Department of Education

First Steps in Mathematics

Space

Developing Geometric Understandings

First steps in Mathematics: Space

Revised edition

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Diagnostic Map: Space

What is the Diagnostic Map for Space?

How students currently think about spatial ideas will influence how they respond to the activities provided for them, and hence what they are able to learn from them. As students' thinking about space develops, it goes through a series of characteristic phases. Recognising these common patterns of thinking should help you to interpret students' responses to activities, to understand why they seem to be able to do some things and not others, and also why some students may be having difficulty in achieving certain outcomes while others are not. It should also help you to provide the challenges students need to move their thinking forward, to refine their half-formed ideas, to overcome any misconceptions they might have and hence to achieve the outcomes.

Most students will enter the Recognising phase between 4 and 5 years of age.

During the Emergent phase:

As students move about their environment and explore the objects in it, they respond perceptually to spatial features, encoding shape and the location of objects they can see within a framework of landmarks.

As a result, they begin to name things they can see and handle in ways that reflect attention to shape and they can match simple shapes in an impressionistic way.

Also as a result, they begin to understand that we can represent the relative position of neighbouring things, for example, placing a toy boy under the toy table to 'stand for' the real boy under the real table.

By the end of the Emergent phase, students typically:

- distinguish shape from other attributes that relate to how things 'look' (colour, size, texture), although they may not do so consciously
- use informal language that indicates they are responding to shape; e.g. 'the pointy one'
- carry out matching tasks by selecting a matching shape from a collection and either posting shapes in boxes or fitting shapes into cut outs
- notice similarity in the shape of familiar things, saying, for example, it 'looks like' a seesaw or a car or a ball
- reproduce simple geometric configurations if only encoding is required; that is, build a matching shape or arrangement to one that is constantly in sight
- draw simple figures by imitating how they have seen them drawn (including letters and numbers)
- give directions from one landmark to the next when retelling a journey or places in a story; e.g. 'go to the pond, go on the bridge, go home'.

As students move from the Emergent phase, to the Recognising phase, they:

- refer to objects by their everyday or toy name (blocks, bricks, witch's hat) rather than their shape
- may not think to turn a figure over or around in order to match or post in cut outs
- may have difficulty in matching a shape by feel alone (e.g. in feely bags) as they grope and pat objects rather than explore in a way oriented to discerning shape
- given drawings, will not distinguish, for example, triangles from 'almost triangles', relying on an impressionistic match
- may be able to copy a figure such as a square with matchsticks but not be able generally to copy one from a ready made drawing unless shown how (that is, they have difficulty in dissecting the parts and deciding the sequence or route to bring the components together).

These are the learning challenges for the Recognising phase.

Most students will enter the Describing phase between 6 and 7 years of age.

During the Recognising phase:

Students' exploration of objects and space through touch and sight gradually becomes more regulated as they attend to spatial features and construct mental and visual representations of shapes and arrangements in space.

As a result, they can copy simple figures and recognise figures of 'the same shape', constructing visual images or prototypes of what people mean when they refer to common figures and objects; e.g. *This is a rectangle because it looks like a rectangle.*

Also as a result, they construct visual images of familiar objects and of where objects are within familiar spaces and locations.

By the end of the Recognising phase, students typically:

- describe figures and objects using terms that are evocative of shape, such as 'corner', 'pointy', 'lopsided', 'slanty'
- learn the names of some shapes (triangle, cube), although which names they know will depend upon the frequency and naturalness of everyday use at school and home
- describe conventional figures and objects by reference to prototypes they 'look like'; e.g. 'It's a door shape'.
- select ready-made materials that 'look right' to make recognisable models of parts of their environment (e.g. circular pieces for wheels, a cylinder for a tree trunk)
- remember what some families of shapes look like and produce recognisable versions; e.g. draw a figure that resembles a triangle with three lines that more or less join at their end points or as a continuous curve with three 'straight' sides and three corners
- remember key aspects of the way things look and try to reproduce them in their drawings; e.g. drawing circles for wheels, putting two eyes, a nose and a mouth on a face
- begin to give simple explanations that relate shape to purpose (e.g. circles for wheels, blocks to stack)
- relate the position of objects to each other in familiar settings using terms such as 'behind', 'near'
- draw or make simple 'route' maps and models that show a sense of spatial relationships and order, although only for local settings that they have freely explored.

As students move from the Recognising phase to the Describing phase, they:

- explore objects using touch (e.g. in a feely bag) oriented to shape as a whole, and are not generally focused on the parts
- while implicitly knowing some features of a familiar type of figure (triangles have three sides, triangles are pointy), do not recognise them in that way; a triangle is a triangle because it looks like one
- can identify familiar shapes singly but not within complex configurations or in non-standard orientations
- use terms such as 'corner', 'pointy', 'lopsided', 'slanty' vaguely and inconsistently
- are not consciously aware of properties; e.g. they could produce a recognisable rectangle without realising that it had right angles
- although able to perceive the difference between a 2D and a 3D thing, may think the word 'shape' refers to a 2D attribute and so may say there is no difference in the shape of a ball and a hoop, since both are called by the same shape name 'circle'
- when drawing a 2D figure (e.g. a circle) to represent a 3D object (e.g. a ball), think of the region inside the circle as inside the ball rather than as the surface of the ball
- in trying to represent what an object is rather than how it happens to look, may draw what they know to be there; e.g. they show a hidden handle on a cup, or draw more sides of a cube than one could possibly see
- will often show a mixture of viewpoints in the same picture; e.g. side view of the legs of the table and top view of the table top and the items on it.

These are the learning challenges for the Describing phase.

Most students will enter the Analysing phase between 8 and 10 years of age.

During the Describing phase:

Through their own physical and perceptual action on spatial configurations focused on interpreting, describing and representing the parts making the whole, students make sense of the spatial relationships within figures, objects and arrangements and in the visual representations of them.

As a result, they identify the features of particular figures *(This has four sides and two of its sides are equal.)* and objects *(This has six faces and they are all rectangles.)* and construct 3D meanings for the 2D representations of 3D that are conventional within their culture.

Also as a result, students pay attention to the shape and placement of component parts when they draw, match, make and copy things and are able to think of objects in positional relationship to each other rather than in relation to themselves.

By the end of the Describing phase, students typically:

- respond to a request to 'tell me about the shape of this …' using language such as 'flat', 'curved', 'side', 'round', 'face', 'edge', 'square', 'angle', 'base'
- compare and contrast geometric figures
- are able to identify the faces, edges and vertices of a geometric object and hence select component parts to make it in various forms (skeletal, hollow)
- understand that the word 'shape' refers to or signifies both a 2D and 3D attribute and so understand, for example, that a cube and a square are different shapes and have different names even if they cannot recall the names
- when using a 2D figure (e.g. a circle) to represent a 3D object (e.g. a ball), interpret the region inside the circle as representing the surface of the ball
- match the 2D shapes with the faces of standard 3D shapes
- select nets that have the right component parts to match a simple object
- pay attention to the shape and placement of component parts when they interpret and make drawings
- observe the component geometric parts within pictures and patterns and the movements needed to produce them
- attempt to produce visual reality in drawings by only drawing the objects or parts of objects that can be seen
- rearrange and combine a few shape pieces (e.g. tangrams) to make another specified shape, such as a square
- repeat multiple copies of a figure in a systematic way to create a pattern
- recognise repetitions of the same shape embedded within arrangements and patterns
- identify component parts to show that a shape or arrangement is symmetrical
- are able to describe one thing being between others and put key features in order on a map
- attempt to show a bird's eye view of familiar settings with a 'rough' sense of proximity.

As students move from the Describing phase to the Analysing phase, they:

- may think of a figure as a 'picture' of a shape and so may, for example, think of a slant parallelogram as a rectangle looked at from 'the side' (as in 2D drawings of 3D)
- may keep the same name for a transformed figure; e.g. a rhombus is just a pushed over square so it is still a square, an ellipse is a stretched out circle
- may use descriptive terms in ambiguous or incorrect ways; e.g. using 'side' to mean 'on the side' as distinct from 'top' or 'bottom'
- may still respond to figures by their overall appearance, and may therefore continue to be tricked when a shape is presented in an unfamiliar orientation; e.g. a square drawn 'on its point' may not be recognised as a square, others will say that will be a square 'if you turn it around'
- will be aware of some properties related to a common figure, but these properties may continue to play no detectable role in the recognition of the figure and students do not generally call upon properties to justify why a figure is or is not in a particular class
- may give vague descriptions that could apply to a number of different shapes, perhaps focusing on only one feature of a figure or object
- although now understanding which objects or parts of objects are seen or not seen, do not yet understand how they are seen (shape, orientation, size) and so have difficulty making their drawings 'look right'
- even when provided with a drawing to copy, are influenced by how they think about or describe the object to themselves, so if told a drawing is a cube, they make the top face more square, if told it is a house they make the top more slanted
- select or draw nets that have the right component parts to make a particular object, but often ignore the relationship of the parts to each other so do not position them correctly to fold into a net
- may have little overall sense of relative position or scale in their plans and maps; e.g. they may draw their own desk larger than those of other students in the class.

These are the learning challenges for the Analysing phase.

Diagnostic Map: Space

Most students will enter the Relating phase between 11 and 13 years of age.

During the Analysing phase:

As students consciously compare and contrast spatial configurations, they form generalisations about relationships both within and between figures, objects, movements and arrangements. Through their own experimentation, they realise that when an object or arrangement is transformed, relationships between its component parts may be preserved or not, and they try to visualise 'what happens' when things are represented or moved.

As a result, students establish that shapes and movements they recognise as in the same class have features in common, thus the term 'triangle' can now be interpreted as a collection of properties (a closed figure with three sides) that can be represented by many figures.

Also as a result, students try to ensure that desired relationships are preserved when they make (e.g. produce a net of an object, make a scaled copy), represent (e.g. draw a map or a diagram of an object) or move things (e.g. look from a different view, fold and unfold, turn).

By the end of the Analysing phase, students typically:

- give a detailed list of properties in their descriptions of shapes, confidently asserting, for example, that rectangles always have four sides and always have right-angled corners
- select figures and objects based on geometric descriptions such as 'has five faces and nine edges'
- know from the properties of a rectangle that a slanted parallelogram cannot be a rectangle even though it is what a rectangular face on a block 'looks like' from 'the side'
- understand that lines drawn obliquely to the horizontal suggest depth and incorporate this into their drawings of objects
- use mathematical conventions to represent objects in different types of drawings
- match suitable nets to prisms and pyramids that are actually present (not drawn) by considering the shape and placement of the component parts
- produce their own nets for geometric shapes that they can see and handle
- visualise the folding process to say which of a number of potential nets that have the right number and shaped components will actually fold up to form a cube or prism
- select and build arrangements of geometric figures to match information in drawings and plans
- describe characteristic features of mirror symmetry; e.g. may explain that for mirror symmetry matching parts of figures are the same distance away from the mirror line
- visualise and reproduce the folds and cuts needed to produce symmetrical designs
- explain why they think a shape will not tile by focusing on the corners
- identify the particular rotations, reflections and translations that relate the component parts of simple arrangements and patterns
- understand that when figures are rotated, reflected and translated the position and/or orientation change but the size and shape do not, so the original figure can be superimposed on the transformed (or moved) figure
- understand that when figures are enlarged or reduced, the shape stays the same but the size changes (the position and/or orientation may change); that is, 'scaled' figures keep the same shape and so look 'the same but smaller' or 'the same but bigger'
- understand the use of 'scale' on a map to preserve proximity between things being represented; that is, to show the relative distance between things
- recognise and use a top view to represent familiar locations on plans using order and relative proximity among landmarks.

As students move from the Analysing phase to the Relating phase, they:

- may include irrelevant features as properties of families of shapes if they have only experienced shapes in upright orientations; e.g. they may think that the 'top' and 'bottom' sides of a trapezium have to be parallel
- may have a correct verbal description for a concept and 'know' they should use it but be unable to override a strong visual image or prototype; e.g. they may reject a square resting on its point as a square, even though it fits their definition
- may not generally understand that one class of shapes can be included in another, so that while they can be taught to recite that squares are special rhombuses, most do not understand why or how
- may have a well-developed concept of a particular shape, but their definitions may not provide sufficient features to define the shape; e.g. they may say that a rectangle is a shape with four sides and opposite sides the same length
- may define a class of geometric shapes and include many of the known features, not simply a sufficient set, thus providing redundant information
- may see properties as distinct from each other and so not see that some properties are a consequence of others; e.g. if a quadrilateral has four right angles it must have opposite sides the same length
- cannot typically coordinate all the components and measurements needed to plan a net completely from imagination or from specifications (except for simple well-rehearsed objects).

These are the learning challenges for the Relating phase.

During the Relating phase:

Students develop coordinated mental representations of spatial configurations in relation to their component parts enabling them to mentally manipulate and transform figures, objects and arrangements. Through investigating *properties* of shapes and movements and inter-relationships between them, their use of visual images becomes constrained by their more abstract verbal knowledge of the properties.

As a result, students are able to visualise the result of systematically moving or folding figures or moving objects or themselves in relation to an object and to represent transformations. They also integrate distance and direction in their descriptions of paths and locations and can represent them on coordinate systems.

Also as a result, students come to recognise relationships between properties and between common classes of shapes; e.g. *This square is also a rectangle because it has all the properties of a rectangle.*

By the end of the Relating phase, students typically:

- understand what a definition is and use counter examples to show that a definition such as 'a rectangle is a shape with four sides and opposite sides the same length' is not adequate because it does not exclude some shapes that are not rectangles
- use properties to convince themselves and others why a figure or object belongs to a class; e.g. *This is a square because it has four equal sides even though it is not resting on a 'flat bottom'.*
- understand that knowing just a few properties of a figure or object enables us to work out (deduce) other properties
- understand relationships between properties of figures; e.g. if a triangle has two equal angles then it has two equal sides
- understand class inclusion and so can classify figures and objects hierarchically; e.g. all squares are rectangles but not all rectangles are squares
- produce their own nets, considering in advance the level of precision needed to ensure the shape is correct in form and size, where tabs will be, and so on
- predict which face on nets will match which face on corresponding objects
- predict the effect of particular movements (translations, rotations and reflections) on the orientation and position of figures and objects
- visualise an object or scene in different orientations, drawing other possible views of an object from information in 2D drawings.

Contents

Foreword

The ***First Steps in Mathematics*** Resource Books and professional development program are designed to help teachers to plan, implement and evaluate the mathematics curriculum they provide for their students. The series describes the key mathematical ideas students need to understand in order to achieve the mathematics outcomes described in the Western Australian *Curriculum Framework* (1998).

Each Resource Book is based on five years of research by a team of teachers from the Department of Education and Training, and tertiary consultants led by Professor Sue Willis at Murdoch University. The ***First Steps in Mathematics*** project team conducted an extensive review of national and international research literature, which revealed gaps in the field of knowledge about students' learning in mathematics.

Using tasks designed to replicate those in the research literature, team members interviewed students in diverse locations. Analysis of the data obtained from these interviews identified characteristic phases in the development of students' thinking about major mathematical concepts. The Diagnostic Maps—which appear in the Resource Books for Number, Measurement, Space, and Chance and Data—describe these phases of development.

It has never been more important to teach mathematics well. Globalisation and the increasing use of technology have created changing demands for the application of mathematics in all aspects of our lives. Teaching mathematics well to all students requires a high level of understanding of teaching and learning in mathematics and of mathematics itself. The ***First Steps in Mathematics*** series and professional development program will enhance teachers' capacity to decide how best to help all of their students achieve the mathematics outcomes.

The commitment and persistence of many teachers and officers of the Department of Education and Training, who contributed to the research and development of ***First Steps in Mathematics***, is acknowledged and appreciated. Their efforts have resulted in an outstanding resource for teachers. I commend this series to you.

Paul Albert
Director General
Department of Education and Training
Western Australia

CHAPTER 1

What Are the Features of this Resource Book?

The ***First Steps in Mathematics: Space*** Resource Book will help teachers to diagnose, plan, implement and judge the effectiveness of the teaching and learning experiences they provide for their students.

This Resource Book includes the following elements.

- Diagnostic Map
- Mathematics Outcomes
- Markers of Progress
- Pointers
- Key Understandings
- Sample Learning Activities
- Sample Lessons
- 'Did You Know?' sections
- Background Notes

Diagnostic Maps

The purpose of the Diagnostic Maps is to help teachers:

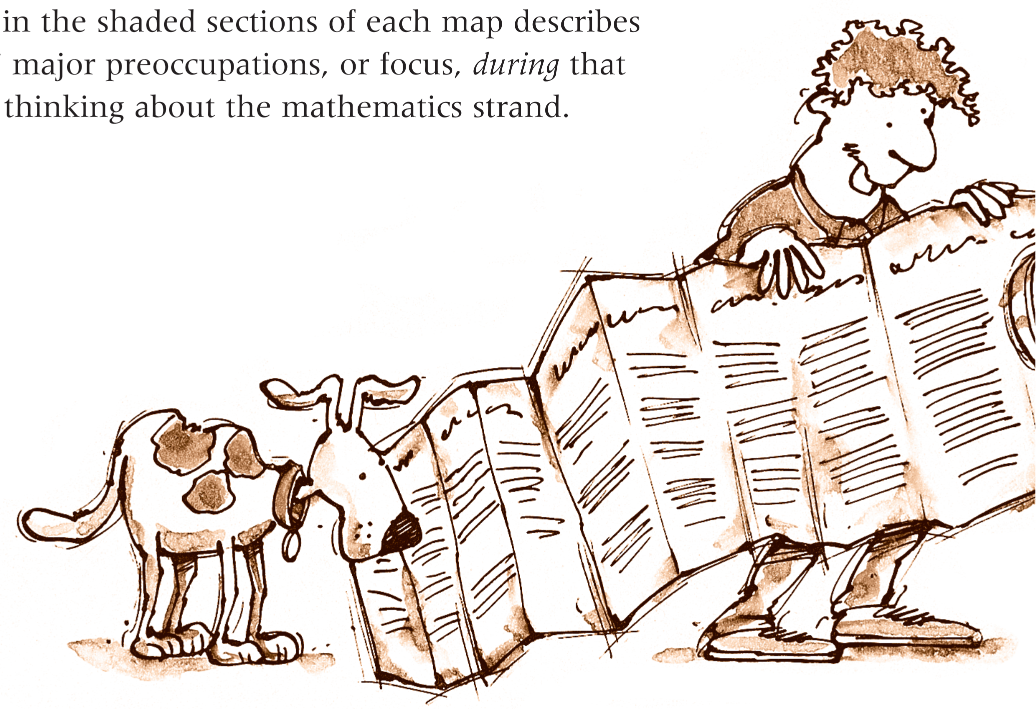
- understand why students seem to be able to do some things and not others
- realise why some students may be experiencing difficulty while others are not
- indicate the challenges students need to move their thinking forward, to refine their preconceptions, overcome any misconceptions, and so achieve the outcomes
- interpret their students' responses to activities.

Each map includes key indications and consequences of students' understanding and growth. This information is crucial for teachers making judgments about their students' level of understanding of mathematics. It enhances teachers' judgments about what to teach, to whom and when to teach it.

Using the Diagnostic Maps

The Diagnostic Maps are intended to assist teachers as they plan their mathematics curriculum. The Diagnostic Maps describe the characteristic phases in the development of students' thinking about the major concepts in each set of outcomes. The descriptions of the phases help teachers make judgments about students' understandings of the mathematical concepts.

The text in the shaded sections of each map describes students' major preoccupations, or focus, *during* that phase of thinking about the mathematics strand.



The 'By the end' section of each phase provides examples of what students typically think and are able to do as a result of having worked through the phase.

The achievements in the 'By the end' section should be read in conjunction with the 'As students move from' section. The 'As students move from' section includes the preconceptions, partial conceptions and misconceptions that students may have developed along the way. These provide the learning challenges for the next phase.

Together, the 'By the end' and 'As students move from' sections illustrate that while students might have developed a range of important understandings as they passed through the phase, they might also have developed some unconventional or unhelpful ideas at the same time. Both of these sections of the Diagnostic Map are intended as a useful guide only.

Mathematics Outcomes

The mathematics outcomes indicate what students are expected to know, understand and be able to do as a result of their learning experiences. The outcomes provide a framework for developing a mathematics curriculum that is taught to particular students in particular contexts. The outcomes for Space are located at the beginning of each section of the Resource Book.

Markers of Progress

There are Markers of Progress for each mathematics outcome. The ***First Steps in Mathematics*** Resource Books provide five markers of progress towards these outcomes, covering the typical range of achievement in primary school.

The Markers of Progress describe the development of student understanding towards full achievement of the outcomes. Each student's achievement in mathematics can be monitored and success judged against the Markers of Progress.

As the phases of the Diagnostic Map are developmental, and not age specific, the Markers of Progress provide teachers with descriptions of the expected progress that students will make every 18 to 20 months when given access to an appropriate curriculum.

Pointers

Each Marker of Progress has a series of Pointers. They provide examples of what students might typically do if they have developed the skills outlined in the marker. The Pointers help clarify the meaning of the mathematics outcomes and the differences between the Markers of Progress.

Key Understandings

The Key Understandings are the cornerstone of the ***First Steps in Mathematics*** series. The Key Understandings:

- describe the mathematical ideas, or concepts, which students need to know in order to achieve the outcome
- explain how these mathematical ideas relate to the markers of progress for the mathematics outcomes
- suggest what experiences teachers should plan for students so they achieve the outcome
- provide a basis for the recognition and assessment of what students already know and still need to know in order to progress
- indicate the emphasis of the curriculum at particular stages
- provide content and pedagogic advice to assist with planning the curriculum at the classroom and whole-school levels.

The number of Key Understandings for each mathematics outcome varies according to the number of 'big mathematical ideas' students need to achieve the outcome.

Sample Learning Activities

For each Key Understanding, there are Sample Learning Activities that teachers could use to develop the mathematical ideas of the Key Understanding. The activities are organised into three broad groups.

- Beginning activities are suitable for students 4 to 8 years old.
- Middle activities cater for students 8 to 10 years old.
- Later activities are designed for students 10 years and older.

If students in the later years have not had enough prior experience, then teachers may need to select and adapt activities from earlier groups.

Sample Lessons

The Sample Lessons illustrate some of the ways teachers can use the Sample Learning Activities for the Beginning, Middle and Later groups. The emphasis is on how teachers can focus students' attention on the mathematics during the learning activity.

'Did You Know?' Sections

For some of the Key Understandings, there are 'Did You Know?' sections. These sections highlight common understandings and misunderstandings that students have. Some 'Did You Know?' sections also suggest diagnostic activities that teachers may wish to try with their students.

Background Notes

The Background Notes supplement the information provided in the Key Understandings. These notes are designed to help teachers develop a more in-depth knowledge of what is required as students achieve the mathematics outcomes.

The Background Notes are based on extensive research and are more detailed than the descriptions of the mathematical ideas in the Key Understandings. The content of the Background Notes varies. Sometimes, they describe how students learn specific mathematical ideas. Other notes explain the mathematics of some outcomes that may be new or unfamiliar to teachers.

CHAPTER 2

The Space Outcomes

The Space strand focuses on the spatial features of objects, environments and movements; their positions, their transformations, their properties and how to draw and model them. As a result of their learning, students should be able to visualise, draw and model shapes, locations and arrangements and to predict and show the effect of transformation on them. They should be able to reason about shapes, transformations and arrangements to solve problems and justify solutions.

This requires the ability to attend to the shape and placement of parts when visualising, drawing and modelling 2D and 3D shapes and an understanding of maps and plans, including the words and symbols used on them. It requires a sound understanding of the conventional words and symbols we use to talk about shapes and an understanding of the way in which we classify them using conventional geometric criteria. This also includes an understanding of how patterns and arrangements can be created through the systematic movements (or transformations) of shapes. Thus learning experiences should be provided that will enable students to represent locations, represent shapes, represent transformations and to reason geometrically.

As a result of their learning experiences, students should be able to achieve the following outcomes.

Represent Spatial Ideas

Visualise, draw and model shapes, locations and arrangements and predict and show the effect of transformations on them.

There are three parts to this outcome—Part A: Represent Location, Part B: Represent Shape, and Part C: Represent Transformation. Each is dealt with in a separate chapter.

Reason Geometrically

Reason about shapes, transformations and arrangements to solve problems and justify solutions

Integrating the Outcomes

The outcomes suggested above for Space are each dealt with in a separate chapter. This is to emphasise the importance of each and the difference between them. For example, children need to reason about the properties of shapes (Reason Geometrically) as well as being able to draw or model 2D and 3D shapes (Represent Shape). Sometimes, however, we focus on the former and give insufficient attention to the latter, assuming that knowing how to name and classify shapes will enable children to draw and model them using mathematical conventions and understandings. By paying separate and special attention to each outcome, teachers can make sure that both areas receive sufficient attention and that important ideas about each are drawn out of the learning experiences they provide.

This does not mean, however, that the ideas and skills underpinning each of the outcomes should be taught separately or that they will be learned separately. The outcomes are inextricably linked. Consequently, many of the activities will provide opportunities for students to develop their ideas about more than one of the outcomes. This will help teachers to ensure that the significant mathematical ideas are drawn from the learning activities so that students achieve each of the outcomes for Space.

A Snapshot of the Markers of Progress in Space

Students should not always be expected to be at the same level of understanding for each of the outcomes in Space. Students vary, so some may progress more rapidly with several aspects of Space than others. Teaching and learning programs also vary and may, at times, inadvertently or deliberately emphasise some aspects of Space more than others.

