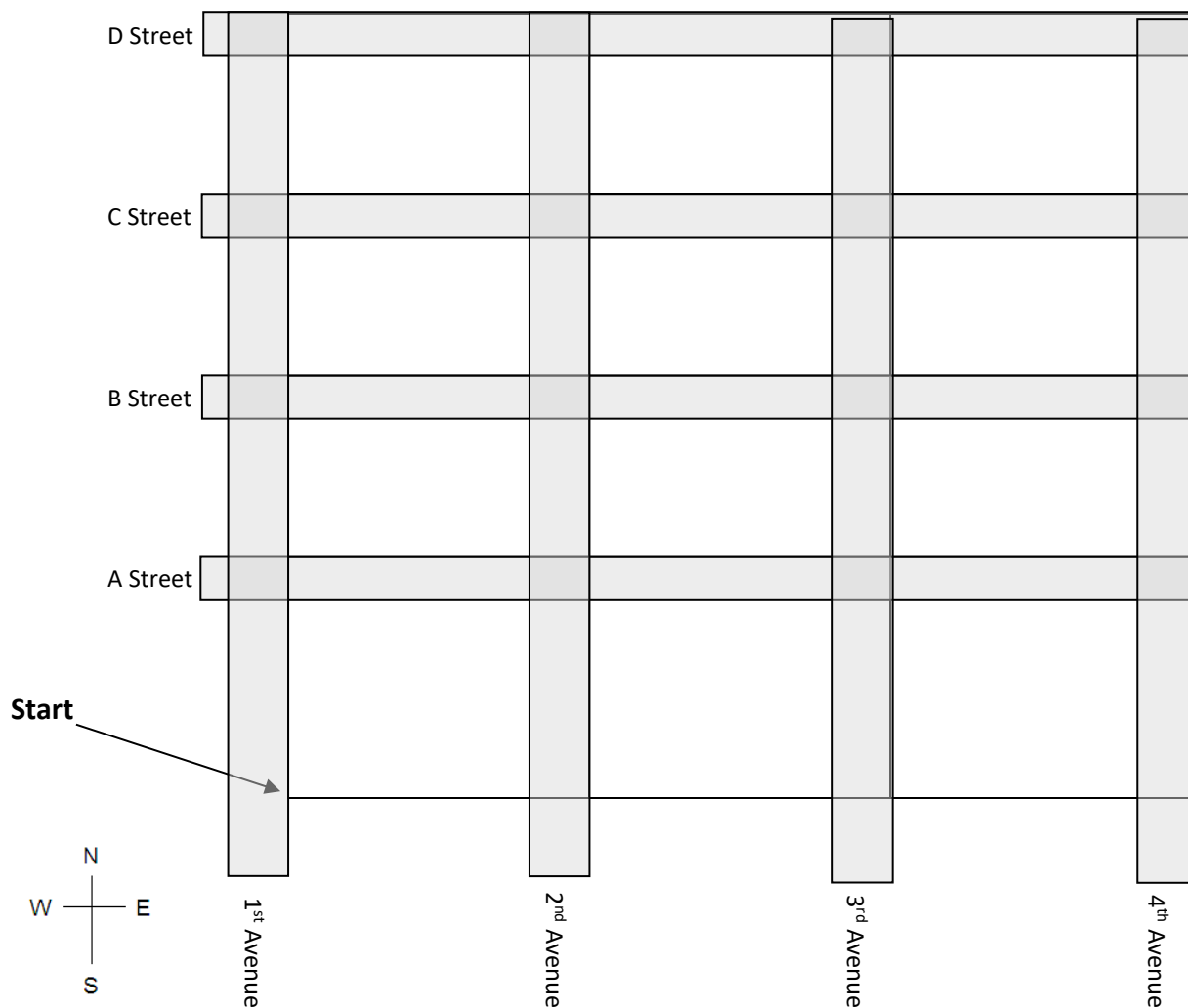
Notes: In this task, students explore the concept of chance events in a game setting. They need sufficient playing time to decide if the game is fair or not, and then further time to change the rules if they need to make it fair. Although students will know how to take turns and play a board game, they could need assistance in analysing the probable outcomes of the game. The task can be given at any time of the module. Students have the opportunity to use a tree diagram to show their working – possible outcomes.

Presenting the problem:

(a) Demonstrate the rules where the players start at 1st Avenue and A Street. The games piece is played on the lines or roads, not on a rectangle as this is a housing block.

(b) Students need to explain their thinking fully. Encourage students to make sketches, charts, tables or lists to make their reports clear.

Assessment criteria:

(a) The reasoning used to decide if the game is fair.

(b) How clearly they present their thinking.

(c) If they decide the game is unfair, whether they are able to change the rules to make the game fair.

Prompts to get students started:

(a) Who is more likely to win? Can you explain why?

(b) Which player would you prefer to be? Why?

(c) What makes the game fair or unfair?

(Rich Task modified from: Westley, J. (1994). *Puddle questions: Assessing mathematical thinking (Grade 7).* Creative Publications, California, USA)

Rich task 2: Baby boys

In 1998, the news that 22 boys were born in a row at King Edward Memorial Hospital hit the headlines. Doctors worked frantically from 3:30 am on August 20 to 10:53 pm on August 22, to deliver the same gender one after the other. This was a great surprise, so many boys and not a single girl delivered during this short period.

Problem: Discuss the probability of such an event happening. Design a simulation, which looks at a similar problem of 50 consecutive births, and determine how many times a run of three boys would occur.

Planning:

    (a) What do you need to know before you begin?

    (b) What do you need to do to solve this problem?

    (c) What assumptions do you need to make?

    (d) What do you predict you will find?

Solving: Follow your plan to solve the original problem.

Results:

    (a) Accurately summarise your results. What did you discover?

    (b) Was your prediction correct? Why? Why not?

    (c) How do the girls' results compare with the boys' results?

    (d) How could you extend this investigation further?

Prep to Year 9: Resource 7 – **Statistics and Probability**

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