Nevertheless, while the outcomes for Space are dealt with separately in these materials, they should be developing together and supporting each other, leading to an integrated set of concepts within students' heads.

The Markers of Progress for each mathematics outcome indicate the typical things students are expected to do at the same time. Generally, students who have access to a curriculum that deals appropriately and thoroughly with each of the outcomes reach a particular level at roughly the same time for each outcome in Space.

> *A student has demonstrated a marker of progress towards a* ***particular outcome*** *when he or she is able to do all the things described consistently and autonomously over the range of common contexts or experiences.*
>
> *A student has achieved a* ***set of outcomes*** *when he or she consistently and autonomously produces work of the standard described.*

Judgment will be needed to decide whether a student is progressing towards a particular outcome. When mapping and reporting a student's long-term progress, a teacher has to find the marker of progress for a particular outcome or the marker for the set of outcomes that best fits the student, in the knowledge that no description is likely to fit perfectly.

The Space Markers of Progress are elaborated upon on pages 210 to 227.

CHAPTER 3

Represent Location

This chapter will support teachers in developing teaching and learning programs that relate to Part A of the outcome:

Visualise, draw and model locations and arrangements.

Overall Description

Students understand and use the everyday language needed to describe paths, locations and arrangements. They interpret and construct a range of maps and diagrams that are familiar within their communities, understanding that different maps emphasise different features of the space or objects being represented. They visualise and sketch paths, regions and arrangements to meet specifications. Perhaps they show their route to school, plan a visit to the zoo, plan an optimal route for a tour of country towns by a rock group, sketch the region covered by a special sprinkler system, or interpret a kinship network. In representing location and arrangements spatially, they use a range of conventions, including coordinates and networks.

Markers of Progress	Pointers Progress will be evident when students:	
Students use and interpret familiar everyday language for the position of things, movements of them, and paths between them.	• respond appropriately to the language of position and orientation such as 'under', 'behind', 'in front of', 'below', 'on', 'alongside', 'near', 'right' and 'left'; e.g. carry out the request to 'Put the dolly near the bears.' • use some of the language of position, movement and orientation for themselves; e.g. *The book is under the desk. The bee flew around my head.* • respond appropriately to the language of movement such as 'back', 'forward', 'around', 'past', 'turn' and 'up'	• draw pictures or make things that illustrate the meaning of words relating to position and orientation; e.g. a picture with 'the crocodile behind the boy' • move themselves to illustrate the meaning of words relating to position, orientation or movement; e.g. in drama or dance
Students attend to order and betweenness on informal maps and in descriptions of locations and paths.	• place important things in their environment in order on their map; e.g. the taps, toilet and shady trees in the school yard • describe the location of an object as 'between' two other locations • find paths on informal maps and mazes; e.g. a path between two parts of the playground or through computer-generated mazes	• give instructions for creating paths on squared paper or computer screens that require movements to be done in order; e.g. 'forward 3, then turn right, then forward 7, then turn left' • follow directions that use betweenness and order to indicate position; e.g. find the lost keys when told they were dropped between the two trees, but past the tap
Students understand a map or plan as a 'bird's eye view' and use order, proximity and directional language associated with quarter and half turns on maps and in descriptions of locations and paths.	• attempt to provide a bird's eye view of familiar locations such as their classroom • order and show a sense of the proximity of things in locating key features on maps; e.g. correctly place the bus stop between the park and shop but closer to the shop • attend to the order and proximity of things in giving directions; e.g. *As soon as you get to the tree, look to the right …*	• use directional language associated with quarter and half turns (such as north, south, east, west, right angle, quarter turn, right, left) to describe a route; e.g. *Go east for 20 metres and then due north for …* • predict the effect of following a rule involving a simple sequence of movements by imagining looking down on the path mapped out; e.g. 'Two steps forward, turn right, two steps, turn right, two steps, turn right, …'
Students use distance, direction and grids on maps and plans and in descriptions of locations and paths.	• give unambiguous instructions for moving and locating objects in their environment or on models, maps or plans using distance, direction, including angle multiples of 45°, and common map grids • place or describe key features on a map or path with sufficient care so that others can use them; e.g. provide a tour map of their school for visitors • interpret maps relevant to their school, home and community to find their way around the actual environment; e.g. use a plan of the library to find a particular book	• use conventional maps to find locations and paths that meet everyday specifications, such as the closest post office or the safest route • use the informal idea that a particular map is 'to scale' in interpreting it, e.g. 'the map shows that the river is about ten times further away than the road' • draw maps and plans that show a sense of scale • use whole-number scales such as 1 centimetre for each metre
Students use coordinates, bearings and scale on maps and plans and in descriptions of locations and paths AND they identify the essential features of a location or arrangement needed to serve a purpose and represent in networks and other diagrams.	• sketch a path on a map or model, or follow a path through a location, given directions based on distance, angle and compass bearings • give unambiguous instructions for describing locations and paths using distance, angle and compass bearings; e.g. write a set of directions to assist visually impaired students find their way round a part of the yard • follow and give directions for locations and paths using ordered pairs; i.e. coordinates • interpret scales expressed as ratios when using maps and plans; e.g. 1:100 • make scale maps and plans where the location of components is mostly based on straight lines and right angles	and when students: • decide on the key features of a location that need to be identified by considering the purpose • draw diagrams to represent familiar places that they can see or walk around; e.g. use a network-type diagram to represent the location of main buildings in the school • represent in diagrams the systematic movements needed to change an arrangement under certain constraints; e.g. solve problems such as 'River Crossing' (see page 36)

Key Understandings

Teachers will need to plan learning experiences that include and develop the following Key Understandings (KU), which underpin achievement of the outcome. The learning experiences should connect to students' current knowledge and understandings rather than to their year level.

Key Understanding	Stage of Primary Schooling—Major Emphasis	KU Description	Sample Learning Activities
KU1 We describe where things are in relation to other things. There are special words, phrases and symbols that help us with this.	Beginning ✔✔ Middle ✔✔ Later ✔✔	page 12	Beginning, page 14 Middle, page 17 Later, page 20
KU2 Some maps or diagrams show the order of things and what comes between what. Others also represent distances and directions between things.	Beginning ✔✔ Middle ✔✔ Later ✔✔	page 26	Beginning, page 28 Middle, page 31 Later, page 34
KU3 Plans show the placement and relative size of things from a top view.	Beginning ✔✔ Middle ✔✔ Later ✔✔	page 38	Beginning, page 40 Middle, page 43 Later, page 46

Key
✔✔✔ The development of this Key Understanding is a major focus of planned activities.
✔✔ The development of this Key Understanding is an important focus of planned activities.
✔ Some activities may be planned to introduce this Key Understanding, to consolidate it, or to extend its application. The idea may also arise incidentally in conversations and routines that occur in the classroom.

KEY UNDERSTANDING 1

We describe where things are in relation to other things. There are special words, phrases and symbols that help us with this.

When we think about and describe where something is, it is generally in reference to other things. We say it is 'near the telephone', 'south of the river', 'first on the left after you turn right into Brown Street' and 'over there'. Students should develop the everyday language and some of the special mathematical ways of talking about location and arrangement, including the use of a coordinate grid.

When describing where things are, young students attend largely to closeness or proximity, readily using words such as 'on', 'in' or 'under' and 'near'. Over time, they learn that there are different degrees of 'nearness' and that we can describe the location of things in relation to each other even when they are far apart or cannot both be seen from the same position. Many students initially use a narrative style, recounting a kind of journey from one item to the next: 'There's the truck, then the car and then the bike.' Later, they are able to use language that focuses on the relationships between objects: 'The car is between the truck and the bike.' They eventually build up a kind of mental map that enables them to simultaneously think about many locations and describe the possible routes between them: 'Walk 10 metres and turn right.'

Students will need many experiences arranging and rearranging familiar objects both freely and by following oral instructions, and will need opportunities to try out the language for themselves. Their early ideas about 'where it is' should broaden to encompass directional information, such as 'facing the door', 'to the left of', or 'north of'. They should develop the language associated with direction (up/down, clockwise/anti-clockwise) and movement (forward/backward, go around, turn) developing more technical spatial terms, symbols and methods of representation (angle, north, NWW, parallel, grid, coordinate, (3, 4), 60°).

Grids and coordinates provide the major mathematical contribution to representing location and direction, the essential idea being that we can use numbers (usually pairs of numbers) to describe where something is. Often students' experience with locating things on grids involves focusing on the squares where rows and columns intersect. In order to take advantage of the numerical dimensions of a coordinate system (for distances to make sense), students need to be able to focus upon points as the intersection of two grid lines. (See Did You Know? page 22).

The language of direction and the kinds of reference points used for finding our way around will often differ with local customs and the type of environment. These could range from high-density inner cities where street names, locally named corners and right or left turns dominate to small island communities where landmarks like a rocky outcrop or the direction to the closest mainland harbour might be used. Students should be encouraged to make use of what they already know as a starting point for developing new ways of navigating and communicating direction. When their descriptions of positions and paths serve real purposes, students are more likely to realise when their descriptions lack clarity or precision.

Progressing Through Key Understanding 1

Initially students use the everyday words of position (under, behind, in front of) and movement (back, forward, turn) and so they could 'put the bear *behind* the chair' and tell you that the 'book is *under* the table' and that they 'walked *past* the taps'.

As students continue to progress they will attend to and describe 'betweenness' and order relationships among a number of objects or places, but may not consider relative distance and direction. Next, students will know that direction and distance is important and will try to convey this, but may include non-essential information and personalised knowledge not shared by the listener.

As students progress further they give clear if simple directions for moving and locating things in the actual environment or on models, plans, maps or computer screens using both distance and direction and conventional map grids. They also plan and describe routes on a road map to fit specifications such as the shortest, the safest or one that does not involve retracing one's steps. Later, students interpret and use technical terms and measurements associated with degrees of turn, compass directions and bearings, distance, scale and coordinates.

SAMPLE LEARNING ACTIVITIES

Beginning ✔✔

Fitness and Drama

Have students follow oral instructions that involve directional language during activities that involve physical movement, such as sport and drama. Use terms like 'in front of', 'behind', 'next to' in relation to themselves and to others. For example, during sport or drama, ask students to wave their hands behind, above, at the side of themselves; ask them to jump backwards, sideways and forwards.

Simple Simon

Invite students to follow specific oral directions involving words like 'between', 'under' and 'next to' to place themselves in relation to other objects. For example, when playing 'Simple Simon', say: Simon says, stand next to the door. Simon says, put your hand under your chair. Extend this to include directions that involve students thinking about the position of more than one object. For example, say: Simon says, stand between the wall and the desk. Simon says, put your hand under your chair and next to the leg.

Instructions for Others

After playing games like 'Simple Simon', where students have been following instructions, invite students to give the instructions for a partner to follow (e.g. how to set the table, how to make a car from blocks). Later, extend this by having students direct the movement and positioning of a third object (e.g. *Put teddy in the car next to the toy box*). Encourage students to use position words to clarify what their partner means (e.g. *Do you want the teddy in, on or next to the toy box?*)

Computers

Use questions and modelling to make students' use of positional language more specific. For example, when students are using drawing software to create a picture, ask: Where will you put the bird? Will it be *in* a tree? *On* the fence? What can it be *near*? Suggest other objects to refer to. For example, when students say their bird is 'over there', ask: What can you say about the bird and the rock? Is the bird near (on, over, next to) the rock? (See Sample Lesson 1, page 23.)

Visiting Classrooms

Encourage students to use the word 'between' to describe location. For example, move around the school, visiting classrooms so that the order in which rooms are visited does not match their physical arrangement. Ask: Which room is *in between* the music room and the cafeteria? Which room did we visit *between* visiting the music room and the cafeteria? Ask: What does the word 'between' mean? Which room was next to the music room and the cafeteria?

Mystery Object

Invite one student to select an object in the classroom and give others clues to its location, such as *It's on the bookcase. It's near the whiteboard. It's under the window.* Ask the student questions: Is it to the left of the clock? Is it facing the wall? Encourage students to ask similar questions.

Compass Walls

Label the walls or corners of the classroom with north, south, east and west. Refer to these directions incidentally and when giving directions in the classroom. Extend 'Mystery Object' by encouraging students to use the four directions in their clues and questions. Vary the game by asking one student to leave the room while an object is hidden. When the student returns, encourage the rest of the class to give directions to the hidden object using only north, south, east or west. Ask: Which direction is Elen facing? Which direction does she need to be facing to find the mystery object? (See Key Understanding 2.)

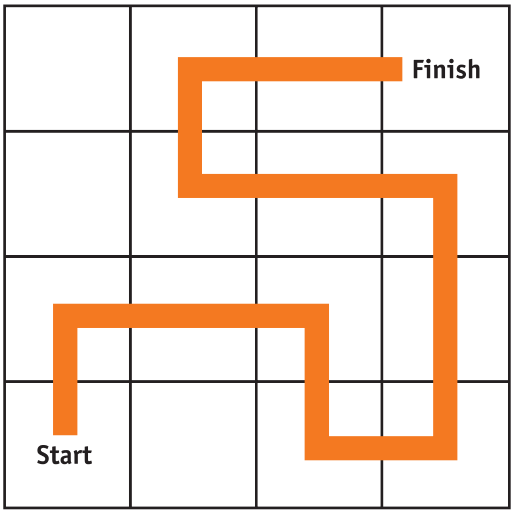
Aerial View

Display a large aerial photo of the local area. Have students practise describing routes using 'left' and 'right' (or north, east, south and west) in relation to the streets to go from one place to another. For example, *To go from Stephen's house to school, turn left at the first corner, and then left at the next corner and then right.* Take a neighbourhood walk and match where you have just been to the map. For example, ask: Which way did we turn when we got to Harper Street? How many streets did we pass before we needed to turn left onto Robert Avenue? (See Key Understanding 3.)

Beginning ✔✔

Moving on a Grid

Have students follow instructions to move around a grid system. For example, make a 4 x 4 grid on the floor using 16 carpet squares and make cards with paths drawn through the grid. Invite one student to take a card and call out directions for another student to follow the path through the grid (e.g. *one forward, two to the right, one backwards*). Encourage students to check each other's instructions and movement through the grid. Extend the activity by asking students to label the rows and columns to make giving instructions easier (e.g. *Go to row red, column 2*).



Grid of Roads

To introduce grid coordinates, make a grid of roads using paper tape on the floor. Label each of the roads using known street names. Have several students stand on the intersections of two streets. Ask: If you wanted to meet Ilya and Rhial, where would you find them? When a student names one street (e.g. High Street), ask: How far along High Street? Could we tell where they are by naming two streets? Invite three or four students to walk down the streets. When they meet someone at a corner, ask: What are the two streets that make the corner?

SAMPLE LEARNING ACTIVITIES

Middle ✔✔

Mystery Object

Extend Beginning Sample Learning Activity 'Mystery Object' by having students play in pairs and use directional language (e.g. left, right, north, east, south, west, forwards, backwards, half turn, quarter turn) to lead partners to the object. Explain to them that they can direct their partner to the object only by saying how many steps to take and in what direction.

Obstacle Course

Take digital photos of obstacle courses students have designed and walked through. Encourage them to use the photographs to write directions explaining how to complete the course. Invite them to give the instructions to a partner to test.

Virtual Tour

Invite students to plan a series of locations to visit, such as the library or the cafeteria. Have them walk from one to the next, taking digital photos as they go. In the classroom, record students' oral descriptions of the route they took. Use drawing software and the photos to create a 'virtual tour' of the school. If possible, include it on the school website for visitors to use. Ask: Could you use some other words to help make your description clearer? What sort of words would be helpful?

Aerial View

Using copies of an aerial photograph of the local area, invite students to tell a partner how to get to their house (the local shop, the library) from the school. Ask partners to draw a line on the map to show the pathway described. Invite students to compare the pathways drawn to the pathway they had visualised. Ask: What direction words were the most helpful in deciding where to go or which direction to move? (See Key Understanding 3.)

Show Time

Give students maps of a showground, showing the amusement rides along with an entrance and an exit. Invite them to choose the rides they would like to see and describe their pathway through the showground to their partner, using directional language (e.g. left, right, north, east, south, west, forwards, backwards, half turn, quarter turn), as their partner draws the pathway onto their map. Encourage students to 'read back' their maps to see how they match. Ask: What could you have said to make your pathway clearer to your partner?

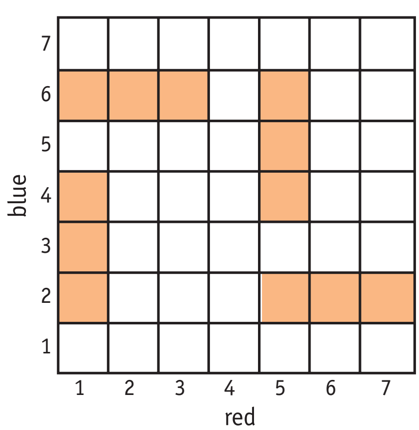
Middle ✔✔

Local Excursions

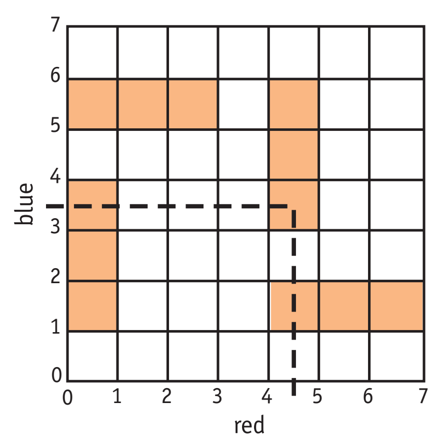
Give pairs of students a map to decide on the best route to take to get to a venue for an excursion. Encourage students to suggest a route to take, giving oral instructions for their partner to draw the route onto the map. Invite students to compare it with others in the class and discuss why they think their chosen route is the best one to take.

Battleships

Give pairs of students 6 x 6 grids and ask them to number the columns and rows 1 to 6 (grid references) and label the rows blue and the columns red (see Battleship Grid 1—using grid references). Have them secretly place two battleships onto their grids by choosing three adjacent squares for each ship. Players have three turns to 'shoot' their partner's ships by calling out grid references (e.g. red 3, blue 2). Two 'hits' are required before a ship is sunk. Repeat the game, this time labelling the lines not the spaces (grid coordinates) (see Battleship Grid 2—using grid coordinates). Have students again choose three adjacent squares for each ship. Ask: How will this change the game? (more possible hits) What counts as a hit? (anywhere the ship is hit) Would any of the intersections be a hit? (yes) Should we call a hit one that lands inside the ship? What numbers would we need to use for that to happen? Where would you find red $4\frac{1}{2}$?, blue $3\frac{1}{2}$?



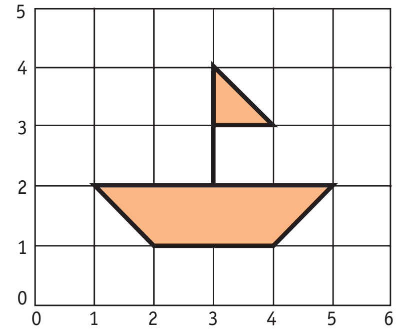
Battleship Grid 1—using grid references



Battleship Grid 2—using grid coordinates

Grid Picture

Invite students to draw a simple line picture onto a grid and label the end of each line using coordinates. Have them describe to a partner how to draw their picture by giving the coordinates of points to be connected. For example, *Start at (1,2) and join it to (5,2)*… Ask: Why do we need to label the lines and not the spaces for this activity?



Drawing Shapes

Have students describe a series of commands to create simple shapes by walking its boundaries. For example, invite students to think about the properties of a shape (e.g. square, triangle, circle), act out the movements required to create the shape and write down the commands (e.g. *Walk forward 10 paces, turn clockwise a quarter turn, walk forwards 10 paces* ...) Have another student follow the commands. Ask: How are the directions for making a square different from those needed to make a rectangle?

Did You Know?

Many young students initially think of direction in relation to their own bodies and therefore find directions like 'turn left' and 'move to the right' easier to learn than those associated with fixed external reference points like 'turn east', or 'turn towards Carnarvon'. However, in some Indigenous Australian communities, north, south, east and west may be learned at a very early age and are often used to give directions that non-Indigenous people would find very difficult to interpret; for example, 'Slope your writing a little more to the east', or 'Look for my pencil on the north side of the desk'. In such communities, students' mastery of complex directional and spatial relationships is taken as a sign of intellectual prowess in much the same way that counting skills are among some non-Indigenous Australian families.

Some Indigenous students will come to school already understanding the use of fixed external references when giving or interpreting directions. These students may initially find the idea of left and right directions difficult and confusing. Conversely, non-Indigenous students who have learned left and right directions quite early, may have difficulty understanding and using compass directions.

It is therefore important that the navigational strengths of all students are recognised and built upon, and that what is typical for the majority culture is not assumed to be the norm for all students, with the consequence that difference is interpreted as deficit.

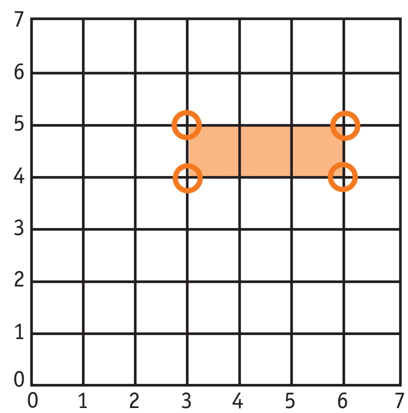
SAMPLE LEARNING ACTIVITIES

Later ✔✔

Battleships

Extend Middle Sample Learning Activity 'Battleships' so that students use standard grid coordinates (rather than grid references). Help students to define their battleships by the four corner points, using the convention of naming the horizontal axis first; for example, (3,4) (3,5) (6,4) (6,5). Ask: What coordinates would be a hit? What is the range of coordinates that would count as a hit for this battleship? (The first number between 3 and 6, and the second number between 4 and 5.) Invite students to use fractions and decimals to give coordinates that will count as a hit. Ask: How many different coordinates could there be? Draw out that there are an endless (infinite) number. Ask: Why do we need a zero line?

(3,4) (3,5) (6,4) (6,5)



Street Maps

Refer to both Middle and Later Sample Learning Activity 'Battleships' when examining street directories and maps to describe the kinds of referencing systems used. Give students maps of the local area that have both grid references and grid coordinates shown (some road maps have both systems). Encourage them to practise giving directions to find a location using both reference systems. Ask: What information can one give that the other cannot? Why might a grid reference system (rather than coordinates) be on a street map?

Latitude and Longitude

Invite students to use a map in an atlas to find a place they are interested in. Have them give the latitude and longitude coordinates to a partner who then locates it. Ask: How is the reference system in an atlas different to a street directory? Are the lines or the spaces numbered? Draw out what is the same about the standard grid coordinates (see Did You Know? page 22) and the latitude and longitude numbering system. Ask: Where are the zero lines on the globe? Why do we need to use the compass directions north, east, south and west when referring to latitude and longitude? What will you find at 23°30'S? (Tropic of Capricorn) What town is closest to 28°117'E? (Mt Magnet)

Where in the World?

When students understand latitude and longitude coordinates (see 'Latitude and Longitude'), use a globe to play a 20 questions game where one student tries to locate another's choice of country or capital city on the globe by asking up to 20 questions requiring yes or no answers. For example, *Is the place in the eastern hemisphere? The northern hemisphere? Is it between the Tropic of Capricorn and the equator? Is it between 0°E and 90°E?*

Vacation Map

Have students indicate places of interest on a map of your state or territory and decide on a route for a vacation. Encourage them to use the map's scale to decide how far they could travel each day and still have time to look around. Have them prepare a written itinerary saying where, how far and in which direction they would travel each day. Invite them to give their itinerary to a partner to draw the route on the map.

Mud Map

Invite two students to sit back-to-back. Have one student use a street map and explain how to get from one point to another while the second student sketches a map as the instructions are given. Compare this mud map to the street map. Ask: How is the mud map different from the street map? What problems might be encountered if the area was unfamiliar?

Car Rally Navigator

Organise students into pairs. Invite one student, the 'navigator', to write directions for the route of a car rally using a road map. Have their partner, the 'driver', follow the directions, marking the route on the map, hidden from the navigator's view. Encourage them to compare the intended destination with the actual destination. Ask: How could your directions be more accurate?

Grid Picture

Extend Middle Sample Learning Activity 'Grid Picture' by suggesting to students that they could name places within the squares by using fractions or decimals. For example, *Start at (1,2) and join it to (4.5, 6.5)*. Later, give students 1-millimetre grid paper, and invite them to label the centimetre lines with whole numbers. Challenge students to draw curves by naming grid points in sequence using tenths. Ask: Is tenths precise enough to draw a curve?

Antarctica

Have students use an atlas or search the internet to find a map of the Antarctic bases. Ask: Why are the grid lines different from most other maps of countries? What do the circular grid lines represent? What do the grid lines radiating out from the South Pole represent? How is this the same as or different from rectangular coordinates on other maps? How would you navigate using this map? (Link to Key Understanding 2.)

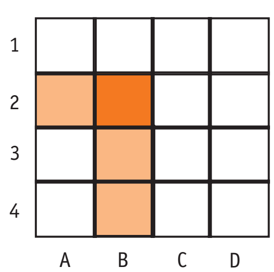
Later ✔✔

Walk on a Bearing

Have students face north and tell them that north is 0°. Call out various angles, including the major compass points (e.g. 90°, 45°, 270° north, south, east, west) and encourage students to estimate the angles by moving their right arm in a clockwise direction. Invite students to choose a particular angle or bearing (e.g. 45°) and work out which direction this is from where they are standing. To do this, they will need to find north and then work out where their angle is from north. Encourage them to find a landmark in the distance to use to ensure they stay at the right angle, and then walk on their bearing. Later, engage students in orienteering activities. (Link to Key Understanding 2.)

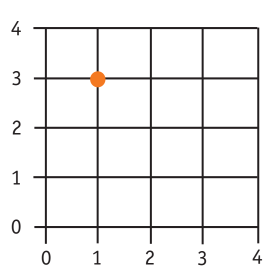
Did You Know?

Grid references describe the cells or spaces between the grid lines, and use numbers as labels not quantities (see ***First Steps in Mathematics: Number***, Understand Number, Key Understanding 3). Letters or other symbols would be just as appropriate as numbers to label the columns and rows; e.g. B2 on this diagram indicates the dark shaded cell.

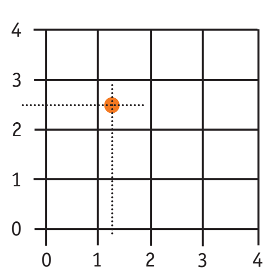


A grid reference system cannot show different positions within a cell. While this is a useful way of labelling an area, as in road directories or atlases, it is important that students understand that this is not a coordinate system, and that numbers are used in a different way when labelling grid coordinates.

Maps that use grid coordinates number the lines instead of the spaces. Pairs of numbers are used to indicate position; e.g. (1, 3) shows the exact point where the gridline at 1 on the horizontal axis crosses the gridline at 3 on the vertical axis. The convention used when reading and writing grid coordinates is to name the horizontal axis first and the vertical axis second.



Coordinates can be named using decimals (or fractions) to give more precise locations within a map; e.g. (1.25, 2.5) gives the location of the point shown to the left.



The numbers used in a coordinate system are not just labels (as in a grid reference), they are points on a number line and therefore can show measurements of things like distance and degrees; e.g. latitude and longitude lines show degrees.

Understanding how a grid coordinate system works is important as it underpins coordinate geometry, which students will meet in secondary school.

SAMPLE LESSON 1

Sample Learning Activity: Beginning—'Computers', page 15

Key Understanding 1: We describe where things are in relation to other things. There are special words, phrases and symbols that help us with this.

Learning Purpose

I had noticed that my students' use of directional language was somewhat limited. When asked where objects were kept in the classroom, they would point or use general terms such as 'over there', or 'near the door' instead of using more specific positional language, such as 'under the bookcase' and 'next to the table'. I wanted to develop this positional language, while helping them to see the value of using these words to be more precise when describing locations.

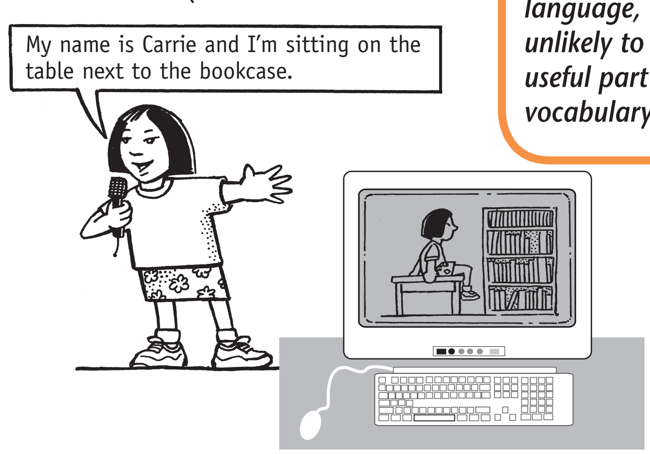
Engaging in Action

I initially engaged the group in lots of physical movement, giving them opportunities to position themselves in various locations around the room while I modelled the language they needed to develop. I gave directions such as, 'Krista, stand in front of the bookshelf', and then described their positions, 'Krista is in front of the bookshelf, next to the door.'

Students need to hear the language of position and have the opportunity to respond to the language before they can begin to use the language for themselves.

After a number of similar sessions, we decided to produce a 'talking book' about what we'd been doing. We used our digital camera to take photos of students in different locations around the room, and then we all looked at the pictures on the computer and decided on the appropriate oral text to describe the student's position in each one. The student concerned then recorded this text using Kid Pix Deluxe® Slide Show (software such as PowerPoint could also be used) to complete the book. Students had the opportunity to use the language of position in several contexts—to suggest positions to be photographed, to discuss which oral text would describe each position the recorded voice-over, and actually record a description of their own position in a photograph.

Unless students are given a real purpose for using the appropriate language, they are unlikely to make it a useful part of their vocabulary.

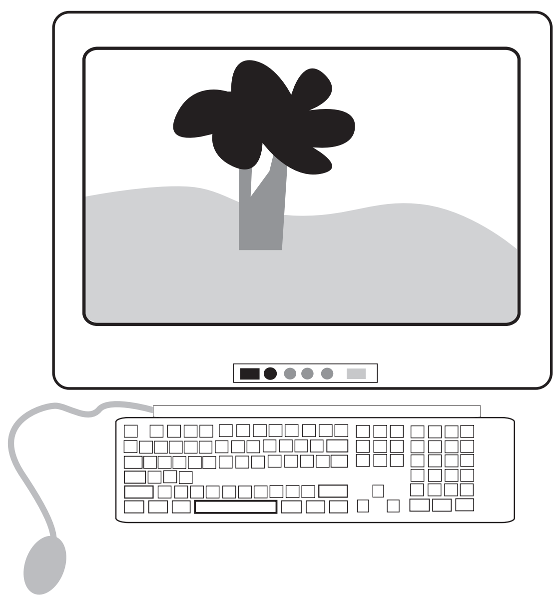


This required students to externalise the actions in order to successfully carry out the instruction and, for many, this was challenging.

We had been taking part in a travel-buddy project, and had a toy raccoon in the classroom that had been sent to us from a class of students in America. Several students were keen to make a similar talking book about Rocky the Raccoon to send to our pen-pals. I asked students to place Rocky in various locations around the room in response to the kinds of directions used in the earlier activities. This time, instead of saying, 'Krista, stand in front of the bookshelf', I said, 'Krista, can you put Rocky in front of the bookcase?'

Extension of Individual Language Skills

The following day, I worked individually with each student at the computer to help them to create a picture using Kid Pix Deluxe®. Each picture began with a simple horizon and a tree, and students used a 'stamp set' to add animals in varying positions. Other locations like ponds, rocks and buildings were added as desired.



Some children were using language that suggested they saw the picture only as a 2D image and related location words to the screen itself (e.g. 'The birds are at the top'). Others used language such as 'The rabbits are jumping away to a shed', which indicated they were visualising the screen space as a 3D environment. The need to see a 2D image as representing 3D space underlies the ability to interpret maps and therefore the attainment of the Location outcome.

I asked Robert where he wanted to put his frog.

'There,' he said, pointing to a location near the tree.

I replied, 'Ah, yes, next to the tree.' We continued to add several frogs and some birds to the picture, with Robert continuing to point at the screen when asked where he wanted to put the animals.

I wanted Robert to begin talking about the position of his animals in his own words, so I said, 'Tell me about your picture.'

Robert said, 'There's a tree and some birds and some frogs.'

'Let's begin with the birds,' I said. 'Tell me where you put the birds.'

'There,' he said and pointed at the screen.

'Try to say where they are in your picture without pointing at them, so I can write it down,' I said, but he didn't respond. 'Well, I can see a bird up in the tree. Where did you put the other bird?'

'In the sky.'

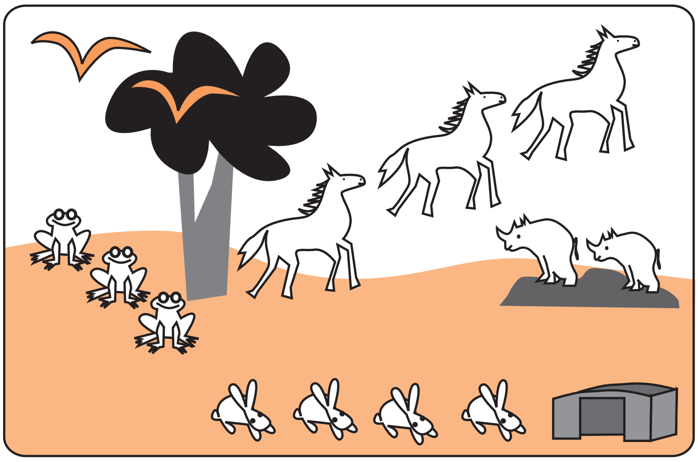
'Is it flying high in the sky or near to the tree?'

'It's near the tree.'

'What should we write down, then, about the birds?' I asked. 'We could begin with "There is a bird ... " '

'There's a bird in the sky near the tree, and there's a bird in the tree as well.'

These final responses were written beneath the pictures and later shared with the rest of the group and parents.



There is a bird in the sky near the tree,
and a bird in the tree as well.
There are 3 frogs jumping forwards.
There are 4 rabbits jumping away to a shed.
There are 2 rhinoceroses sitting on a rock
watching the other animals.
There are 3 horses and they are galloping
into the sky.

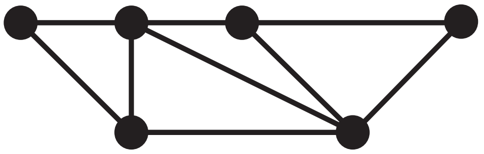
I was pleased to see students practising the language during free play activities in the block corner. Many students constructed houses, boats and such for their teddies and then spontaneously used location language in their talk: 'I'm putting my teddy in the car, you put your teddy next to mine.'

KEY UNDERSTANDING 2

Some maps or diagrams show the order of things and what comes between what. Others also represent distances and directions between things.

There are a wide range of maps and diagrams that provide two-dimensional visual representations of locations, regions and arrangements. All maps have one common feature. They represent (or preserve) what comes 'between' what; that is, the arrangement or order of objects within the environment. Thus, all maps should enable you to tell whether you can get from one place to another, whether you have to go through another place on the way, which place comes first or second, and what routes are possible.

Some maps seek only to preserve 'betweenness' and do not indicate direction or distance. Examples include the network diagrams typical of rail and subway systems and air routes. These schematic network diagrams do not enable you to tell how far it is from one place to another or in what 'true' direction you are going.



Other maps, while correctly representing 'betweenness', also correctly represent (or preserve) some measurement information about the location or region. Different maps will preserve different measurements and distort others. Thus, the geographic maps called Mercator projections preserve direction (based on lines of longitude and latitude) but distort areas, so that, for example, Greenland looks much larger in area relative to Africa than it is. Other geographic maps keep areas accurate but distort shape (see page 36). Most common road and regional maps preserve road distances and directions; that is, they provide a sort of scale drawing of the location.

Students should develop the basic concepts underlying the mathematical representation of arrangement and location by exploring and describing the layout and position of things in their environment and paths and movements within it. Their work with maps should begin with those most familiar within their home communities and gradually expand to include those that are less familiar or more conventionally mathematical. They should use sketches of their locality or road maps to describe the position of local features, understand and use bearings to define direction, and specify location by using simple coordinate grids and distances and directions. They should learn to relate direction and angle of turning to compass directions and use a magnetic compass to determine simple directions. In addition, they should develop intuitive ideas about pathways and networks.

While it is important for students to develop the ability to produce accurate scale maps, making this the sole focus of mapping activities may lead students to the conclusion that maps and diagrams are always drawn to scale when this is clearly not the case.

Progressing Through Key Understanding 2

Initially, students' maps of, for example, the route from home to school may be like a narrative or 'story' of a route, starting at the beginning of the trip and moving through the journey ('and then, and then, …'). As students continue to progress, their maps will show order and correct placement of one thing between two others on their journey and may even look like a bird's eye view, however, they may not match the actual locations of the places. Like informal mud maps, there may be little overall sense of orientation or scale. Next, students become less egocentric in this respect and are able to think of a plan or map as a bird's eye view. They will attempt to represent orientation and proximity in maps of familiar locations.

As students progress further they can use simple scale to interpret and make maps and attend to direction but may not connect this with the general idea of angles and degrees of turn. Later, students use coordinates, bearings and scale to interpret and draw maps and plans. For example, they may use a map of their state or territory to estimate distances between towns and the bearing of one town from another. They produce diagrams, such as networks, that show key locations in a familiar environment where what is ' key' depends upon the purpose of the 'map'. They also use diagrams to represent arrangements and movements when solving problems.

SAMPLE LEARNING ACTIVITIES

Beginning ✔✔

Little Red Riding Hood

After reading a familiar story, such as Little Red Riding Hood, have students suggest events and landmarks and describe the pathway taken by the characters as the teacher draws the events and connects them, making a story map. For example: Ask: Where was Little Red Riding Hood at the start of the story? What happened next? What do I need to draw to show that she walked through the woods to get to Grandma's house? Where did she go after she got to the front door? What happened next? When the story map is complete, ask questions about the order and position of landmarks and events in the story, in relation to other landmarks and events. For example, ask: Did Little Red Riding Hood see the wolf before or after she got to Grandma's house? How did we show that on the map?

Story Map

Extend 'Little Red Riding Hood' by having students suggest other stories that they are familiar with and, in pairs, construct a map of another story. After students have drawn their own story maps, pin their map up and have them retell the story, referring to their maps to help and using a pointer to show where they are in the map as the story progresses.

Rolling the Ball

Have students recall the order of actions, turns or throws that occur in a game and show them in a diagram illustrating the people and pathways taken. For example, set up a small group in a circle and have a pair of students with a marker and a large diagram showing the names and positions of the students in the circle. Invite students in the circle to roll a ball backwards and forwards to different students and have the pair of students draw the path of the ball on their diagram. Ask: Did anyone miss out on a turn? Which pairs had the most turns? Encourage students to look at the diagram to find the answers. Extend the activity to having students record a game of catch or having them design, use and illustrate an outdoor fitness or climbing circuit.

Stretchy Connections

Join six or seven students together using lengths of elastic looped over their wrists. Invite them to move around so that they are no longer standing so close to or so far away from the students they are connected to. Ask: Now that you've moved over there, who are you joined to? How come it is the same people? Say: Before you moved, I could get to Shay by following the path from Jessie to Guy to Lee and then to Shay. Is that order still the same? What has changed about the path? Draw out the idea that changing position doesn't necessarily change the order.

Landmarks

On a large whiteboard, mark in the approximate position of several landmarks in the local area (e.g. the school, the local shopping area, a playground, a lake). Invite students to direct you to the approximate position of their home by asking them questions. For example, ask: Is your home further from the school than the shopping area? Is it on the same side of the playground as the school? Is it closer to the school than Karen's home?

Playground

Begin to develop students' understanding of mapping.

1 Have students help make a map of the school playground on a large sheet of paper in the classroom, using 3D representations of the playground equipment and using their memory and language (e.g. near, next to, between) to establish positions. When they are satisfied with the placement, mark each with a cross and label each piece of equipment on the map (or draw a little picture for younger students). Discuss and mark where they think other key features will be on the map; for example, ask: Where will the gate be on our map? Where will our classroom be? Where will the big tree near the oval be? (Link to Key Understanding 1.)

2 Take students to the playground with their map to match it to the actual equipment. Ask: Where are the swings on our map? Which way do you have to look to see the classroom door when you're on the swings? Pretend you are tiny and can sit where the swings are on our map. Where would you be sitting on the map? What would be in front of you? What would be next to you? What would be behind you? Is that how it is when you sit on the real swing?

3 With students' help, make corrections on the map while in the playground in response to their suggestions. When back in the classroom, redraw the map more carefully, marking and labelling locations more accurately. Laminate it and put it in the play corner for students to use in their play.

4 Use the map and blocks to plan where temporary play equipment could be placed for sport or outside play activities. Mark the placement on the map and invite students to follow the map to set out the equipment in the right place. Ask: Where are the hoops on the map? Is the place Kim put the hoops really between the slide and the fence in the playground?

Beginning ✔✔

Looking at Maps

Give students a variety of maps with different scales (e.g. a map of the school grounds, a page of the local street or area map, a map of the state, a map of the country, a map of the world). Tell them the maps are all about where they live. Encourage students to ask questions about the different maps, and pin them up on the wall for incidental discussion (e.g. *How come the map of the town has numbers on it but the school map doesn't? If I need to find out whether the shopping centre or the swimming pool is closer to the library, which map would help me?*)

Compass Walls

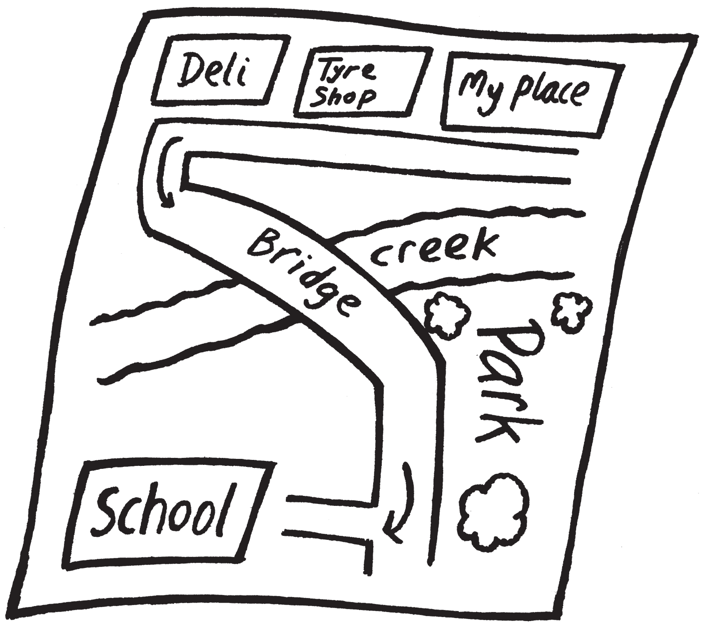
Extend Beginning Sample Learning Activity 'Compass Walls' (Key Understanding 1) by drawing a large plan of the school grounds with the classroom clearly marked. Place the map on the floor, correctly oriented to the classroom, and ask: Which direction is the playground? Which direction is the playground on the map?

SAMPLE LEARNING ACTIVITIES

Middle ✔✔

Mud Maps

Invite students to draw a mud map of their pathway to school, marking in the landmarks passed on the way. Have them compare their maps to see if any have the same order of landmarks. Ask: Does anyone follow a different pathway and still pass the same landmarks? (e.g. both go past the tyre shop, over the bridge, past the deli, then to school, but one takes a shortcut between the tyre shop and the bridge) Could you follow someone else's pathway to get to school? How do you know? Draw out the idea that a mud map can appear to be different when the landmarks are in the same order. They can also look the same but the distances travelled between landmarks may be different.



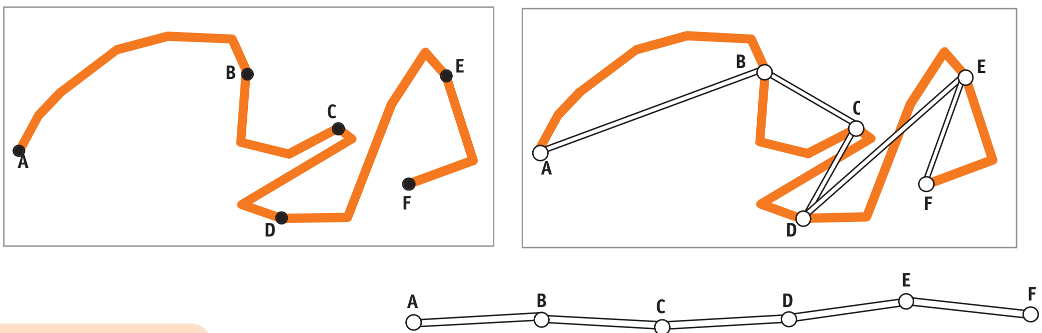
Scale Map

Extend 'Mud Maps' by asking students to modify their map of their route to school to take account of relative distance. Ask them to record the distance between landmarks (in footsteps if walking, kilometres and metres if driving or riding). Ask: Is the distance from home to the supermarket further than the distance from the supermarket to the school? Does this match your mud map? What do you need to change on your map to make it a closer match to the real distances?

Middle ✔✔

Stretchy Connections

Extend Beginning Sample Learning Activity ‘Stretchy Connections’ to show how landmarks on a map can be repositioned in a way that does not affect the way the landmarks are connected. Use an enlarged map of a familiar bus or train route. Mark a small number of ‘stops’ with large pins on the map and use elastic to connect the pins, to show how they are connected by roads or rail. Reposition the pins and the elastic in a straight line below the map. Ask: Has moving the ‘stops’ into a network diagram changed the way they are connected? Is the order of the stops still the same? What could you see on the original map that you can’t see on this network diagram? Encourage students to examine other bus or train network diagrams that show the order of the stops.



Sight Seeing

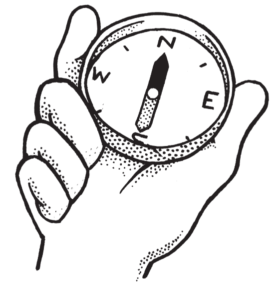
Have students design a pathway to follow to visit a selection of sights in their local suburb, town or city. This could be done after an excursion or for a familiar tourist location. In Kings Park, Perth, for example, the sights to see are the karri log, the Pioneer Women’s Memorial Fountain, the State War Memorial, the Botanic Garden, the DNA Tower and the Lotteries Family Area. Ask: What information do you need to help you design the route that requires the least walking? What information would you need to include in your diagram for people to know how to get from one sight to the next?

Flow Charts

Have students use flow chart software to investigate the effect of changing the shape of flow charts. For example, when studying life cycles in science, construct diagrams to show the different stages in a circular diagram, and then display it in line view. Re-tell the cycle both ways. Ask: What has changed? What has stayed the same?

Local Maps

Give students a map of the local area, with the school shown in the centre. Find north on the map. Invite them to draw on the map the approximate amount of turn in direction (half, quarter, three-quarter) between various landmarks and the northern direction of the school. Ask: If you are facing north at school on the map, how far would you have to turn to face where the



shops are? In the classroom, use a compass to help determine north, and have students use the maps to help them turn to face the direction of the landmarks they marked on their map. Ask: What did you have to think about to match your turns to the map? Draw out that you need to know where north is both on the map and on the ground to know where you start your turn from.

Playground

Extend Beginning Sample Learning Activity 'Playground' to further develop mapping skills by having students create a map of the school grounds, using symbols for the different fixed positions of objects. Invite them to create a map to use for a science nature walk to visit sites like an ant's nest, a special plant or a spider's web, maintaining relative distance and direction as much as possible. Ask: How can we show where things are when there are no signposts or nearby fixed landmarks to help? Would compass directions help? How can you show on your map that you need to change direction from facing the tree, which is in an easterly direction, to facing the basketball hoop, which is in a south-easterly direction? How can you show on your map how far to walk in that direction? Encourage students to follow each other's maps. Ask: What things made following some maps easier than others?

Fun Run

Have students use a map of the local area to plan the route for a fun run, marking the route on the map. Encourage them to think about the distance travelled compared to other distances in the locality; for example, ask: How does it compare to the distance from the school to the local shopping centre? (It's about twice as far.) Ask students to create another simplified map of the run, making sure there is sufficient information on it for runners to know which way to go, where to turn and so on. Ask: How can you show on your map that the runner needs to change direction after going over the bridge? How will they know that the distance between the first and second checkpoint is three times as long as the next leg of the run?

Looking at Maps

Extend Beginning Sample Learning Activity 'Looking at Maps' by starting with the world map and locating the position of the school, noting that we can only see a dot for the whole city or that it's unmarked if a town. Look at the map of Australia, then the state map, then the local area map, and lastly the school playground. Ask: Does our school appear on each map? Why? Why not?

Scale Information

Invite students to locate the scale information on two differently scaled maps of the local area. Ask: How they are different? Which map shows a bigger area on the same-sized page? How does the scale affect the amount of area shown on the map?

SAMPLE LEARNING ACTIVITIES

Later ✔✔

Mud Maps

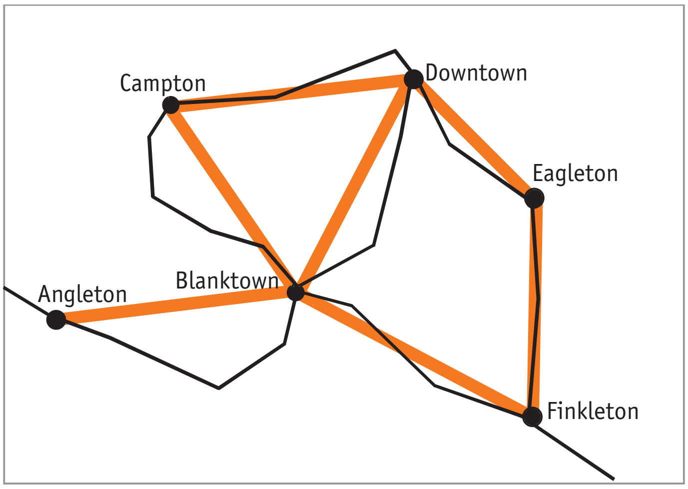
Extend Middle Sample Learning Activity 'Mud Maps' by asking students to compare actual distances on their mud map to a road map or street directory of the local area. Ask: What would you need to change on your mud map to make it a more accurate guide to the distance and direction you travel to school? Encourage students to redraw their mud map for a person who only has the map as a guide to get from their home to school and is unfamiliar with the area. Ask: What did you change from your original mud map? Why?

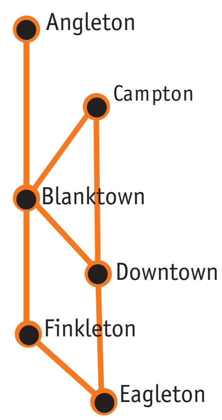
Sorting Maps

Give students a range of different maps and plans, including networks such as bus or train routes, and nature walks from parks, and ask them to sort them, giving reasons for their groupings. Ask: How is each map in this group the same? What are the differences between the groups? Which maps tell you the order of the places? Which maps tell you how far apart places are? Which maps show direction?

Stretchy Connections

Extend Middle Sample Learning Activity 'Stretchy Connections' by placing labelled pins into places on a country road map on the notice board. Use elastic to join those places that have connecting roads. Move the pins with elastic attached off the map and onto a blank area of the board. Ask: What has changed and what has stayed the same? Draw out that even though the positions of the pins have been rearranged, the relationships between have not changed. Draw the network so that it fits neatly down the side of a page and place names can be listed next to the network. Ask: Would this map be useful? How?





Fun Run

Ask students to use a map of the local area to plan the route for a 5-kilometre fun run. Invite them to mark the route on the map and use the scale to calculate the distance of each section. Ask: How do you know the overall length will be 5 kilometres? Have students draw a simplified map for the runners, giving full directions for distance, direction and turns (e.g. *run north for 50 metres along Westbourne Road and then turn east down Jacksons Lane* ...)

Local Maps

Extend Middle Sample Learning Activity 'Local Maps' by asking students to imagine they are in their classroom facing north. Invite them to use the map to decide how far and in which direction they would need to turn to face various landmarks, drawing lines on the maps and using their protractor to measure angle as needed. Test directions by having a student face north, turn as directed, then match the actual direction to the direction on the map.

Looking at Maps

Extend Middle Sample Learning Activity 'Looking at Maps' by asking: What's the same about each map? What's different about each map? Draw out that each covers wider areas of the same location.

Scale

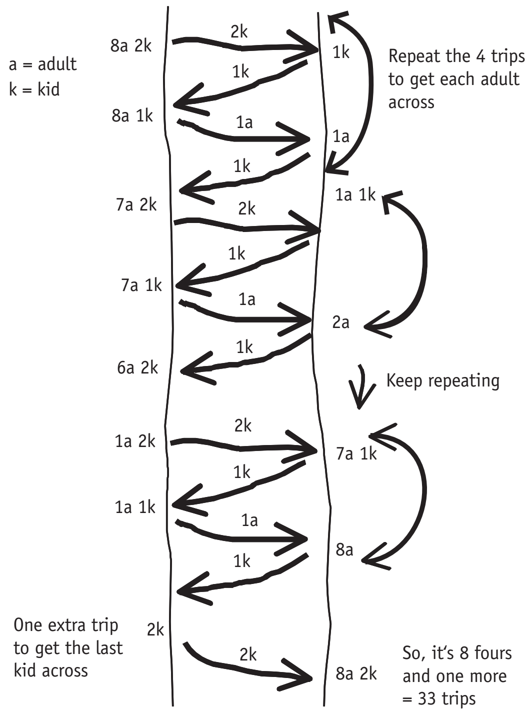
Invite students to examine the scale used on a road map of the local area. After determining real distances between locations using the scale on the map, have students use the scale to determine the *width* of one of the roads (with assistance if needed). Compare this to the actual width of a similar road. Ask: How wide would the road be if it was drawn to actual scale? Why would the map maker draw the road to a different scale?

Orienteering

Use orienteering activities to explore the relationships between direction and distance as shown on orienteering maps. Ask: How is distance shown on the map? How is this converted to real distance? How is change in direction shown on the map? How is this converted to actual change of direction?

Later ✔✔

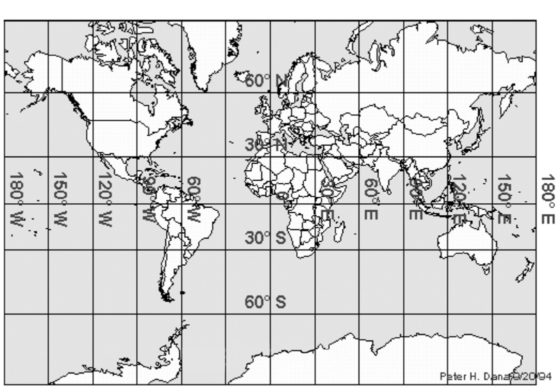
River Crossing



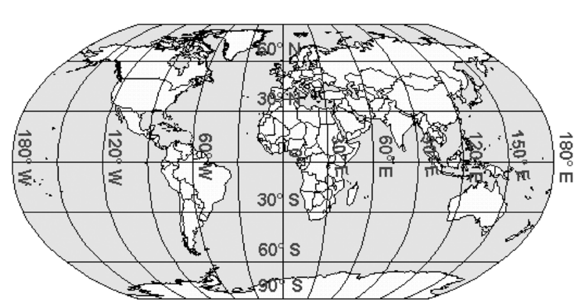
Have students use pathway diagrams to solve problems not related to maps or locations. For example, say: A group of eight adults and two children want to cross a river. Their boat can hold just one adult or up to two children, but not an adult and a child together. What is the minimum number of trips needed for all to cross the river? Invite students to draw a diagram to help them solve the problem. Ask: How does your diagram show what is happening? How does it help you solve the problem? (Link to ***First Steps in Mathematics: Number***, Reason About Number Patterns, Key Understanding 2.)

Map Projections

Have students investigate how map makers have dealt with the problem of creating a flat map from the curved surface of the earth. For example, provide atlases for students to select and compare different types of map projections of the world, such as the Mercator projection, the Lambert equal area projection and the Robinson projection. Invite a student to use a globe to find two locations along the equator and two locations with similar distance and directions near the Arctic Circle. Encourage students to compare this to the same locations on the different maps. Ask: How are they different?



Mercator Projection



Robinson Projection

Internet Maps

Search the internet for maps using the word 'map' and the name of the town and state or territory as search parameters. Print off a range of maps for whole-class comparisons. Ask: How are direction and distance shown in each map? Which symbols and information relate to real-world features (e.g. rivers, roads, mountains) and which to imaginary features (e.g. longitude, latitude)? How are these symbols the same on different maps? How are they different?

Did You Know?

When making a mud map, we often describe the journey while we draw. It is the combination of the marks on the page and what we are saying (go along this road, there is a big tree here, and you turn left down this track ...) that communicates our intended message. Someone who has not heard the conversation may not be able to interpret the map.

In some cultures, this is how all maps are intended to be read. Some maps are designed to be 'sung', and the accompanying chant conveys extra information that is necessary to make sense of the drawings. Other maps will contain markings that will be interpreted differently depending on where the map has come from. It is not possible to make sense of the map without knowing where it was produced or who has helped to produce it. Students whose cultural background includes only experiences with these kinds of maps may see no reason for including conventions such as keys and universal symbols on more formal maps—they may assume that anyone reading the map will automatically know what the drawings on the map mean.

KEY UNDERSTANDING 3

Plans show the placement and relative size of things from a top view.

A plan has many things in common with the kinds of maps we find in road directories (see Key Understanding 2). Plans, too, show a top view and are drawn to scale, so directions and relative distances between objects are preserved. In a map, however, objects are usually not drawn to scale but rather are represented by symbols and labels. In plans, we are interested in *how* things are positioned as well as where they are, so we also draw the objects to scale and angle them correctly in relation to each other and the space they are in. Thus, plans can show how buildings are positioned in relation to roads and fence lines (e.g. town or block plans); how parts of a building are positioned in relation to each other (e.g. house plans or shopping centre layouts); and how furniture is positioned in relation to a room (e.g. floor plans and layouts).

Students need a lot of experiences making, comparing and talking about their own and others' plans of a diverse range of environments. They need opportunities to draw and redraw their plans to refine their ideas about orientation and scale. It is also important that their plans be drawn and used for real purposes, like rearranging the furniture in their classroom, planting a vegetable patch in a corner of the schoolyard, or planning a model village. Drawing attention to the way things look smaller and closer together when seen a long way off can develop their initial ideas about scale. Photographs of their toy towns taken from atop a ladder, and aerial photos of known environments can reinforce this idea. They need to make judgments about which features are needed and which are irrelevant. For example, showing compass points on some plans might be useful, for others a grid reference system might be appropriate, while for others the frame of reference might simply be the edge of the space represented on the plan.

Progressing Through Key Understanding 3

Initially students will correctly place objects in the right general order on a simple plan, but their sense of how close things are together and their sense of scale may not be particularly well developed. Their plans are likely to be somewhat egocentric so that they may draw their own desk larger than those of other students in the class.

Next, students will recognise and use a top view to represent familiar locations on plans. They notice order and relative proximity among some objects but may not attend to scale or the overall frame of reference used in a plan.

As students progress further they understand that plans represent a given space containing an arrangement of particular objects. They informally attend to general direction and relative size but are not able to make use of formal scale relationships in making or interpreting plans.

Later, students use formal scale and relative angle to represent size and position accurately when making and using plans for a wide range of purposes.

SAMPLE LEARNING ACTIVITIES

Beginning ✔✔

Making Models

Provide students with baseboards of imaginary environments on which are drawn various natural and man-made features and boundaries, such as rivers, trees, lakes, road networks and fences. Invite them to use building materials to create appropriate models on the baseboards. Ask: Why did you decide to make a shop there? How will the people walk around your zoo? Where would a farmhouse go? Encourage students to draw their arrangements for others to understand. Ask: Can you make a drawing that someone else could use to build the same town?

Dolls' Rooms

Provide students with doll-house furniture and shoe boxes and invite them to set up different rooms in the boxes. Take top-view photographs and have students help match them to the models. Ask: Which parts of the furniture can you see and which are hidden? Have students trace over the photographs to make plans of the 'rooms'. Ask: How are the plans different from the models? Draw out that the space between furniture looks smaller as well as the furniture. Mix the plans and have students match them with the original models.

Mixed Up Dolls' Rooms

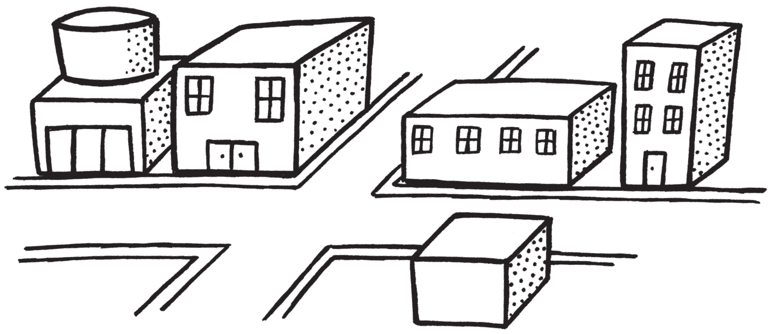
Extend 'Dolls' Rooms' by secretly changing the arrangement of doll-house furniture in the 'rooms' and drawing plans of the new arrangements. Mix the plans and ask students in pairs to match a plan to a 'room'. Ask: What gave you clues about which arrangement was the right one?

Remaking Dolls' Rooms

Further extend 'Dolls' Rooms' by removing all the furniture and asking students to put the furniture back as it was, using the plans. Have others look at the arrangements and the plans and check for correct placement. Ask: How did you know the table was not in the right place? Encourage students to draw plans of their own arrangements, then, after removing the furniture, ask their partners to set up the furniture again, using their plan. Ask: What do you need to think about when you draw your plan? How do you know which way to put the bed?

Model Town 1

Invite students to build a town with recycled materials or blocks, placing buildings around the classroom floor. Encourage them to decide on the identity of different buildings and add roadways with masking tape and other features such as ovals and a swimming pool. Invite students to pretend to fly over the town and say what parts of their buildings they see. Ask: What does the skyscraper look like from above? Which way is the shop facing? If you were flying over a real town, which other things would you see? Which things stay in the same place? Which things move around? (cars and people)



Model Town 2

Extend 'Model Town 1' by taking a series of photos, some from close to floor level and others standing on a ladder or desk directly above the model. Display the photographs for students to identify which groups of buildings the photos depict. Talk about bird's eye view compared to other views. Using enlargements of the top-view photos, help students trace the outlines of the buildings to create plans. Ask: How do the plans look different from the models? What can you see from the side view that is hidden from a top view? What would change if the photo was taken from even higher above the models?

Model Town 3

Extend 'Model Town 2' by having students draw and cut out top views of their buildings and features (e.g. an oval, a swimming pool). On a notice board, have students recreate the physical arrangements by pinning up the cut outs. Ask: Which side of the shop is your house? How close was your house to the shop? Which way was the shop facing? Was the oval on the same side of the school as the shop?

School Buildings

Make an enlarged plan of the school buildings with the rooms, doorways, windows, verandahs and other features clearly shown but not labelled. Lay the plan on the ground, correctly oriented, on an open area of the playground. With students gathered around, point to a room on the plan and challenge students to point to where it is in the real world. Send pairs of students to check the number of the room indicated. Ask: How did you decide which room? Encourage students to solve how to match the plan to real space by pointing out important features; for example, *There's a door on the verandah side on the plan. It's the second room from the end. It's the room with the windows on this side.*

Beginning ✔✔

Aerial View

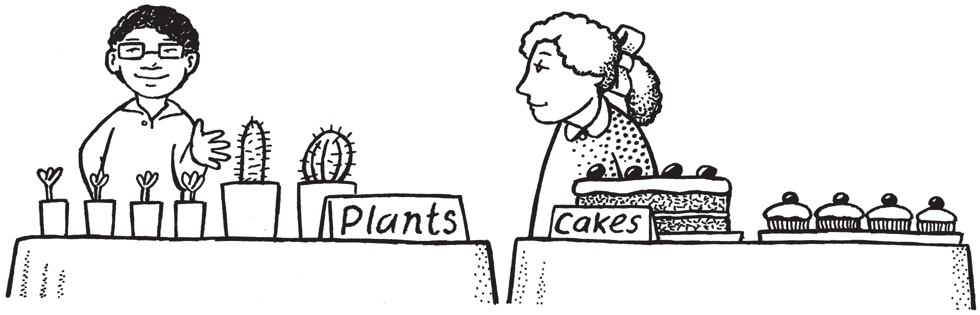
Ask students to help locate and label the school, relevant streets and local landmarks using a whiteboard marker on a large-scale aerial photo of the local area covered with plastic. Invite them to draw a top-view representation of their house so that someone else could recognise it. When they think it looks the right size for the plan, have them pin it on, along with their house number. Draw their attention informally to relative scale. Ask: Is your house bigger than the shopping centre? (See Key Understanding 1.)

Classroom Arrangements

Prepare some top-view cardboard cut outs of the desks and other furniture in the classroom, drawn to scale, and draw to the same scale an outline of the room on a large sheet of paper, showing the door and windows. Place the plan on the floor, using the same orientation as the room. Invite students to place the pieces of furniture correctly. Ask: Is the bookcase touching the door, or is there a space? Should those desks be that close to the teacher's table? Refer to the actual arrangement to check. When everyone is satisfied, draw around the furniture cut outs and label them on the map. Display for discussion.

Real Plans

Have students draw informal plans for real purposes, such as planning a vegetable garden, the arrangement of desks for a parent morning tea, the set-up for circuit activities or the stalls for a fair. Give them opportunities to draft and redraft their plans if they wish. Have others try to interpret the plans with the help of the author of the plan. Base the actual arrangement on one or more of the students' plans.



SAMPLE LEARNING ACTIVITIES

Middle ✔✔

Classroom Plans

Invite students to make a bird's eye view map of the classroom, imagining they are a fly on the ceiling or a miniature person on the light fitting. Compare positions on their plans to the real positions. Ask: How far is your desk or the bookcase from the door? Is it the same on your map? When you sit at your desk, which direction is the teacher's table? Is it the same on your map? How can you check?

Classroom Arrangements 1

Extend Beginning Sample Learning Activity 'Classroom Arrangements' by having students use the furniture cut outs to produce a plan for a different classroom arrangement. Draw around the cut outs and display the plan. Mix up the furniture and ask students to use it to reproduce the plan. Ask: How did you decide how close to put those desks? Which way does the bookcase have to face?

Classroom Arrangements 2

Extend 'Classroom Arrangements 1' by inviting students to draw their own plans to show how they would like the desks in the classroom to be arranged. Take turns for groups of students to rearrange the desk cut outs according to the plans. Follow up with a class discussion. Ask: What made some plans easy to follow? What caused difficulties?

Classroom Arrangements 3

Extend 'Classroom Arrangements 2' by having students draw classroom plans for special purposes. Ask: How should we arrange the desks to display work for parents' night (a parent lunch, drama activities, a visiting class)? Consider and discuss the practical requirements before planning; for example, for parents' night, room would be needed for people to come through the door and move around without crowding in one place. Use a student's plan to rearrange the furniture for those purposes.

Aerial View

Extend Beginning Sample Learning Activity 'Aerial View' by cleaning off the aerial photo and pinning a large sheet of tracing paper over it. Over time, have students trace roads, buildings and fixed features, labelling as they go. When they have completed this, remove the tracing paper and compare it to a local road map of the area. Ask: What have we included in the class plan that is missing on the street map? How are they the same? How are they different? (See Key Understanding 2.)

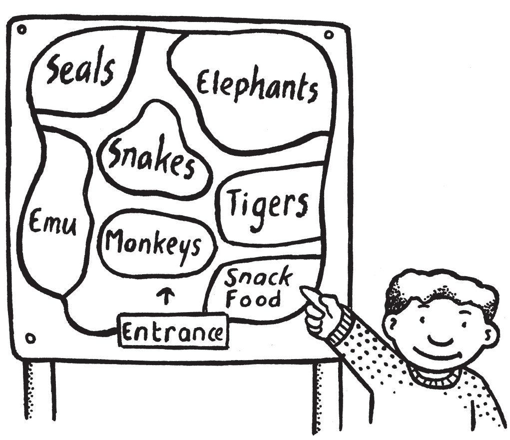
Middle ✔✔

School Layout

Provide an unlabelled plan of the school buildings with only the classroom and one or two main features labelled. Help orient students to the layout outside their classroom door (e.g. by making sure they are facing the office with their plans correctly oriented), then have them move around the school in pairs, labelling the classrooms on their plan. Encourage students to compare their labelled plans and, as a class, transfer the information to a large copy of the plan. Pair any students who have conflicting information and have them recheck the rooms. Ask: How do you know that classroom is next to ours on the plan? (See Sample Lesson 2, page 49.)

Visits

After a visit to the museum or zoo, invite students to recreate a plan of the layout from memory, visualising the different arrangements of the exhibits. After completing their plan, ask them to join with a partner and compare their maps. Encourage them to make a new map, combining their memories of the position and orientation of the layout. As a class, combine features of the individual maps into one large map. Ask: What do you remember that can help us decide which way the shop was facing? Can you imagine standing outside the snake exhibit and looking around? What could you see? (See Key Understanding 1.)



Dolls' Rooms

Following Beginning Sample Learning Activities (p. 40) 'Dolls' Rooms', 'Mixed Up Dolls' Rooms' and 'Remaking Doll's Rooms', invite students to use doll-house furniture or other suitable classroom materials to display on a sheet of paper the rooms they have in their home. Encourage them to draw the walls and doors on the paper, using their informal sense of scale. Have students draw top-view plans of their models and display them on the notice board without naming them. Take photos of students' models that show them from a different angle. Invite students to match the photos with the plans.
Ask: What clues did you use to make the match?

Ideal Playgrounds

Invite students to draw an informal plan of their ideal playground. Encourage them to consider which pieces of equipment they would like to include and where each needs to be in relation to the others, and to the boundary of the play area. Ask: How do you know the slide has enough space at the end for students to get off safely? How do you know there is enough space in the area for all of the pieces you want? Compare their ideas to the actual school play area using students' intuitive sense of scale. Ask: Is the space for the climbing frame in the school playground bigger than the space for the swing in the school playground? What about on your plan?

Real Plans

Extend Beginning Sample Learning Activity (p. 42) 'Real Plans' by focusing on the overall orientation of their plans to boundaries. Ask: How have you shown where your arrangement is? How will the garden beds fit in the school yard? How have you shown where the fair stalls will be in relation to the school buildings? Have students display their plans and then ask the class to choose which one they want to use, giving reasons for their choice.

Shopping Centre

Have students list all the different types of shops they might find in a shopping centre, then invite them to design a shopping centre themselves, ensuring the shops are all accessible and that the size of the different shops 'look right'. When they are finished, have them draw a grid on the plan and use grid references to show where to find different shops.

Architects and Builders

Following activities like 'Shopping Centre', invite an architect or builder to visit the class, bringing in plans and talking about how such plans are produced and used in real life. Focus on how the rooms or shops are arranged. Encourage students to find out how the builder knows where to begin and which way the buildings will face.

Design a Shopping Centre

Build on 'Shopping Centre' and 'Architects and Builders' by having students draw plans for a model shopping centre or village of their own, using the class collection of boxes and cartons for buildings. Encourage them to use their plan to show how the buildings are arranged and where they are in relation to where they will be built (e.g. on a desk in a corner). After they use their plans to complete the buildings, ask: How useful were your plans for placing the buildings? Did you need to change the plans to make some buildings fit? Why? What would you do differently next time you draw a plan?

SAMPLE LEARNING ACTIVITIES

Later ✔✔

Circuit

Invite students to design a physical education circuit and draw a plan showing the position of equipment, starting point and the order. Ask: How can you show how far apart to place the equipment? Would arrows and measurements help? How can you show direction and where the starting point is in relation to the school building? Would looking at other maps and plans help? If you are going to set up the equipment from the plan, what will you need to do first? Why?

Classroom Arrangements

Extend Middle Sample Learning Activities (p. 43) 'Classroom Arrangements' by helping students use a simple 10 centimetres = 1 metre scale to draw and cut out their desk and chair from card. (This is useful as a first scale because 1 millimetre = 1 centimetre. Students can construct a 'scale ruler' by renaming decimetres and millimetres as metres and centimetres on a normal ruler, or can create one on card.) On a display board, draw a scaled outline of the floor of the classroom, showing placement of doors and windows. Invite students to pin their desk and chair to the plan in the correct position and orientation to classroom walls. Ask: How did you decide where your desk should be? Invite students to make cut outs of the other furniture in the classroom and place it on the plan. When all the furniture cut outs are on the plan, help students to check that distances between furniture and the angles of placement are correct and to scale. Ask: If the space between the teacher's desk and the wall is 1 metre 25 centimetres, how much should it be on the plan? How much is the desk 'turned' to be sticking out at that angle from the wall? (Link to ***First Steps in Mathematics: Measurement***, Indirect Measure, Key Understanding 3.)

Rearranging the Classroom

Extend 'Classroom Arrangements' by having students use the display board plan to design classroom arrangements of their own and for special purposes, such as parents' night, a parent lunch or morning tea, an evening social event, or drama activities. When they are satisfied with their arrangements, have them trace or copy the plans onto a large sheet of centimetre grid paper and keep until it is used to guide rearranging the furniture for those occasions. Ask: What did you need to think about when you copied the plan?

House Plan

Enlarge a house plan (that includes the layout of furniture) from a newspaper housing section, and give copies to each student. Invite students to decide what each of the shapes represent. Ask them to colour the floor to contrast with the furniture. Ask: How did you know which sections were floor and which were not? Is it possible to put the furniture in different positions in the rooms? How do you know if it will fit? Encourage students to trace the floor plan and draw in new arrangements.

Rearranging the House

Extend 'House Plan' to include the use of scale information to work out arrangements of furniture. Help students to use measurement to determine scale information on the house plan and work out if new arrangements of furniture will fit. (Many house plans reproduced in newspapers use a scale of between 0.5 centimetres and 1 centimetre to 1 metre. Students can use the known width of a typical passage or bedroom to help determine a rough scale.) Ask: About how much room is needed between the bed and the door? Will there be enough room to move the chair back from the table? (If students are not confident with using measurements to help determine scale, have them continue to use their intuitive sense of scale. Ask: Does it 'look right'? Does the distance between the door and the desk look as much as the width of the bed? Would that give you enough room to move your chair out in real life, do you think?) (Link to ***First Steps in Mathematics: Measurement***, Indirect Measure, Key Understanding 3.)

Aerial View

Extend Middle Sample Learning Activity (p. 43) 'Aerial View'. After tracing the aerial photograph and comparing it to a local road map of the area, obtain a copy of a council survey plan for the area and compare it to the other two maps. Ask: What do all of these maps have in common? What features are different? What are the purposes for the road map? What are the purposes for the council plan? How are they different? What reference system does each map use? How do you know where the area is in relation to the rest of the city or region?

Real Plans

Extend Middle Sample Learning Activity (p. 45) 'Real Plans' by having students use scale (formal or informal according to each student's current skills) to show the appropriate placement and distance from boundaries for the various plans. Encourage them to use compass directions or grid references as appropriate to convey position and the orientation to the larger area or boundaries. Use student plans for the given purposes.
Ask: How useful were the plans?

Later ✔✔

Changing the Scale

Have students try redrawing simple plans and changing the scale. For example, provide house plans from the newspaper drawn in 5 millimetres = 1 metre scale and ask students to draw a larger scale version, such as 2 centimetres = 1 metre, or 4 centimetres = 1 metre. Ask: How big will the sheet of paper need to be for the new plan? What kinds of things do you need to think about when converting to a larger scale? What about the width of the walls? Why might you be able to show more detail on the larger scale plan than on the smallest scale plan? How could a protractor help you make your plan accurate?

Shopping Centre

Extend Middle Sample Learning Activity (p. 45) 'Shopping Centre' by inviting students to use their knowledge of scale to draw a plan of a shopping centre. Have them use a local road map to decide where their shopping centre will be and encourage them to include a site plan that shows the surrounding buildings and parking area and to use actual directions from the road map to indicate compass directions on their plan. Ask: How did you work out which way your shopping centre would face? What measurements did you take to decide on the size of the parking area bays and the entry roads? What problems did you find when you were drawing your plan?

Entries and Exits

Compare and discuss how overall position and orientation has been established on plans such as seating plans for an entertainment venue, a fire escape plan for a hotel, a layout of a local shopping centre, or the museum. Ask: How would you know where to enter or exit? How could you find where your seat might be? How have they shown how the building is positioned in the larger area?

Plans on Computer

Use design or architect software to help draw plans for a range of purposes, such as model towns, or layouts for the annual fair or playground equipment. Ask: How do you get the program to show the distance and direction you want on the plan? How does the program make it easier for you to draw plans than doing it by hand? What information and measurements would you need to have before you could use the program to draw a plan?

SAMPLE LESSON 2

Sample Learning Activity: Middle—'School Layout', page 44

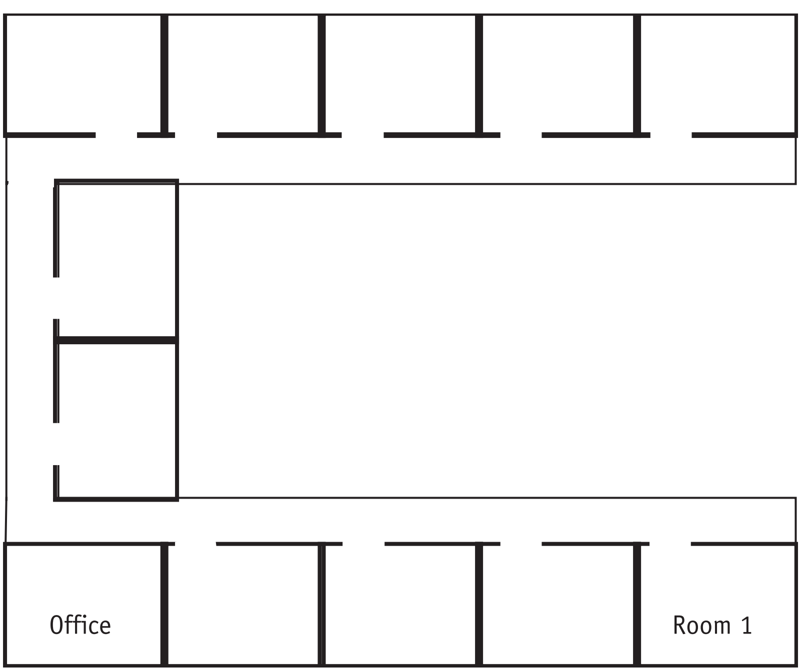
Key Understanding 3: Plans show the placement and relative size of things from a top view.

Motivation and Purpose

Lachlan's class of Year 3 students had just moved from the junior primary building to the separate primary school U-shaped block. His students had been practising giving and receiving oral directions to move around their environment, and he decided to use a 'getting to know your building' activity to develop some ideas about plans and how they can be interpreted.

Connection and Challenge

Students were given the floor plan below, with only the office and their own room (room 1) indicated. (Rooms are numbered in sequence around the U, with room 5 being the library, and room 6 the music room.)



Many students initially have difficulty understanding the conventional bird's eye view used in plans and maps. Much of this understanding is developed in the society and environment learning area, but mathematical understanding is embedded in the interpretation of scale and arrangement in such plans.

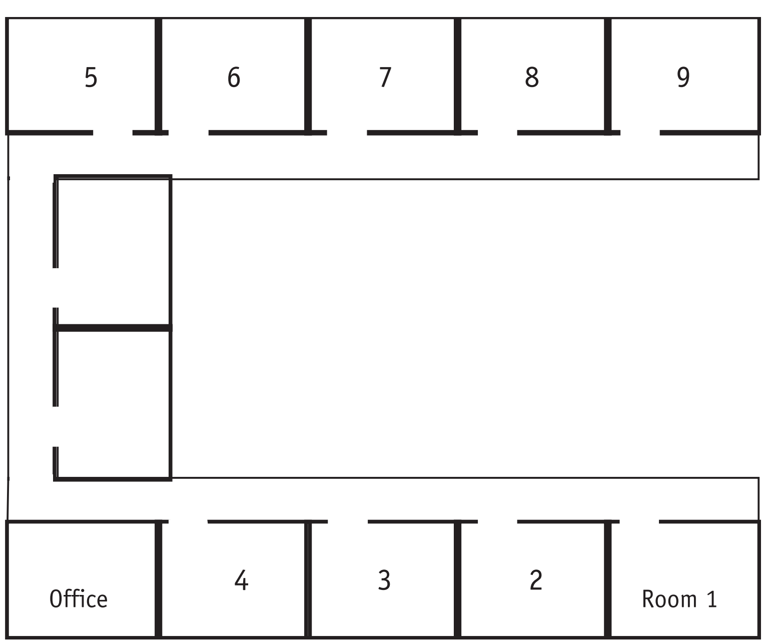
Students can often correctly interpret quite complicated plans simply by using cues like shape and colour to match the parts of the plan to their real world counterparts, and do not need to attend to the position of things. Plans like this one, which include a number of locations that look identical, force students to rely on these positional relationships.

Lachlan told them they could move around the building however they wished, but on each of the rooms on their plan he asked them to write the corresponding room number and the teacher's or room's name. (These numbers and names were displayed on each door.) To get them started, Lachlan made sure they were all facing the office with their plans correctly oriented, and checked that they could point to where they were standing on their plan. He also made sure they could point to the office both on their plan and in real space. Students moved off to carry out their task while Lachlan observed their strategies.

Lachlan talked about what he discovered from observing students:

I was really surprised how few students understood that the placement of rooms on the plan had to reflect the order and relative position of rooms in the building block. For example, Carmen ran diagonally across the playground to room 7, her brother's room, then labelled room 7 next to our room 1. She continued to label adjacent rooms on the plan in the order in which she visited the rooms, even though none were visited in that sequence. A number of students used this strategy.

Aaron correctly numbered the rooms along the first wing, but missed labelling two rooms next to the office because the doors were on the other verandah. Instead he carried on labelling the rest of the rooms on his plan in numerical order, but came to me worried that he'd run out of places on his plan for rooms 10 and 11, the last two rooms on the second wing.



Many other students produced unique room sequences that had some personal significance to them, but little relationship to the actual order of the rooms. When Lachlan questioned students, he found that they did know that the rectangles represented the individual rooms and could point to where the doors and the verandahs were shown on the plan. However, many did not see the significance of the position and order of the rooms. Lachlan realised that he needed to focus explicitly on this with students in order to make progress.

Challenging Existing Ideas

Back in the classroom, Lachlan produced a larger version of the blank plan of the school block and asked students to look at their plans so they could help him label his plan. In working through this process with them, he was able to use language he knew they understood about locations in the real world to draw out the corresponding relationships in the placement of the rooms on the map. He also asked pairs of students with conflicting plans to go and check on the actual placement of rooms and report back. For example:

'Point to the room on the plan which is **next** to our room. Whose room is really **next** to our room. Irah, go **next** door with Sean and check the room number.'

'Well, it's room 2, so what must we write on the room **next to** ours on the plan?'

'Where is the office on our plan? Which two rooms is the office **between**? Katie has it **between** 2 and 8, Angie has it **between** 4 and 10 and Corrie has it **between** 4 and 5! Well, I think the three of you can go and have a look which two rooms it really is **between**.'

After a little time, they were able to match locations on the plan to the arrangement of rooms in the school block, and Lachlan was satisfied that students were beginning to understand the relationships between locations shown on the plan.

Opportunity to Practise

The next day, Lachlan removed his plan and gave the students a new blank plan each. He asked them to draw a path from room 1, visiting at least four other locations before returning to their room again, without writing any of

It is quite a mental feat to orient oneself to locations and direction in a plan or map. It requires relationships between size, shape and angle to be understood. That is, when shape is maintained and only size changed, the angles are identical to the original, therefore relative position and direction are maintained in a scale plan or map, even though the sizes have been greatly reduced. Students in junior and middle primary do not yet understand these relationships, but can develop an intuitive understanding about the consistencies between positional relationships in plans and the environment, using their everyday language of position: 'next to', 'behind', 'in front of', 'between', and so on.

the room numbers on their plan. They had to secretly list their four locations in the order they were visited on a separate piece of paper. Students paired up and swapped plans, then tried to follow their partner's plan, listing the locations visited in their correct order. Together, they then wrote in the room numbers on the plans and compared their lists of locations with their partner's secret list. Pairs tried to identify reasons for any discrepancies and their opinions were shared with the class.

Background Notes

Maps and Plans

For many people, the concept of a 'map' may be limited only to the types of maps they see in atlases and street directories. These maps typically show many aspects of the real world, including distance, angle, direction and scale. Other maps, however, may or may not include all of these aspects. Aboriginal dreaming maps, for example, may show the relative positions of things using a series of concentric circles that indicate order but may not show direction. A bus route will often show the stops in order but not the distance between them. In order to make sense of a map, we must first make some inferences about what has and hasn't been included.

The features of some common map types are listed below. Some maps will not fall neatly into one particular category, and it is not intended that students be taught to name map types, or recall the features of each. But it is important when planning learning experiences to expose students to the full range of maps and assist them to discover the features that are included. Knowing the features of different types of maps will help teachers to do this.

Mud maps are informal sketches that are often drawn on the spot to show how to get to a particular place. Unlike scale maps, which are usually intended to be used by many people for a range of different purposes, they generally have a specific, immediate use (to show how to get to the church, or a town, for example). Thus, they tend to show only a few key landmarks that are relevant to the particular journey they describe. While they are not drawn to scale, and things are not angled correctly, there is often still some reference made to distance and direction, through the use of symbols (e.g. arrows) or spoken words (e.g. 'Go along this road for 100 metres or so and when you get to this tree turn left.')

Networks only represent the order of objects and connections between them (e.g. the flight routes between particular cities). Each object is represented with the same symbol (usually a dot, or node) and connected objects are joined by lines (paths) that are not drawn to scale or in any particular direction. Arrows are sometimes used to show whether these connections are one-way or two-way relationships.

Plans and ***scale maps*** use scale (which may be measured precisely or estimated) to represent the distance and direction between objects. Thus, the position of objects on the map or plan replicates their position in real life. On ***plans***, the objects themselves are also drawn to the same scale. ***Scale maps***, however, are normally of much larger areas, so a smaller scale is needed to represent the distances between objects. For this reason, objects themselves cannot be drawn to the same scale, and may be represented using a different scale (e.g. the roads in street directories) or by symbols.

Map Type	Representations	Distances between objects (proximity)	Relative position of objects (order and direction)
Mud maps	Key features relevant to a particular journey drawn informally	Not drawn to scale but symbols or verbal instructions may be used	Landmarks usually drawn in order; informal indication of direction may be used
Network diagrams	Dots (nodes) represent the feature of interest; lines indicate connections between items	Not shown	Order preserved by lines that indicate connections between items; direction not shown
Scale maps	Key features within certain boundaries are represented (mainly with symbols)	Drawn to scale	Bird's eye view showing relative position
Plans	Key features within certain boundaries are drawn to scale	Drawn to scale	Bird's eye view showing relative position and orientation

CHAPTER 4

Represent Shape

This chapter will support teachers in developing teaching and learning programs that relate to Part B of the outcome:

Visualise, draw and model shapes.

Overall Description

Students recognise shapes in different orientations, sections and diagrams. Thus, they can predict the different shapes of the faces produced when a carrot is cut in different ways, how an object will look if the viewer walks around it, and whether a particular net will fold up to make a planned container. In drawing shapes and spaces, they use a range of the mathematical techniques available to show 3D things in 2D form. These include, for example, using ellipses to represent circles, showing things further away as smaller, and using dots to indicate lines that cannot be seen but must be there. They can also compare these different forms of representation and choose appropriately from among them. They construct full-size and scale models of places and structures; for example, they may use junk equipment to build a model of their local community centre, produce a careful scale drawing of their classroom, or plan a net to make a container to hold three tennis balls.

Markers of Progress	Pointers Progress will be evident when students:
Students pay attention to shape as they make or draw things that they remember, imagine, see or handle.	• attend to general spatial features when making an object; e.g. select circular pieces for wheels, choose rectangular pieces for table tops • attend to the essential spatial features of pictures or objects when copying them; e.g. copy a witch's hat made from triangles and a circle, choose building blocks to make a house • attend to shapes when making and drawing things from memory; e.g. draw a seesaw with a long flat board balancing on a 'point' roughly at the centre • draw or make a thing from an oral description that involves spatial language or implies shape • convey the essential spatial features of common mathematical figures in their drawings; e.g. draw a shape that resembles a triangle with three 'roughly straight' sides that 'more or less' join
Students fulfil simple criteria relating to shape or structure as they make and draw things, making recognisable copies of arrangements of shapes.	• make things that meet criteria or needs relating to function; e.g. 'will roll', 'will stack', 'will stand up by itself', 'can remove a piece to make a door' • select materials and methods to achieve the ends they have in mind; e.g. select objects that will stack in order to build a model of the walls of their house from memory • draw figures that show the essential spatial features of named geometric shapes; e.g. draw five straight sides that join end-to-end for a pentagon • copy pictures composed of simple geometric figures so that the main components are recognisable in shape, position and orientation • draw, from memory, an arrangement of several shapes, either freehand or with a template; e.g. look at an arrangement of four shapes made by a partner, before covering and making a sketch of it • select a set of 2D figures from a collection to match the faces of a provided 3D shape
Students attend to the shape and placement of parts as they match, make and draw things, including matching 3D models that they can see and handle with conventional drawings of them and their nets.	• make polyhedra in solid (e.g. with clay), hollow (e.g. with provided nets) and skeleton (e.g. with straws) forms and discuss which features of 3D shapes are emphasised and best represented in each form • match clearly different right prisms and pyramids with nets by considering the number, shape and placement of the faces • talk about what they can and cannot see of an object from different positions and attempt to draw what they see rather than what they know to be there • match standard geometric models with realistic drawings and conventional diagrams; e.g. match a prism with a conventional drawing of it • imagine and draw different cross sections of simple 3D shapes and then check and improve the drawings by observing the cross section; e.g. by slicing carrots at different angles
Students attend to the shape, size and placement of parts as they match, make and draw things, including making nets of 3D models that they can see and handle and using some basic conventions for drawing them.	• make a box to match a provided wooden block by drawing around each of its faces to make a net • visualise to match faces of an object with parts of its net; e.g. colour the faces of a prism or pyramid and, without folding, colour parts of the net to match • select and cut suitable lengths to make a skeleton of a provided 3D shape; e.g. cut straws to the correct lengths to make a triangular prism • use cubes to copy other structures made from cubes, attending to what cannot be seen but must be there • draw prisms, pyramids, cylinders and cones recognisably if not with a high level of precision • use some mathematical conventions in drawings; e.g. draw things that are further away smaller, ellipses for circles, parallel lines for parallel edges • select actual objects to match exploded drawings or front, back and side views of various art forms; e.g. given a tray of objects, select one to match a particular drawing and explain why
Students visualise and make models of 3D shapes and arrangements and interpret and produce conventional mathematical drawings of them.	• design their own nets to construct 3D models from imagination or drawings, taking care to ensure the component parts fit together • visualise and plan essential details when constructing figures and objects; e.g. match lengths and angles, ensure the hole is in the centre of the circle, place tabs on a net so the solid will be held together • predict which hexominoes (figures made of six squares) can be used as nets for a cube and visualise the relationship of the squares to each other in the folded cube; e.g. colour the faces of a cube different colours, and, without folding, indicate where the colours could be on the net • visualise an object or scene in different orientations and draw possible 'other views' of an object from information contained in 2D drawings • use conventions for oblique or perspective drawings consistently when drawing common 3D shapes • construct an arrangement of cubes from an isometric or oblique drawing and then draw the arrangement from a different point of view • consider what cannot be seen but must be there when constructing an arrangement of cubes from a diagram; e.g. produce a complex shape made with cubes from a front, back and side view

Key Understandings

Teachers will need to plan learning experiences that include and develop the following Key Understandings (KU), which underpin achievement of the outcome. The learning experiences should connect to students' current knowledge and understandings rather than to their year level.

Key Understanding	Stage of Primary Schooling—Major Emphasis	KU Description	Sample Learning Activities
KU1 When we copy and make figures and objects, we need to think about how the whole thing looks and about how the parts relate to each other and to the whole.	Beginning ✔✔✔ Middle ✔✔ Later ✔✔	page 58	Beginning, page 60 Middle, page 63 Later, page 66
KU2 The net of an object has to have the same component parts as the object and the parts have to be in the right relationship to each other.	Beginning ✔✔ Middle ✔✔ Later ✔✔✔	page 72	Beginning, page 74 Middle, page 76 Later, page 78
KU3 To understand drawings of objects we need to combine what we can actually see with what we think is there. Special drawing techniques emphasise different aspects of an object.	Beginning ✔✔ Middle ✔✔ Later ✔✔✔	page 84	Beginning, page 86 Middle, page 89 Later, page 92

Key

✔✔✔ The development of this key understanding is a major focus of planned activities.

✔✔ The development of this key understanding is an important focus of planned activities.

✔ Some activities may be planned to introduce this Key Understanding, to consolidate it or to extend its application. The idea may also arise incidentally in conversations or routines that occur in the classroom.

KEY UNDERSTANDING 1

When we copy and make figures and objects, we need to think about how the whole thing looks and about how the parts relate to each other and to the whole.

We often make models of things that only represent certain features of the original object. A stage setting looks 'right' from some directions but not others; a model aeroplane built to be tested in a wind tunnel is different from one built to show the interior design, and both look different from the 'real' plane. The emphasis within the Space strand, and for this Key Understanding, is in the production of models that preserve shape; that is, they *look* the same as the real thing no matter how we view them. A copy of a 3D object will be another 3D object of full or part scale. Similarly, a copy of a 2D figure will be another 2D figure of full or part scale.

As described in Key Understanding 4 of Reason Geometrically, we initially recognise figures and objects by what they look like *as a whole*, just as we recognise people's faces. As for faces, in order to make recognisable copies, we need to be able to focus on the component parts, the 'spatial features'. Young students drawing a triangle will often draw a continuous curve as they would a circle, but containing corners. In order to draw a triangle, students need to see that the sides are always straight, that there are always three of them and that they are arranged in relation to each other in a particular way. Thus, to produce a triangle means to produce three straight sides that touch end-to-end to form a closed shape. To reproduce a particular triangle means to produce a triangle with specified side and angle measurements. To produce a scaled version of a particular triangle means to produce a triangle of the same shape but possibly of different size.

In producing a shape or structure, simple or complex, 2D or 3D, students must:

- 'see' the component parts of the whole thing

- produce each component part (in the right shape and size, either full or scaled) and
- put them together in the right relationship to each other and the whole thing (in the right proximity and orientation to each other).

In the early years, students build quite complex structures to model their environment, but they do so by arranging ready-made pieces, such as off-cuts of wood, boxes, paper cylinders, twigs and packing foam. As they progress, they see a need for more precision and the emphasis shifts to constructing component shapes for particular purposes. Students will need many opportunities to make standard 3D shapes. Although the product may appear less complex, the task may be considerably more difficult, since it is easier to make a tower by stacking cardboard boxes and cylinders than it is to make a box from a sheet of paper. Students need to analyse the component parts that form the object—their shape, size and placement, considering how the components fit and hold together. They will need to learn from their mistakes by observing what goes wrong when insufficient attention is paid to details of shape, size and placement.

Progressing Through Key Understanding 1

Initially students recognise and attend to the shape of things in their models and drawings. Thus, their squares will have four roughly equal sides that 'more or less' meet and their circles will show a continuous closed curve and no deliberate corners. As students continue to progress they copy pictures composed of geometric shapes (e.g. a table setting) so that the main components are recognisable in shape, position and orientation. They can also match 2D figures to the faces of a provided 3D shape.

Next, students make standard 3D shapes in solid (clay), skeletal (straws) and hollow (paper) forms and compare these forms to say which features are emphasised in each. As students progress further they attend to shape, structure and scale in making recognisable models of things (e.g. a television set, a netball ground).

Later, students attend to essential details when constructing figures and objects, match lengths and angles to make copies of 3D geometric shapes, and combine them to make complex structures.

SAMPLE LEARNING ACTIVITIES

Beginning ✔✔✔

Junk Box

Invite students to handle objects (e.g. toy vehicles, houses, animal cages, animals, telephones) and say what packages and containers from their junk box could be used to make a model of the object. Focus students on the parts, such as the neck of a giraffe, by asking: Which object could be used to make this part of the giraffe? Where will this attach to the rectangular tissue box that you have chosen for the body?

Vehicles

Have students reproduce simple diagrams from a mixed collection of 2D shapes. For example, show them pictures of vehicles (animals, houses, trees) from a 'big book' and invite them to select the shapes they need to make copies of the vehicles (animals, houses, trees). Ask: Is that a circle or an oval shape you can see there for the wheel? What other shapes will you need to collect? Which shape best matches the back of the truck? (See Sample Lesson 1, page 69; link to Reason Geometrically, Key Understanding 2.)

Copy a Prism

Provide each student with rectangular prisms (either boxes or blocks) and a larger number of pre-prepared figures made from card than they will need. Ask: Which shapes will you need to copy your prism? Encourage students to match the faces of their prism with card shapes as a way of finding out which shapes they will need. Ask: What shape is at each end of your prism? Can you find a shape just like that one? What do you need to look for?

Build a Picture

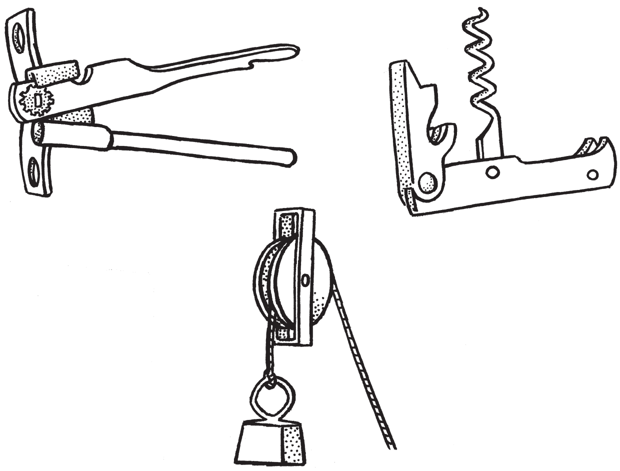
Organise students into pairs. Invite each student to build a picture with pattern blocks and then draw their partner's picture on paper. Ask them to check one another's pictures to see that the shapes in the picture are drawn correctly (e.g. the triangles have three straight sides that meet, the circles have no corners, the squares have four sides that meet at the corners). Encourage them also to check to see if the shapes are in the right positions. Ask: Is the triangle really that close to the square?

Copying a Bike

Give students ready-made 3D shapes to construct copies or models of objects around the classroom or playground. Ask: If I was going to make a model of the bike, what objects would I need to get? How could you arrange them to look like the bike?

Simple Machines

Provide a collection of simple machines (e.g. tin opener, corkscrew, pulley, rocker balance, bike) and invite students to describe the shapes of the component parts and how they move. Have recycled material available and ask: What could you use for this part? What could we use to make this part move up and down? Would this shape be any good? Why? Why not?



Finding 2D in 3D

Have students look for 'flat' (2D) figures within structures (3D objects) they have made. Provide them with a variety of materials (e.g. popsticks, straws) to make structures (e.g. bridges, playground equipment, cages). Focus students on the 2D shapes created by the straws and how these shapes fit together to make the whole object. Ask: What 'flat' shapes can you see that go together to make your bridge?

Buildings

Invite students to make clay or play dough models of buildings. Focus on the faces of the model. Ask: Has anyone found a way of making the sides of the house look right? Is it okay to have the sides of your house bulging? Do the corners need to be straight? As a class, make a list of the different-shaped roofs students have seen. Ask: Which shaped roofs would be the best to have in heavy rain or snow? Why?

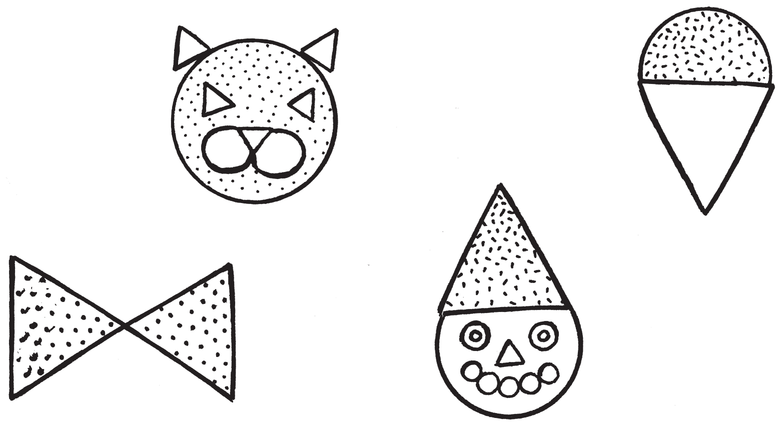
Shape Bingo

Invite each student to select a 3D object (e.g. cylinder, triangular prism, rectangular prism) and a small piece of play dough. Choose one student to be the caller and give them a cloth bag of small cards with 2D figures drawn on them (e.g. rectangles, circles, triangles). Have the caller take out a card and call out the shape that is on it. Encourage the rest of the students to respond by looking for that shape on their chosen object. If they have that shape, have them attach a dab of play dough to it, and when all of the faces on their object are marked, call out 'Bingo!'

Beginning ✔✔✔

Circles and Triangles

Provide students with a number of identical circles and equilateral triangles, either pattern blocks or made from card. Brainstorm the pictures that can be made using only these two figures (e.g. ice-cream in a cone, a clown's head, a bow-tie, a cat's face with ears). Display the pictures and have students make them, using their circles and triangles. Ask: Why do all the pictures look different even though they have the same shapes?



Toothpick Shapes

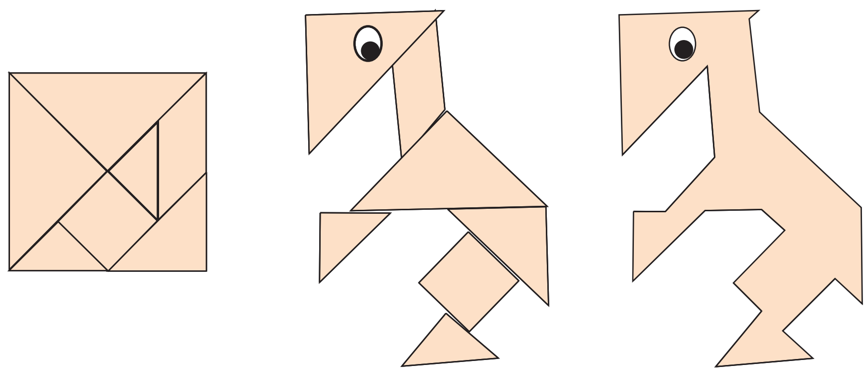
Show students triangle shapes, rectangle shapes, semicircle shapes and heart shapes and ask: Which of these can be made from toothpicks? How do you know? Use toothpicks to make the figures. Ask: Why can some shapes be made with toothpicks and others can't?

SAMPLE LEARNING ACTIVITIES

Middle ✔✔

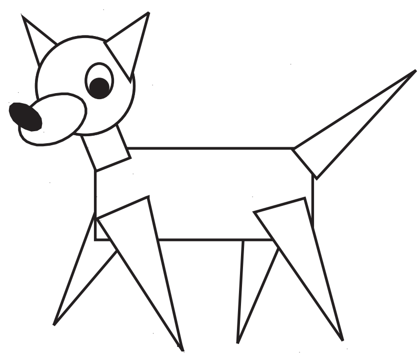
Tangrams

Invite students to create and glue a picture made from tangrams onto card. Have them trace the outline of the picture onto a sheet of paper and swap their outline with another student. Encourage them to use a second set of tangram shapes to recreate their partner's picture. Ask: What shape could (could not) possibly fit into this space?



Freehand Drawings

Invite students to make freehand drawings of pictures composed of simple shapes, such as a shape drawing of a dog. Ask: What shapes did you see in the picture? Are the triangle legs in your diagram in the same place as in the picture? How do you know? Are the other shapes drawn in the same place as the shapes in the picture?



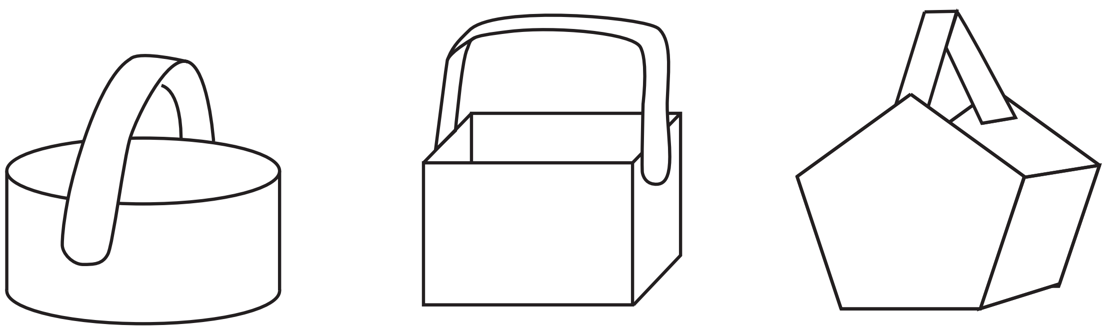
Simple Machines

Show students a collection of simple machines (e.g. tin opener, corkscrew, pulley, rocker balance, bike, transformers) with moving parts. Ask them to describe the shapes of the component parts and how they move. Invite them to suggest recycled material that could be used for each part. Ask: What shape could be used to make this part move up and down? Why wouldn't this object work?

Middle ✔✔

Baskets

Show students a range of paper basket styles already constructed (at Easter time) and invite them to describe the parts and their shapes. Have students choose a basket to copy by folding card or paper, cutting, scoring and joining parts. Ask: What parts did you see in the basket you were copying? How did you make that part out of card? How did you know where to put it in your model?



Models

Have students work in small groups to make models of toys or structures. For example, invite them to make a model of a bicycle (space station) using clay, straws and paper. Ask: Which material best suits each part of the model? Would clay, paper or straws be best to make the bike seat? What would be best to use to make the frame? What materials could you use for the wheels? How is the shape (size, placement) of the parts influenced by the material? (solid, skeletal, hollow)

Make a Ramp

Have students consider how appropriate various materials are for making 3D objects for a given purpose. For example, invite them to make a triangular prism that could be used for a ramp (a frame for a roof, a tent). Ask: Why did you choose this to make the ramp (the frame, the tent)? Does it suit what the ramp (the frame, the tent) will be used for?

Hollow Models

Have a group of students use paper or card to make hollow models of a structure (e.g. house, bridge, tower) and objects (e.g. tricycle, teddy bear, chair) in their environment. Invite them to identify the 3D shapes that go together to make up their structure or object and decide on ways to fold, cut and join the materials to construct that part. Have each student in the group make one part and then put the parts together to make the object or structure. Ask: What 3D shapes do you need to make? How can you cut or fold to make the shape that you need?

Function of Shapes

Have students investigate the function of shapes within structures in the environment. Invite them to use materials such as popsticks (straws) to make models of real bridges, playground equipment, buildings and simple machines (e.g. tin opener, corkscrew, pulley, bike). Focus students on the shapes created by the popsticks (straws) and how these shapes fit together to make the whole structure. Ask: Are there some shapes that are stronger than other shapes? How do you know? Help students identify and investigate the use of the strongest shaped components in structures in the environment.

Containers

Provide a single sheet of coloured paper and challenge students to make a container without a lid that they could use for a purpose they choose. When they have finished, invite them to swap containers with a partner and challenge them to make another container as close to the original as possible. Ask: What did you need to think about when making your container?

SAMPLE LEARNING ACTIVITIES

Later ✔✔

Polyhedrons

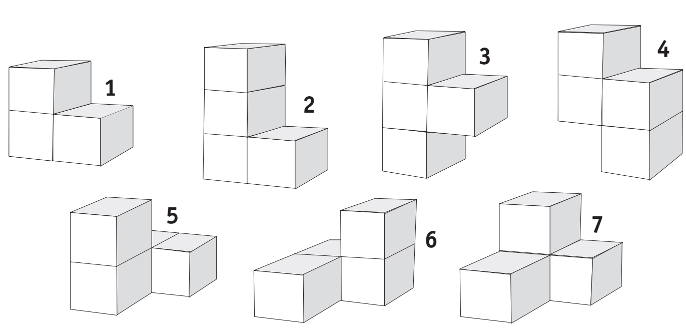
Give students a range of polyhedrons (e.g. cube, square, pyramid, tetrahedron) to examine to make sets of polyhedrons for other classes. Invite them to choose from a selection of materials (e.g. play dough, straws, cardboard). Invite students to share and solve construction problems related to attaching the component parts. Ask: What part of the shape do you need to focus on when you are using straws (play dough, cardboard)? Which features of the objects are highlighted by the different materials? Why aren't the corners fitting quite right? Is it the lengths of the sides or angles that need checking? (See Key Understanding 2.)

Solar House

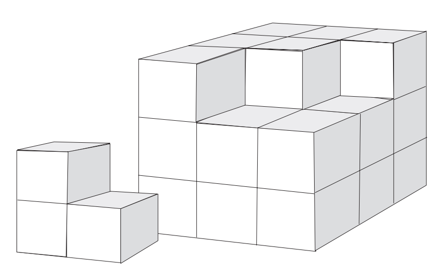
When copying a solar efficient house or other 3D models, encourage students to decide which components can be made from junk materials and which components will actually have to be constructed. Ask: If an upturned tissue box is used as the base of the house, what will be the shape and dimensions for the roof?

Soma Cube

Invite students to use 2-centimetre cubes and glue to copy and construct the seven component parts of a 3 x 3 x 3 cube known as a soma cube. Check to see that all the component parts have been placed correctly by using the completed components to construct the cube. Ask: How do you know where to place each of the blocks (each of the pieces)? If you turn the piece, will it fit? (See Key Understanding 3.)



The seven soma shapes—all the possible irregular solids formed by joining no more than four cubes.



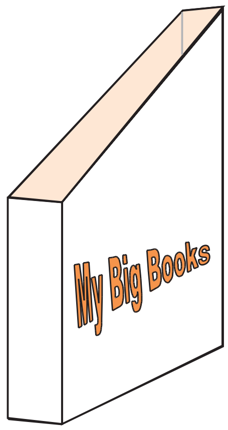
A soma cube puzzle almost completed.

Replicas

Have students produce replicas of artworks by identifying shapes within the picture as a guide to how to draw their copy. Invite them to explain how they produced each feature of the drawing in terms of the shapes identified and the way they positioned and orientated the shapes. Ask: How did you know to place the rectangle in that position? Why have you used ellipses instead of circles for the wheels?

Box for Big Books

Give students small pamphlet boxes to examine. Ask: What parts are put together to make these boxes? Invite them to design a similar box to hold 'big books'. Encourage students to explain how they enlarged their box so the big books would fit. Have students decide whether the larger box is functional. Ask: Is the box strong enough? Is it stable? Are there any changes you need to make to your box? (See Key Understandings 2 and 3.)



Models

Provide students with card and thin dowelling to construct miniature playground equipment. First have them investigate the playground and consider what they should make from card and what from the dowelling. Ask: What shaped pieces of card might you use to make the slide? How could you put the tunnel pieces together? (Some students may use scale—link to ***First Steps in Mathematics: Measurement***, Indirect Measure, Key Understanding 3.)

More Polyhedrons

Have students explore different aspects of polyhedrons. For example, ask students to choose one polyhedron and make three models of it: a skeleton model (dowelling or straws), a solid model (clay or play dough) and a hollow model (cardboard). Ask: What did you need to focus on to make each type of model? What is different?

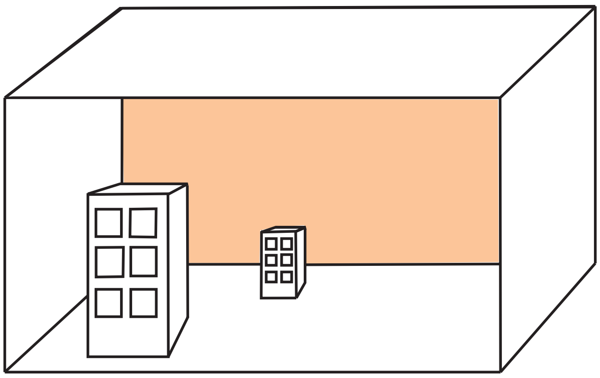
Containers

Challenge students to use a single sheet of paper to make a container without a lid for a purpose of their choice. After they have made the container, encourage them to draw a net or diagram that gives sufficient information for someone else to build the same container. Hide the container and have another person try to build the container from the instructions. Compare the new container with the original and talk about how and why they are different. Ask: What do you need to change to make the container look the same? (Link to Key Understanding 2.)

Later ✔✔

Diorama

Invite students to use balsa wood to make miniature buildings and vehicles for a diorama. Have them plan in advance the parts they will need before they begin to cut out and glue the components together. Ask: What do you need to think about to plan your models? What size should the figures be that are in the foreground (background)? How can you place the figures to give a sense of distance?



SAMPLE LESSON 1

Sample Learning Activity: Beginning—'Vehicles', page 60

Key Understanding 1: When we copy and make figures and objects, we need to think about how the whole thing looks and about how the parts relate to each other and to the whole.

Motivation and Purpose

Katie's Year 1 class had become interested in finding out about different types of vehicles after reading the big book *Toy Town* and playing with an assortment of toy vehicles. Katie decided to have them build model vehicles and to focus her students' thinking on how the whole vehicle looked and the shapes of the parts.

Connection and Challenge

As the students looked at the pictures of each of the vehicles, Katie encouraged them to talk about how each vehicle moved and what it was used for.

'Do all of the vehicles have wheels? Which do? What shape are wheels? Are all wheels the same?' she asked.

Jeff said, 'The bulldozer hasn't got wheels. It's got long things.'

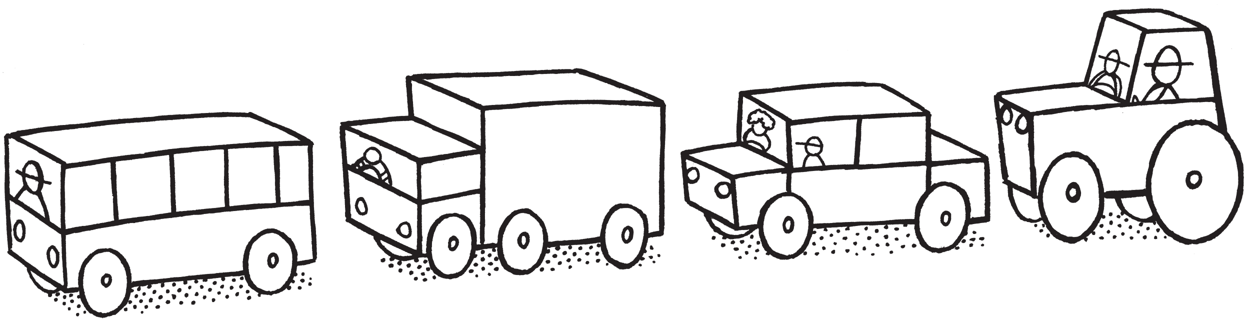
To this, Zoe replied, 'Yes it has, there are those little wheels inside the long thing.'

This comment surprised most of the students and it seemed to focus them more on other parts within parts, such as lights and gear levers.

Katie then moved the children's attention to a collection of toy vehicles. She wanted the students to think about the shapes on them and how they related to each other to make up the whole vehicle, so she pointed to the toy tractor and asked, 'What figures do you see? Does it have the same sorts of shapes as the tractor? What shape are the wheels? How are the front wheels different from the back wheels?'

They looked at how round shapes were used on the different vehicles. To connect their understanding of this with their choices of materials, Katie asked them to sort through the materials to find all the things that could be used for 'round things, like wheels'.

Katie then went back to the toy vehicles and the picture book to find the other shapes commonly used. She decided that the shape of the bus was a good place to start, so asked, 'What shape is the bus? Why is this a good shape to use for the bus? What other vehicles use this shape?'



Several students pointed to the doors and windows on the different vehicles, but they seemed to have difficulty identifying other rectangles. It occurred to Katie that the sections of vehicles that she was seeing as rectangles, the students were not, because they were part of the whole body work of the vehicles.

She pointed to the front section of a car and said, 'I think that the front section of this car is like a rectangle, and the middle section is like another rectangle and the back part of the car is another rectangle. You could make a model of this car by using three different-sized rectangles.'

With this, some students realised that they could look at the parts of the vehicles and 'see' for themselves different rectangular shapes. Katie then asked them to decide which vehicle they would like to make and to choose from the range of packets and containers from the classroom junk box. She asked them to think about which packets would be the correct shapes for the different parts of the various vehicles.

Action and Reflection

As each student began to make his or her own model, Katie circulated and talked to individual students about their choice and arrangement of the component shapes.

Following this lesson, Katie decided to provide the students with the challenge of drawing a plan of their models for another student to use to construct the same vehicle.

She asked, for example, 'Where will you put the wheels for the aeroplane? What will you use to make the wings? What shaped box would be the best shape to use for the wings?', attempting to focus the children's thinking on how the components fit together to make the whole.

After several days, when the models had dried, each student covered their construction with paper squares, then painted it, allowing it to dry at each stage. Before they marked the windows and doors, Katie referred the class back to the toy vehicles, focusing this time on the shapes of these features and their positions in relation to the main components of the vehicle. She asked questions like, 'If you wanted to draw the door, what do you have to think about? What shape is it?'

Did You Know?

Colloquially, concepts such as 'triangle' or 'cone' are sometimes referred to as 'shapes', as are the physical models of triangles and cones ('put the shapes into the box' or 'cut out some shapes').

Using the word 'shape' to describe both the property of the thing and the thing itself can cause confusion. Asking students to 'describe the shape of this shape' highlights one problem. The other problem is that students have to be able to think of 'all rectangles' as being 'the same shape', while mathematically speaking all rectangles are not the same shape. This same ambiguity, of course, exists with colours (we say pass me the 'colours' and so could have to ask 'what colour is that colour'; blue is 'a colour' but all blues are not the same colour). This appears to cause few problems with colour, perhaps due to familiarity and context.

Learning geometrical ideas, however, relies on students developing the precise mathematical meaning for 'same shape'. That is, two figures are the 'same shape' if and only if one is a scaled version of the other, so that all matching angles are the same size, and all matching edges are in the same proportion. Students who think that all rectangles have the same shape may have difficulty with the more correct meaning of 'same shape'. For older students, we should discuss this colloquial dual use explicitly perhaps by comparison with the use of the word 'colour'.

We recommend, however, that teachers model for students the more helpful convention of calling 2D things 'figures' and 3D things 'objects'. Using this informal convention, a circle is a figure but a sphere is an object. We would refer to triangles as figures and also to templates or models of triangles as figures. A tree is an object, but any of the cross sections of the trunk or the front surfaces of leaves are called figures, as are pictures of trees.

KEY UNDERSTANDING 2

The net of an object has to have the same component parts as the object and the parts have to be in the right relationship to each other.

A net is one of the special techniques we use for producing objects. It is composed of figures arranged in a particular flat configuration. The arrangement of these figures does not exactly match the arrangement on the object to be made, as some sides may be some distance from each other. This arrangement is such that when we fold it all the parts will come together in the right relationship.

Students may initially draw nets by focusing on one or two salient features of the object they want to make. Thus, their net of a cube or a rectangular prism may show one or two squares or rectangles connected in some way. Later they begin to recognise that a number of figures go together to form the net of an object and so draw a number of figures connected, but not necessarily the right number or connected in the correct positions. They might omit, for example, the top square of a cube. Through considerable experience that focuses upon the component parts of an object and how they fit together, students learn that the net of any given object will need:

- figures of the right shape and size
- in the right number and
- in the right position relative to each other.

By the upper primary or early secondary years, students should be able to match provided nets to actual objects and to make their own nets for 3D objects. To achieve this, activities will need to begin as early as pre-school and Year 1. As indicated in Key Understanding 1, students need to learn to 'see' the 2D figures in the 3D objects. Activities that involve the students in investigating *which* 2D shapes go together to make a particular prism or pyramid and *how* these shapes are connected to each other will help them to begin to see the parts that make up the whole; for example, asking students to draw the net of a matchbox and then opening up the matchbox and comparing how their drawing is different from the actual net. Activities that involve counting the faces, edges and vertices of prisms constructed from commercially drawn nets are useful for some purposes, but they are *unlikely* to help students to see how the shapes may be arranged (and rearranged) to make the net of an object.

Progressing Through Key Understanding 2

Initially students will begin to remember the shape of objects and their component parts. As students continue to progress they identify the 2D shapes that make up an object and, given a simple 3D object, can list and draw the component faces and say how many there were of each.

Next, students select a suitable paper ‘net’ to make common prisms and pyramids and predict which of different flattened nets fold to make particular clearly different 3D shapes. They do this largely by ensuring that the net and the object have the right number and shape of faces. They are still likely to be confused by subtly different nets, however, or by an arrangement of the faces that will not actually fold to make the object.

As students progress further they draw around simple objects in order to make suitable nets and can visualise the folding process sufficiently well to say which of a number of possible nets of a cube or simple prism will actually fold up to form the cube or prism.

Later, students can construct a net for prisms and pyramids, no longer needing a provided net or the object to copy or trace around. Their capacity to visualise what a net will look like when folded (or a 3D shape when unfolded) has improved to the extent that they can predict which particular figures on a net will become faces on the resulting object.

SAMPLE LEARNING ACTIVITIES

Beginning ✔✔

Open Packages

Have students cut different packages along the edges and open them out. Display the flat nets and ask students to describe and compare the shapes that form the nets. Ask: How is this shape different to this one? What sort of box will this net make when it is folded up? Is there another net that will make the same type of box? Invite students to sort their nets according to the type of boxes each net will make when it is folded and then ask them to describe their categories.

Making 3D Objects

Invite students to freely construct 3D objects using materials such as Polydron™. Provide time for each student to display their structure and say which shapes they used and how they arranged them to create the object. Ask: What flat figures did you need to make your object? How did you put them together to make your object? How is it that you counted five squares on your box, but I can only see four?

Opening 3D Objects

Extend 'Making 3D Objects' by having students carefully open up the objects to make a net of the shape. Encourage them to draw around the net and then remake the object. Ask: Can you open up your object in a different way and still be able to put it back together to make the same object? Is there another way still?

Making Nets

Provide groups of students with boxes so that within a group the boxes are identical (e.g. toothpaste boxes). Invite each student in each group to cut up their box to make a net. Ask: How are the nets different? How are the nets the same? Could we have made some different nets?

Matching Nets

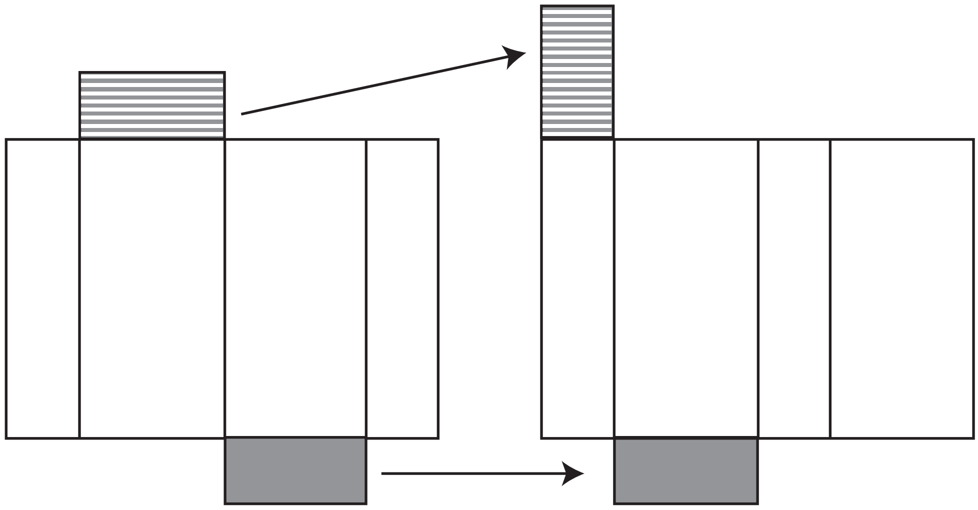
Provide students with a range of 3D objects and the matching range of nets. Invite them to predict what shape each net will fold up to be and then fold to check. Ask: What part of the net helped you to predict which shape it was?

Matching Nets Again

Extend 'Matching Nets' by including nets that will not fold to make some of the objects. Have students list the number and shape of the regions on the net and the number and shape of the faces on the matching object. Encourage them to fold the net over the object to see if the faces match. Ask: Which nets match the object? Which nets do not match the object? Can you see why it doesn't match?

Opening up Boxes

Ask students to open up a box into a net. Invite them then to cut off one face and replace it in another position. Ask: Can your new shape be folded to make your box again? What is the same about your net and the new shape? What is different?



SAMPLE LEARNING ACTIVITIES

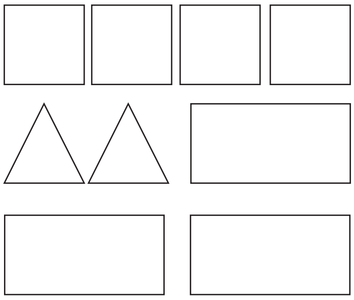
MIDDLE ✔✔

Matching 2D with 3D

Provide students with a choice of 3D objects (e.g. cubes, rectangular prisms, triangular prisms) and a sheet of paper showing a selection of different 2D figures. Invite them to select the 2D figures needed to cover the 3D object. Ask: Why have you chosen this figure for this object? How can you fit the figures together to make the 3D shape?

Tracing

Provide a collection of objects that differ in shape (e.g. matchboxes, dice, product packages, boxes, pyramids). Invite students to trace around all of the faces of each object and present the figures to the class. Display the tracings of the faces (see the diagram for examples). Ask: Which geometric solids have these faces?



Flattened Boxes

Provide a collection of different cardboard boxes. Have students choose one of the boxes, trace around each of the faces and cut them out. Ask them to lay out the shapes so that they would remake the box if folded. Have them find the glued seam, split the box and flatten it out. Encourage students to compare the net of the box with the way they laid out the faces. Ask: Is it the same? If not, how is it different? Does it matter that it is different? Is there a number of different ways of putting the faces together so that they could be refolded to make the box? (See Sample Lesson 2, page 81.)

Predicting 3D from 2D

Provide students with a range of nets (some that will fold up to make closed 3D shapes and some that will not) and a range of 3D objects (e.g. cubes, rectangular prisms, triangular prisms). Invite students to predict what shape each net will fold up to be, then ask them to fold to check. Ask: What part of the net helped you to predict which shape it was?

Envelopes

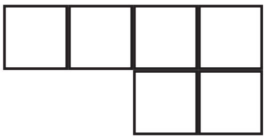
Have students colour each part of an envelope a different colour, and then invite them to draw what it would look like unfolded. Once they have done this, have them unfold the envelope and compare it to their drawing. Ask: Did the colours help you to see the different parts of the envelope? How did they help? How are the different parts connected? Which parts of the shapes meet?

Pass the Nets

Have students explore how many nets they can make for an object. For example, invite them to use materials such as Polydron™ or JOVO to each construct a 3D object. Ask them to make a list of the pieces they have used, and then swap the list and the 3D object with a partner, who uses the list to construct and draw a net of that object. Ask students to then pass everything (list, object and net) to another student, who uses the list to make a net for the object that is different from the first net. Continue in this way until no more nets can be made for each object. Ask: How do you know there are no more possible nets? How can you prove it?

Hexominoes

Have students use six square tiles or paper squares to make as many different hexominoes as possible. Encourage them to record each different shape by colouring squared paper and cutting out the figures. Invite students to predict which hexominoes will and which will not make a cube by visualising the hexomino folded up. Ask: What is it about the arrangement of the squares that tells you that it cannot possibly make a cube? Encourage students to test the predictions by making the cubes.



Pyramid

Display a model of a square-based pyramid. Invite students to sketch all the possible nets for a square-based pyramid. Ask: How are they the same? How are they different?

SAMPLE LEARNING ACTIVITIES

Later ✔✔✔

Flattened Boxes

Extend Middle Sample Learning Activity, 'Flattened Boxes' by repeating it but have students predict by visualising whether the nets will fold up to make the original object before they test by folding it. Encourage them to leave the net flat on the desk and explain to their partner why they think it will fold to make the original object or why they think it won't. Then invite them to fold to check.

Cereal Container

Invite students to design a container for a new breakfast cereal. Discuss the need to consider the construction material, the shape of the packaging, design of the label and so on. Encourage students to draw the net for their new container and then make it up.

Mailing Boxes

Invite students to design and make boxes to send specific objects (e.g. a chocolate bar that is a long triangular prism, a candle shaped like a pyramid) through the mail. Discuss why they chose particular designs and how they determined the nets for these designs. (See Sample Lesson 2, page 81.)

Box for Big Books

Extend Later Sample Learning Activity 'Box for Big Books' (Key Understanding 1, page 67) by asking students to use their perspective and exploded drawings to design a net for a pamphlet box to hold big books in their classroom. Ask: How do you know what size to make each section? Where does it matter whether the sizes are right? Which faces absolutely have to be placed side-by-side? Why? (See Key Understanding 3.)

Dice

Provide students with a dice and a net of a cube. Ask them to place the dots on the cube net so that when it is folded the dots will be in exactly the same place as on the dice. Focus students on the position of the dots in relation to each other on the dice. Ask: How can you be sure the dots are in the right spot on your net?

Different Dice

Extend 'Dice' by providing dice in more complex shapes (e.g. pyramids, prisms) and their nets. Have students place the dots on the net so that when it is folded the dots or numbers will be in exactly the same place as on the dice. Ask: How can you get those three numbers to come together at that point? Which dots were easy to place? Which ones were more difficult? Why?

Moving Faces

Invite students to design nets for 3D objects. Have them cut one face off and place it in another position. Ask: Will the new arrangement fold to make the original shape? Why? Why not? Encourage students to test their prediction.

Coloured Cube

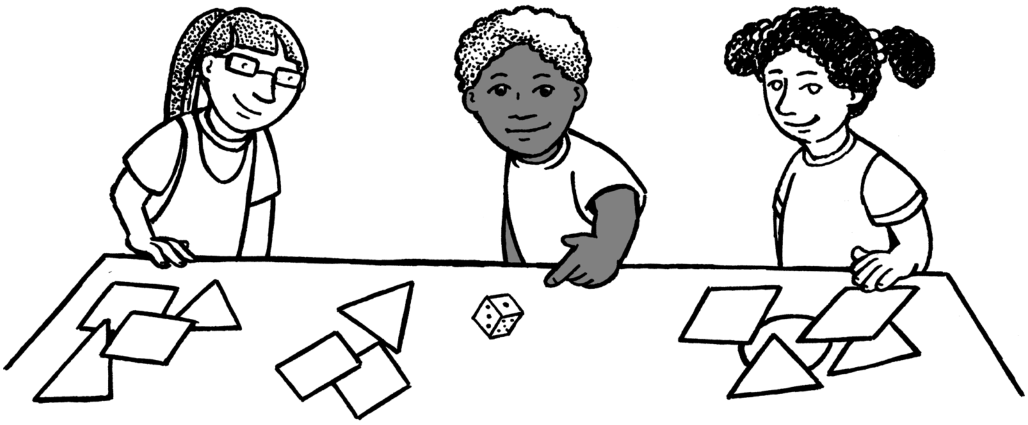
Give each student a cube with each face coloured a different colour and a collection of nets. Invite students to visualise where the colours would be on each face of the net and colour them accordingly. Encourage them to fold the net to check their predictions. Ask: Which part of the net did you colour first? Does the order in which you colour matter? Why?

Templates

Have students construct a set of cardboard figures they can use as templates to produce nets for a number of different polyhedrons. Invite them to examine some polyhedrons and list the common shapes needed for each one. Encourage them to consider the lengths of the sides of the shapes so that the component parts will fit together. Invite students to compare their sets and say which polyhedrons they can construct. Ask: What decisions about the dimensions of the squares and rectangles did you have to make to be able to make both cubes and rectangular prisms from your figures? Are there other different polyhedrons that can be constructed from the set of figures that you have made?

Solids Game

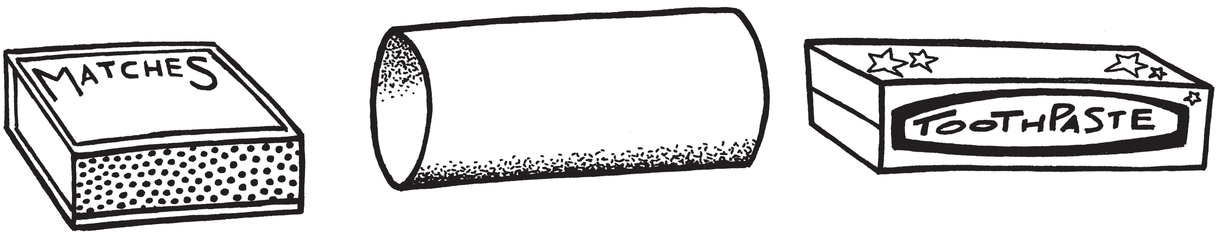
Extend 'Templates' by organising students into small groups to play this game. Have them take turns to roll a dice and trace around a figure from their template set that has that number of sides. For example, if they roll a 4, they might choose their rectangle, rhombus or square. As they build up their collection of figures, have them write a list of polyhedrons that can be constructed from their figures. For example, if they have two triangles and three rectangles, they could have a triangular prism. The winner is the person with the most polyhedrons listed. Ask: Was there a figure that enabled you to construct more polyhedrons than others? If so, what was that figure? Why do you think you were able to use that one more?



Later ✔✔✔

Different Nets

Provide students with a collection of different boxes, for example, a matchbox, a cardboard cylinder and a toothpaste box. Ask students to draw on paper what the objects would look like if they were unfolded on their desk. Invite them to cut out their nets and fold them to see if they were correct. Encourage them to make alterations if necessary. Have students unfold the original objects and compare their drawings to the nets just made from the original objects. Ask: Do they have to be the same? What is it that makes a figure a net of an object?



Folding Nets

Following activities such as 'Different Nets', have students consider where the tabs should be placed so that the faces can be glued together to form the original object. Ask: Why won't it work to have the tabs along that edge?

Identical Boxes

Have students cut identical boxes in different ways to investigate alternative nets for the same box. Ask: Which boxes would be the cheapest to produce? Why? (Link to Represent Transformation; Key Understanding 2.)

Polyhedrons

When building up a class set of polyhedrons ask: Can you think of any polyhedrons we do not have? Encourage students to research what these polyhedrons look like, design the nets on card and fold them to construct the shape. (See Key Understanding 1.)

Message Prism

Ask students to write a word or short message in block letters around the faces of a prism. Invite them then to design a net with the message on it for a partner to cut out and use to make a prism and work out what the message was. Ask: When you were making the net, how did you know where to place the faces? When you were making the prism from the net, how did you know what shape it might be?

SAMPLE LESSON 2

Sample Learning Activity: Later—'Mailing Boxes', page 78

Key Understanding 2: The net of an object has to have the same component parts as the object and the parts have to be in the right relationship to each other.

Motivation and Purpose

My Year 5 class had made biscuits in jars as gifts, so I decided to use this to help the students to learn more about nets. I gave them the following task:

> Make a decorated box for your jar of biscuits so that the jar will not get broken.

I thought the students would interpret this task as requiring a rectangular box, as we had previously constructed them from cardboard nets using pre-drawn templates. None approached the task by planning a suitable net, however, most began by folding or rolling the card stock to fit directly on the jar. For example, James rolled the card around the jar, taped it together, then folded down the ends and taped them.

Making prisms from pre-drawn nets does not help students to focus on the number of shapes, the types of shapes and the way that these shapes are connected to each other.

It became clear that their previous experiences with nets had not equipped them to either choose to use a net for such a purpose or to design one. I realised that the students needed to look more closely at nets to see how the different 2D figures went together to make them.

Connection and Challenge

Using a collection of cardboard boxes, I asked the students to trace around the faces and cut out the figures. I wanted them to think about how these 2D figures could go together to make the original box, so I asked the students to lay out the figures in such a way that if they were folded they would remake the box.

As the students worked, I could see that some were having difficulty finding an appropriate arrangement. I thought it might help them to compare their attempt with someone else, so I suggested that they find someone in the room who had the same shaped box.

The difficulty was caused by several factors. Some students did not have enough shapes, while others had lost track of which shapes they had traced and so had drawn some faces twice. Still others were having difficulty with how to place the shapes together. Each of these problems needed attention.

'Look at each other's arrangement of shapes and see how they are the same.'

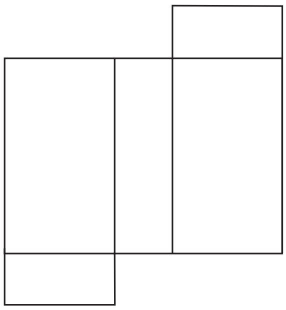
This prompted a lot of discussion as the arrangements were quite varied. Several students were claiming that their arrangement was right and their partner's arrangement was wrong. I could see that in some cases both of the arrangements would fold to make the same shaped box whereas others would not, so I suggested to the students, 'Tell your partner how you would fold up your arrangement to make it into a box.'

To the amazement of many pairs of students, they found that both arrangements would make the box. Rather than focusing on why this was the case, I thought it best to work with the students who had not been able to arrange their figures in a way that would 'work'.

Therefore, I asked the successful group of students to look at how the original box was made by splitting it open and flattening it. I asked them to make a journal entry to explain how their net was the same as the net of this box and how it was different.

Opportunity to Learn

I asked the other students to open out their original box and flatten it. Some students did not have the right number of 2D faces to make the net of their box, so this is where we started in our comparison. I asked them to count the faces in the net of their box and to count the 'faces' they had made. Emma, for example, realised that she did not have enough, so I asked the group to look at Emma's arrangement and the net of her box to say what was missing.

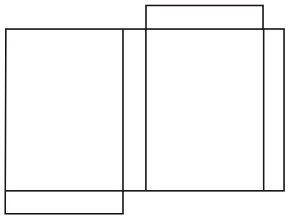


Emma's net

They all agreed that she needed another skinny rectangle.

'So how many rectangles do we all need to make a net of a box?' I asked.

They decided that it was six. I knew that this might not be the number needed for other shapes, so pointed this out. 'Yes, we need six rectangles to make a rectangular prism, but for other objects we might need a different number. Could you all check to make sure that you have the right number of rectangles to make your box?'



Jason's net

It was then time to look at the sorts of figures that the students had cut out, as some had the right number, but had the wrong shapes.

'Look at the net of your box and see if you have exactly the right shapes. For example, Jason's net has two large rectangles and four long thin rectangles, so he needs to have two large rectangles and four long thin rectangles. Check to see whether you have the right shapes.'

I could see several students checking by laying their cut out figures on top of the faces in the net of their box. The other students thought this was a good idea and decided to copy this strategy. This allowed them to easily see which ones they had made two versions of and which ones they had missed out altogether. I allowed them time to make copies of the missing parts before we moved on.

Action and Reflection

'So, now that we have the right number of cut outs and we have the right sort of shapes, how could you put all of these together to make the net for your box?'

I knew that they would probably copy the same arrangement as those in the flattened out box, so, after they had finished this, I challenged them further.

'The other students found that they could arrange their cut out figures in different ways and still fold it up to make the same box. Work with a partner to find a different way of placing your cut outs that will make the same box.'

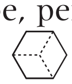
I left them to work on this as I went to talk to the rest of the class.

Working with the class in this way had allowed me to focus the attention of the small group of students on the aspects of the nets that they needed to work on. The rest of the class needed to interrogate why it was possible to make different arrangements of the figures to make the one 3D object. This would have to be the focus of our next session with nets.

In a later lesson, we revisited the original task and students used their new understanding to design a suitable container for a jar of biscuits. This gave me the opportunity to observe whether students had understood the ideas in a way that enabled them to make use of their learning for practical purposes.

KEY UNDERSTANDING 3

To understand drawings of objects we need to combine what we can actually see with what we think is there. Special drawing techniques emphasise different aspects of an object.

There is a wide range of approaches to representing 3D objects on two-dimensional surfaces, each having obvious practical advantages. Representations of 3D shapes are culturally specific and we have to *learn* to interpret them. For example, a common way of representing a cube is like this:  In fact, in a photograph and in a perspective drawing, the parallel edges going back on an actual cube would get closer together. Nevertheless, most adults would say the above diagram looked like a cube and not notice the 'inaccuracy' of keeping all opposite edges parallel. For them, the cube 'pops out' and looks three-dimensional. Young children, however, may not see a cube, perhaps seeing a hexagon  with some lines added.  Asked to reproduce it, they may well produce a flat arrangement comprised of three diamonds. Thus, students have to *learn* how a 2D representation is read within their culture so that it also 'pops out' for them. Of course, cultures vary in the way in which they represent three-dimensional space in two-dimensional forms and, even within the one culture, there may be various forms of representation.

Mathematics provides a number of standard ways of representing space that are in widespread use internationally and students need to learn the conventions for interpreting and producing them. On the one hand, they have to learn that we:

- may represent what can be seen from a particular point of view and ignore things that we know to be there but cannot see (we do not draw all four wheels if we can only see three)
- may distort shape and size to make diagrams look more realistic; for example, drawing circular wheels as ellipses and representing things further away as smaller, as in perspective drawings
- may distort reality in order to emphasise some features, as in oblique and isometric drawings (see page 97).

On the other hand, they also have to learn that we:

- often interpret diagrams by reading beyond them to what is not in the diagram but must have been there (e.g. the wheel at the back)
- begin to 'see' ellipses as circles and converging lines as parallel, so 2D diagrams look like 3D things
- can read beyond the distortion in certain diagrams and understand what they are telling us.

The ability to produce representations of 3D objects drawn from a fixed viewpoint develops slowly and students will need considerable experience in interpreting 2D representations of 3D objects and spaces, varying from photographs and semi-realistic sketches to various geometric diagrams such as the cube above. They might, for example, arrange students and food to match a photograph of a picnic or find the position from which a sketch of a building was made. They may need to 'fill in' the parts of an object not shown in a drawing in order to build a cube structure from an isometric drawing. Over time, they should learn that there are some basic conventions used to represent depth. Students in the later years should learn to use isometric and oblique grids to draw representations of regular polyhedrons, and may begin to use vanishing points for perspective drawing. Investigating the difference between these drawings enables students to see which features of the objects are highlighted in each and so learn to choose which to use for particular purposes.

Progressing Through Key Understanding 3

Initially, students interpret conventional diagrams of objects such as prisms and pyramids, although they may not be able to produce them. They can turn, position and rearrange everyday and geometric objects to match drawings. In their own drawings, they attempt to show what can actually be seen and give some attention to depth.

As students continue to progress they select actual objects to match exploded drawings, or front, back and side views, or various art forms. They draw prisms, pyramids, cylinders and cones recognisably, but not necessarily with a high level of precision, using them to draw other things, such as a building.

Next, students interpret and compare a range of representations of 3D space and produce isometric, perspective and oblique drawings accurately. They can visualise a scene or an object in different orientations and draw possible 'other views' of an object from information contained in 2D drawings.

SAMPLE LEARNING ACTIVITIES

Beginning ✔✔

Stories

Before, during or after reading a story from a picture book, ask students to arrange some 3D representations of the characters and objects from the story to match a picture scene in the story. Ask: Where does this character belong? What is on this side of the character? Where does the tree go?

The Three Bears

When students are illustrating stories such as The Three Bears, provide them with an actual teddy bear to copy. Before they begin drawing, focus the students on the shapes they can see in the ears, face, body and legs. Ask: What shape is the leg? (a long sausage shape or a cylinder) Stand the bear up in front of the group and say: Some things look different when you see them from different places or positions. Ask: What does the leg look like from where you are sitting? Ask students to trace the shape that the leg looks like in the air and then draw what they can see of the leg on the paper. Repeat this with the other parts of the bear so that students draw the bear from their viewpoint. (See Represent Transformation, Key Understanding 3; Link to ***First Steps in Mathematics: Measurement***, Indirect Measure, Key Understandings 2 and 3.)

Only What You See

Have students draw a line drawing of a simple familiar object that has parts hidden from view (e.g. a cup with the handle hidden). Focus students on drawing only what they can see. Encourage students to talk about parts they cannot see but know are there. Ask: What can you see? How have you shown that in your drawing? Why didn't you draw the handle?

Coloured Cubes

Provide pairs of students with a cube that has each face a different colour and a line drawing of a cube. Invite them to colour in the faces to match those on the cube. Ask: Which colours can't be seen on your picture? Are the colours on your picture different from the colours on your partner's picture? Why? Encourage students to swap their pictures and cubes and then match the cube with the view shown on their partner's picture.

Constructing Shapes

Have students examine photographs of basic 3D shapes (e.g. cubes, cylinders, half spheres) and invite them to construct those shapes using play dough or modelling clay. Ask: How did you know the cylinder had two ends, when you could only see one? How did you know those two sides were the same when they don't look the same in the photo?

Pictures

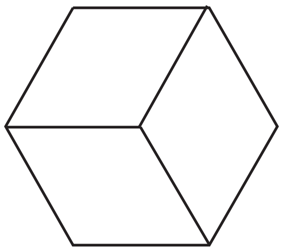
Show students pictures of scenes and discuss what they notice about the shapes in the foreground and background of the picture. Ask: Why do the trees in the background look smaller? Why does this child look bigger than this adult? How has the artist made some things look further away? Encourage students to use these ideas to construct their own picture using pictures cut from magazines.

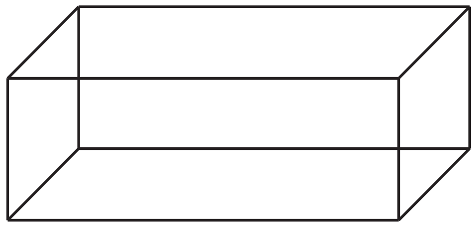
What Can't Be Seen

Give students a familiar object or a picture of a familiar object (e.g. car, television, dog) and invite them to draw what they think the other side of the object would look like. Ask: How do you know what it will look like? What do you know must be there but can't be seen?

Puzzling Pictures

Quickly show students a 2D representation of a 3D object, such as a hexagon made from three rhombuses to represent a cube, or a simple oblique diagram of a rectangular prism. Ask students to say what they saw. Invite them to explain to a partner what part of the drawing 'stood out' so that they knew it was really showing an object, not a flat shape. Show them the representation again and ask: How did you see that figure was a cube (long box)? Which part of the figure showed the front of your cube (long box)? Which part of the figure showed the end of your cube (long box)?

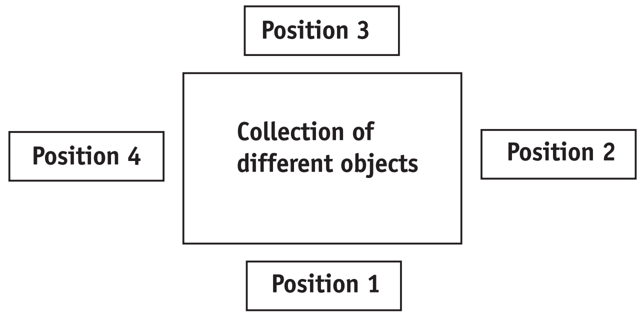




Beginning ✔✔

Different Views

Sit students around a collection of objects (e.g. toy car, cup with handle, jug) and have them take turns to say what they can see. Ask: What is on the table that can't be seen from position 2? Who can see a small part of something and can guess what it is?



Shape Hunt

Have students locate square, rectangular and circular figures in pictures (e.g. the square figure in a table, the circle figure in a wheel). Invite them to place a sheet of plastic over each and trace around the shape. Ask: Why doesn't this look like a square (circle)? How do we know it should be a square (circle)?

Photographs

Give students photos to examine and discuss what it is about the photo that enables them to make sense of the situation. Ask: Is that building larger than that one? How do you know? Which person is at the front? Which animal is furthest away? How do you know? What does it mean when the girl overlaps the boy? Who is in front? Who is behind?

Exploded Drawings

Have students interpret simple exploded drawings, such as those found in Lego® packs or in other kit assembly instructions, in order to work out how the different components fit together to construct an object (e.g. robot, rowing boat, baby carriage). Ask: How do you know which pieces to use? How does the drawing help you know where each piece has to go?

Fruit Bowl

Have students sit in groups around a bowl of fruit, draw line drawings of what they see, and leave their drawings in a pile on the table. Invite them to exchange tables with another group, choose a drawing from that table and match the viewpoints with one of the drawing positions. Ask: How do you know it comes from here? What piece of fruit can you see when you are sitting in this seat? What can't you see? Which seat would you be sitting in if you could see what is shown in this drawing? (Link to Represent Transformation, Key Understanding 1.)

SAMPLE LEARNING ACTIVITIES

Middle ✔✔

Skeleton Shapes

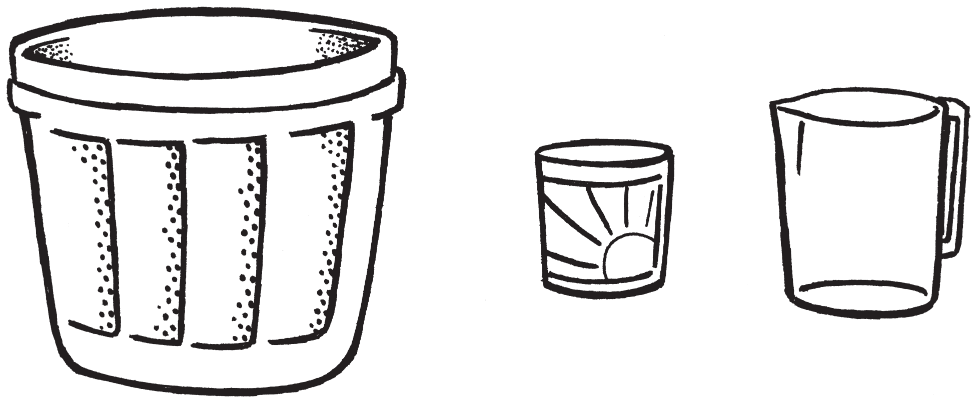
Invite students to make skeleton 3D shapes (e.g. triangular prism, square pyramid) with toothpicks and play dough or modelling clay and then ask them to draw their structure. Display the structures and invite students to choose a drawing and match it with the skeleton shape. Ask: What was it about the drawing that helped you match it with the shape?

Viewpoint

Take photographs of familiar shapes in the classroom or playground from several different positions. Have small groups of students locate the position from where the photograph was taken and explain how they know. Ask: How did you know the photographer stood there? What part of the photo is the same as what you can see? Is there anything different?

Drawing the Top

Have students examine basic 3D shapes and, with assistance, draw what they can see from where they are sitting. For example, invite students to make several drawings of a tall cylindrical shape (e.g. rubbish bin, fruit juice can, water jug), until they are satisfied that the top looks right. Ask: What shape is the top of the can? What shape did you have to draw to make it look right from where you are sitting?



Skeleton Diagram

Show students a diagram of a 3D shape (e.g. prism, pyramid) and invite them to use straws and joiners to construct what they think the diagram represents. Ask: How did you know which parts to build when you can't see it in the diagram? How do you know what the whole shape should look like?

Middle ✔✔

Cross Sections

Have students select a play-dough object from a collection of prisms, pyramids, cylinders and cones. Invite them to imagine and draw the shape they would get if they were to cut it into two pieces. Have them swap their drawings with a partner and make the cut required to get the cross section that was drawn. Ask: Why did your cut through that rectangular prism not match the cross-section drawing? Can you cut it a different way?

Isometric Drawings

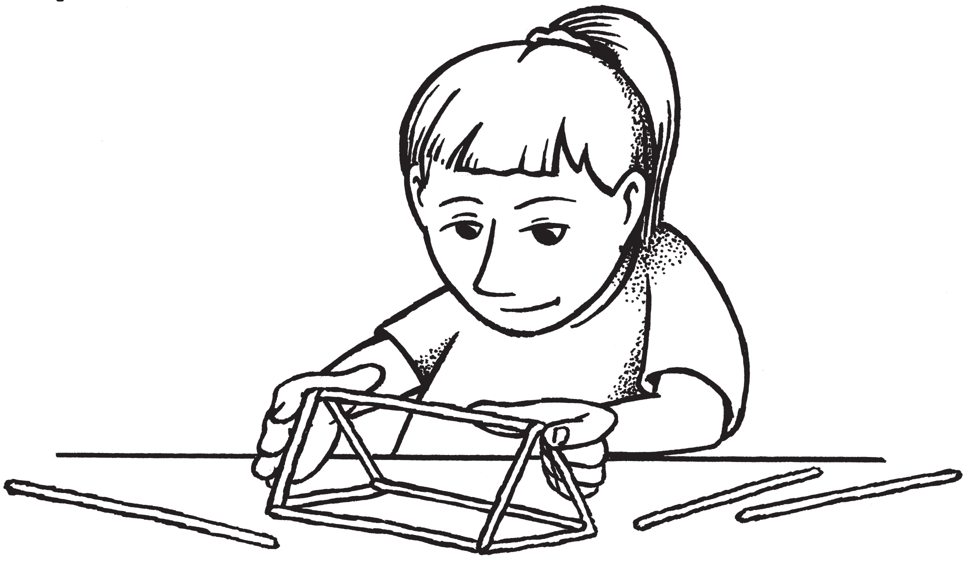
Have students make up different arrangements of cubes from simple isometric or oblique drawings. Discuss how they knew what to do. Ask: How did you know which face of the cube matched each part of the drawing? Which part of the drawing did you use to know what was at the back?

Ten Blocks

Invite students to make an arrangement using ten blocks and then draw their structure. Have them swap their drawing with a partner and make their partner's drawing. Ask: Are the two constructions the same? Why? Why not?

Skeleton Swaps

Invite students to construct skeletal 3D shapes using such things as straws and joiners, pipe cleaners and matchsticks joined with Blu-Tack™. Have each student draw a picture of their model, swap their drawing with a partner and use the new drawing as a plan to construct the model it shows. Encourage students to compare the original model with the copied model, matching each straw in the model to the line on the drawing. Ask: Why doesn't your partner's model look like your model? Is there something that you need to do to your drawing that would help your partner to make a model that looks like yours?



3D City

Have students construct a 3D representation of a photograph of a city (farm, ship), using recycled materials or 2-centimetre cubes. Encourage them to identify the 2D shapes within the picture and say how they decided to represent each using 3D material. Ask: Why did you use the toothpaste box for ...? What was it about the picture that suggested to you that the toothpaste box was the best?

Soma Cube

Invite students to use 2-centimetre cubes and glue to construct the seven pieces of a soma cube set, using a diagram or isometric drawing of each piece. Encourage them to then build the puzzle by constructing a 3 x 3 x 3 cube. (See Key Understanding 1.)

Four-Cube Houses

Invite students to use four cubes to make houses where the blocks are joined at the faces. Ask: How many different designs can you make? Have students draw a diagram of two of the houses from the side, front and top views and use this to explain how the houses are different.

Building

Have students construct a small building using Lego® pieces and draw a diagram of their structure from the side, front and top views. Invite them to exchange drawings with a partner to build each other's structure from the diagrams. Ask: How did you know what to do? How can you check if you are right? What part of the diagram gave the most information about the building? Is the building the same as the original? Why? Why not?

Photographs

Provide photographs and pictures of buildings, roads and other structures. Ask students to mark the lines on the pictures that would be parallel in real life. Attach the pictures to a large piece of butcher's paper. Invite students to draw over and extend the lines to show how perspective drawings come together at a focal point. Ask: Are the lines parallel in the picture? What happens to the lines?

Out of the Window

Invite students to look out of the classroom window and draw a structure as they see it directly onto the glass. Alternatively, set up a large piece of glass vertically on a desk and place objects behind the glass. Have students look at it at eye level and draw exactly what they see. Say: The left side of that building is a rectangle, but you have not drawn a rectangle, you have drawn a parallelogram. Ask: Why is that? What can you see from where you are sitting?

SAMPLE LEARNING ACTIVITIES

Later ✔✔✔

Drawing a Cube

Have students examine a range of 3D shapes (e.g. prisms, pyramids, cylinders, cones). Invite them to sketch what they can see from where they are sitting. Encourage students to make several sketches to come to a closer representation of what they can actually see. Ask: What shape do you know forms the top of the cube? What shape did you have to draw to make the top look right from where you are sitting? Can you sketch the cube again so that the figure at the top of the box is not a square but a rhombus?

Depth

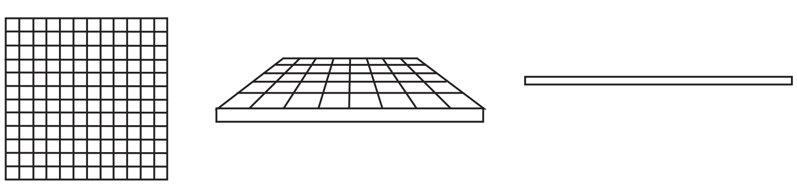
Show students a photograph of a line of telephone posts to see how depth is represented. Ask: How big do the closer telephone poles seem to be? What about the ones further away? What do you notice about the width of the road as it runs into the distance? Encourage students to draw the posts and the road to show depth.

Overlapping Figures

Show students pictures that have overlapping figures within them that show depth or distance. Ask: Which building is in front? How do you know? How can you tell that the car is behind the tree? Give each student a collection of objects (e.g. lunchbox, glue stick, mug) with some placed in front of others. Encourage them to use the same approach to draw the collection to show depth or distance of the objects in the collection. Have them rearrange the objects and swap drawings and places with a partner. Then invite them to use their partner's drawing to arrange the objects as they were. Ask: How did you know that the glue stick was behind the mug but in front of the lunch box? What did your partner do in the drawing to show the positions of each object?

Viewpoints

Have students examine how different viewpoints affect the perspective of their drawings. Place a draughtboard on the desk. Have students draw the board from a bird's eye view. Repeat the drawing standing away from the table. Lastly draw the board with their eyes level with the table. Ask: How does the drawing change with each change of view?



Cross Sections

Extend Middle Sample Learning Activity (p. 90) 'Cross Sections' by having students examine a range of fruit and vegetables, select one and imagine and draw a cross-section view of it. Invite them to exchange the items and the drawing with a partner, then visualise and make the cut to produce the cross section. Ask: How did you know to cut your banana on a slant and not straight across? What was it in the drawing that showed you that the orange was not cut straight down the middle? (Link to Represent Transformation, Key Understanding 1.)

Building Houses

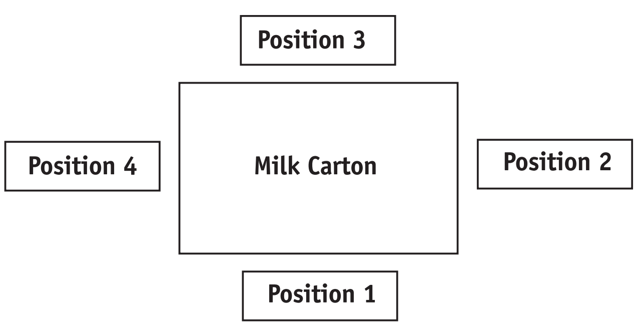
Provide students with architectural sketches showing the different elevations of house plans. Encourage them to identify the basic 2D shapes used for the different elevations of the house and join the shapes to construct the model of the house. Display the houses next to the sketches. Ask: How did you 'fill in' the parts of the building that were not shown in the sketches?

Building Complex Structures

Have students build complex block structures from isometric, oblique or perspective drawings. Ask: How did you fill in the parts of the object that could not be seen in the diagram? Did thinking about rows, columns or layers help you? Which representations are easier to interpret, isometric, oblique or perspective drawings? Why?

Carton and Cup

Place a milk carton with the spout open pointing left in the centre of the table. Have a student sitting in position 1. Say: I want you to draw exactly what you see from where you are sitting. Have the student move to position 2. Repeat the instructions. Do the same for positions 3 and 4. Repeat the activity with a cup on the table. Ask: How does your position change the diagram you drew?



Four-Cube Houses

Invite students to use four cubes to make houses where the blocks are joined at the faces. Ask: How many different designs can you make? Give students oblique grid paper to record their designs to show they have found them all.

Later ✔✔✔

Soma Pieces

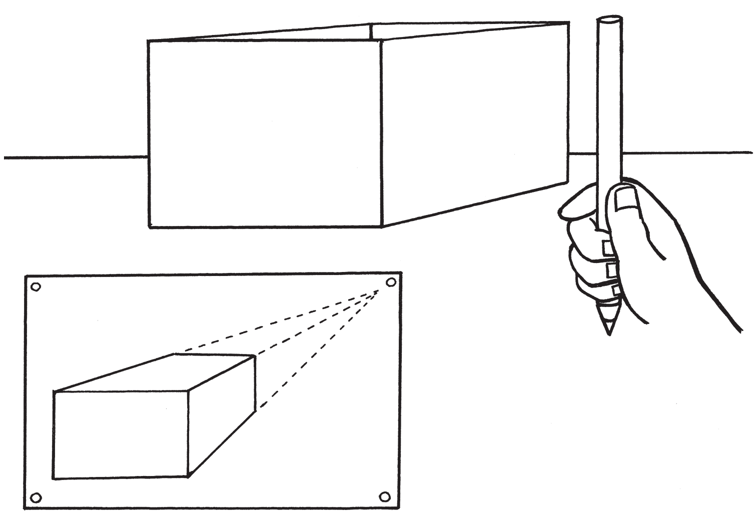
Give students soma pieces to make items such as a chair or a set of stairs. Encourage them to draw an oblique and isometric drawing of their model and say how each is the same and different. Ask: What part of the model is at the front of the isometric drawing? What part of the model is at the front of the oblique drawing? Why is this different? (See Key Understanding 1.)

Soma Pieces Again

Extend 'Soma Pieces' by having students draw an exploded diagram of the same model. Invite them to swap all their drawings with a partner to make the model. Ask: Which drawing was most helpful in making the model? Why?

Cardboard Box

Have students draw what they can see of a large cardboard box that is placed on the floor where all can see it. To help them accurately draw what they see, have them close one eye and slide their thumb along a pencil to compare the lengths of the edges from where they are sitting. Ask: Do all of the sides look the same length? What differences in lengths do you see? While students are drawing, discuss with them how the vertical edge that is further away looks shorter and how parallel lines appear to get closer to each other as they get further away. When several students are satisfied with their drawing, ask them to explain what they did to make their drawings 'look right'. Pin one drawing up and extend the oblique lines as dotted lines to show a 'vanishing point' and explain that they have made a perspective drawing of the box. Encourage students to experiment with using a different vanishing point by drawing the box again from a different angle.



Vanishing Points

Extend Middle Sample Learning Activity (p. 91) 'Photographs' by having students draw over and extend lines that would be parallel (in real life) to identify vanishing points. Ask: Are the vanishing points in front of or behind the shape? Are the vanishing points on the horizon? Are the vanishing points level with each other or is one higher than the other? Sort the pictures according to whether there are one or two vanishing points. Ask: Where would you have to be standing for the vanishing points in each category?

More Vanishing Points

As students produce perspective drawings in other activities, have them examine each other's drawings and make comments about whether it looks right according to what they know about vanishing points. Ask: Why doesn't that bed look right? We know the sides are parallel, but what do we do to make it look more realistic?

Comparing Drawings

Have students make a model using Polydron™ and then draw a perspective, oblique and isometric drawing of it. Encourage them to examine how the shapes of the faces, the angles and the edges are drawn on each and say what is the same and different. Ask: Which part of the model is given emphasis in each drawing? When would we use each type of drawing?

Exploded Drawing

Extend 'Comparing Drawings' by having students make an exploded drawing of their model on a blank piece of paper. Ask: How can you get the angles to be the same as those on the model? How can you get the shapes the same? How can you get the lengths of the edges the same? Encourage students to use protractors, rulers and geometry drawing templates to help.

Box for Big Books

Extend Later Sample Learning Activity 'Box for Big Books' (Key Understanding 1, page 67) by having students sketch a perspective drawing of the box for big books, labelling it with the dimensions required. Invite them to use this drawing to produce an exploded diagram showing the dimensions of each component. Encourage students to use a protractor to get the angles exact. (See Key Understanding 2.)

Soma Cubes

Have students draw each piece of a soma cube on isometric paper. Encourage them to label each shape with a number and have another student use the drawings to construct the components. Compare the original shapes with the new shapes and explain any differences. (See Key Understanding 1.)

Design Problem

Have students draw a diagram of a model to solve a design problem (e.g. making a suitable container for shared drawing materials that is easy to pass around). Ask: Why did you use that particular type of drawing? Could you have used a perspective drawing instead? Why? Why not? Invite them to construct the model from the diagram and then say what other information they needed on their diagram before someone else could use it. Ask: Was the model the right size? Where the different parts the right size in relation to each other? What information do you need to show on your diagram so that the person building the model knows how big to make it?

Later ✔✔✔

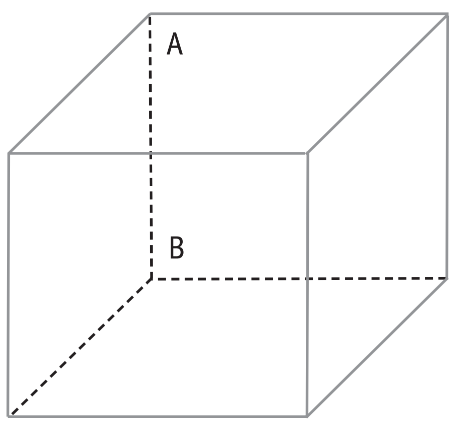
Elevation Plans

Have students make a six-cube house and use isometric paper to record their design. Invite them to hand their design to someone else to construct and discuss any modifications that needed to be made to the design to get the same model. (See Represent Transformation, Key Understanding 3.)

Did You Know?

Often while looking at a drawing of a cube it will seem to 'shift':

*Focusing continuously on some drawing, or attempting to examine the relations among different elements of a 3D object described in a 2D figure, may cause the first 3D image to modify. The back part of the object AB 'pops out' and creates the impression that the object turns inside out or the other way around. This well-known phenomenon, especially related to cubes, is known in psychology literature as 'Necker Cube phenomenon'.**



* From Latner, L. and Movshovitz-Hadar, N. 1999, 'Storing a 3D Image in the Working Memory', Proceedings of the 23rd Conference of the International Group for the Psychology of Mathematics Education (PME23). Haifa, Israel, July 25–30, Volume 3, p. 201.

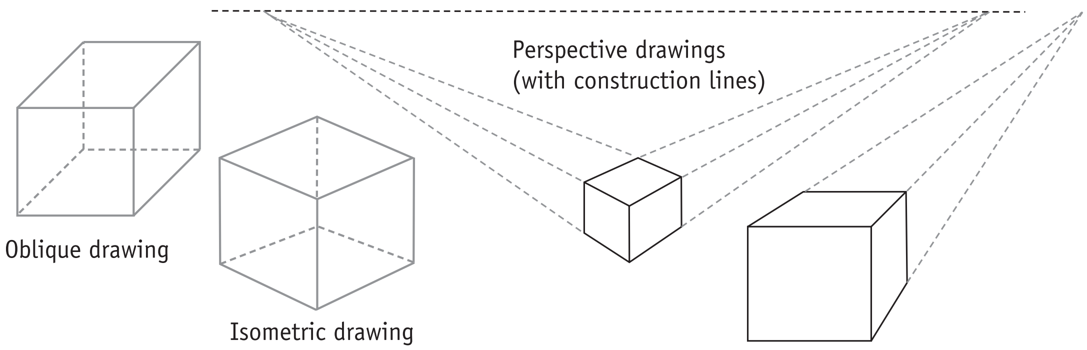
BACKGROUND NOTES

When we draw any object we have the choice of drawing it 'flat' (2D) or as a 'solid' (3D) and our choice will be determined by our purpose. Perspective, oblique and isometric drawings are three common forms of representation which, to varying degrees, 'look like' 3D objects to us. In addition, front, back and side views and exploded drawings look like the objects from particular 'perpendicular' orientations.

We use perspective drawings when we want our drawing to look realistic or 'right'. Perspective drawings look like what you would get if you took a photograph of an object and then traced along the edges. The opposite edges of a cube that recede are not drawn parallel, the edges may not all be the same length and the angles may not be right angles, but the cube will still 'look right'. Students should investigate how we make things 'look right', considering, for example, drawing ellipses for circles and trapeziums for rectangles, and lines coming closer to represent distance.

We use oblique and isometric drawings to draw 3D pictures of objects when the scale of the sides matters; in schematic diagrams used for construction, for example. Oblique drawings have one face of the object positioned at the 'front' so that the shape and angles of this face are the same as the object and the lengths of the edges of this face are to scale. Two other faces are drawn at a 45° angle to this front face, with each maintaining parallelism where it exists in the object being drawn.

Isometric drawings have an edge positioned towards the 'front' of the drawing. Three faces are drawn with the length of all edges to scale and parallelism maintained. However, the shapes of these faces and the angles are not the same as on the object; for example, parallelograms are used to represent square and rectangular faces.



We use exploded drawings to draw 2D pictures of the front, back and side views of 3D shapes to provide three different viewpoints of the one object. Constructing models from these diagrams will help students to learn how to draw them even when they cannot see all three different viewpoints.

Special geometric techniques for constructing 2D figures are generally taught in the secondary years; during the later primary years, however, students should be assisted to develop skill and precision in drawing figures and objects, by using protractors, compasses and other geometric equipment such as geometry drawing templates and Miras.

CHAPTER 5

Represent Transformation

This chapter will support teachers in developing teaching and learning programs that relate to Part C of the outcome:

Visualise transformations and show the effect of them on shapes and arrangements.

Overall Description

Students visualise the effect of transforming shapes and arrangements in particular ways. Thus, they can predict how a shape will appear when reflected in a mirror, how an object will look if the viewer walks around it through a quarter turn, and whether a particular net will fold up to make a planned container.

They understand that there are many transformations, each with its own characteristics and purposes. They know that translation, rotation and reflection all preserve shape and size and that each is the basis of its own form of symmetry. They also know that enlargement preserves shape but not size, and that other transformations may distort shape.

Markers of Progress	Pointers Progress will be evident when students:	
Students repeat, reorient and turn over things when matching shapes and making pictures and patterns.	• match figures one-to-one; e.g. select a figure to match another and place it exactly on top to check • fill in complex pictures and patterns where lines are provided to enable direct matching of component shapes; e.g. picture sticker books • fill in pictures and patterns where no dividing lines are provided to enable direct matching of component parts; e.g. complete a 'fill-in' jigsaw puzzle	• make a copy of a complex picture, pattern or object when provided with a choice of component parts • fit figures and objects together based on shape and orientation • use a fold line to produce symmetrical pictures by drawing freehand, folding, cutting, tracing
Students use multiple copies of shapes to construct repetitive patterns and follow and describe simple movement rules for generating such patterns.	• understand two cut-out figures as 'exactly the same' if one fits on the other exactly and classify cut-out figures into groups that are and are not the same • explain their choice of simple figures that are 'the same' with language such as 'turn', 'turn over' and 'slide along'; e.g. *We turned the tall rectangle around and then we could slide it on top of the wide one.*	• make symmetrical pictures using a variety of means which include cut-out figures and flipping and drawing around templates • repeat multiple copies of two or three figures in a recognisable pattern for decorative purposes; e.g. use three types of pattern block in a symmetrical pattern • use multiple copies of a simple prism to decide whether repetitions will stack or pack and make repeating patterns with those that will
Students recognise repetitions of the same shape within arrangements and patterns and use repetitions of figures and objects systematically to produce arrangements and patterns.	• find repetitions of figures and objects within decorative patterns (e.g. tiling, fabrics), objects (e.g. beehives, blocks of flats, stacked tetra paks) and formations (e.g. dance routines, marching groups) • identify repetitions of component parts in symmetrical objects and arrangements and demonstrate by moving one component over another; e.g. trace around one arm of a windmill and turn the tracing around the centre to show how each arm fits over the others • informally describe the symmetry of a figure or	arrangement; e.g. say that a shape has line (or mirror) symmetry because it can be folded along a line so that one half fits over the other half • use multiple copies of figures to create patterns based on systematic movements of the shape • provided with multiple copies of a simple figure such as a triangle, rectangle or pentagon, decide if it will tile (i.e. tessellate or fit together to cover a surface without gaps or overlaps) and demonstrate • informally explain why they think a figure won't tile; e.g. *You can't make the corners fit together.*
Students recognise rotations, reflections, and translations in arrangements and patterns, and translate, rotate and reflect figures and objects systematically to produce arrangements and patterns.	• use appropriate language of transformation in describing how one shape can be superimposed on another, e.g. *Rotate it at right angles around the centre and slide (or translate) it to the left*. • decide which of rotation, reflection and translation is involved in producing a pattern or formation (in fabrics, tiles, buildings, formations of planes or birds) • decide which of rotation, reflection or translation is involved in producing a symmetrical arrangement and describe it; e.g. explain that a logo will be repeated each third turn and so it has rotational symmetry	• visualise and reproduce the folds and cuts used to make a complex symmetrical pattern; e.g. to copy a frieze or make a 'snowflake' • identify the transformations used to produce a spatial sequence and continue the sequence • provided with a single copy of a shape that will tile, produce a tiling pattern by systematically translating, rotating or reflecting the shape • use appropriate language of transformation in explaining how they tiled a shape or why they think a shape will tile
Students visualise and sketch the effect of straightforward translations, reflections, rotations and enlargements of figures and objects using suitable grids.	• produce designs that exhibit a specified symmetry (rotational, reflection or translation); e.g. fold paper or use mirrors or computer graphics • use an appropriate grid to produce a specified symmetrical shape; e.g. using circular grid paper, rotate a given figure to make a design that has rotational symmetry and that you move six times to get back to where you started • visualise to predict the effect of specified movements on the position and orientation of figures and objects; e.g. imagine what a shape will look like if rotated through a 90° turn clockwise • describe the effect of a translation, rotation or reflection on the position and orientation of a	figure; e.g. experiment with a Mira mirror or computer package and report, *Each bit of the reflection was as far behind the mirror as that bit of the figure was in front.* • use transformations to modify tessellating shapes to produce other tessellating shapes and informally explain why they work; e.g. Escher-type designs based on rectangles • use a grid to enlarge and reduce a figure (whole number and unit fraction scales) and to make distortions (e.g. double widths but not heights) • enlarge models made with cubes to a small whole number scale; e.g. given a model made of six cubes (like a soma cube piece), produce one enlarged by a scale factor of three

Key Understandings

Teachers will need to plan learning experiences that include and develop the following Key Understandings (KU), which underpin achievement of the outcome. The learning experiences should connect to students' current knowledge and understandings rather than to their year level.

Key Understanding	Stage of Primary Schooling—Major Emphasis	KU Description	Sample Learning Activities
KU1 We can imagine how a thing will look after we move all or part of it or change our view of it.	Beginning ✔✔✔ Middle ✔✔✔ Later ✔✔✔	page 102	Beginning, page 104 Middle, page 107 Later, page 110
KU2 We can move things around in space by reflecting, translating and rotating. These do not change size or shape.	Beginning ✔✔✔ Middle ✔✔✔ Later ✔✔✔	page 114	Beginning, page 116 Middle, page 119 Later, page 123
KU3 Some transformations, such as enlargement, change size but leave shape unchanged. Others change shape and size.	Beginning ✔ Middle ✔✔ Later ✔✔	page 130	Beginning, page 132 Middle, page 134 Later, page 136
KU4 Symmetrical things have component parts which can be matched by rotating, reflecting or translating.	Beginning ✔✔ Middle ✔✔ Later ✔✔	page 140	Beginning, page 142 Middle, page 144 Later, page 146

Key

✔✔✔ The development of this Key Understanding is a major focus of planned activities.

✔✔ The development of this Key Understanding is an important focus of planned activities.

✔ Some activities may be planned to introduce this Key Understanding, to consolidate it or to extend its application. The idea may also arise incidentally in conversations or routines that occur in the classroom.

KEY UNDERSTANDING 1

We can imagine how a thing will look after we move all or part of it or change our view of it.

The capacity to manipulate images in the mind and to visualise (or imagine in advance) the effect of particular changes on the shape, size, position and orientation of things is of immediate practical use in understanding, navigating and constructing our environment, including the computer environment. Improving students' capacity to visualise is important both for its direct benefit and because of its helpfulness in learning further geometrical and other mathematical ideas. The essence of this Key Understanding is that students understand that we can imagine the effect of spatial changes without having to actually carry them out, and that if we get 'good at it' we can rely on those predictions.

The ability to imagine the effect of movements should begin to develop as students turn shapes around and over to test whether they will match other shapes (e.g. for shape sorter), fit together (e.g. jigsaws), reproduce an arrangement (e.g. with pattern blocks) or fill or copy a shape (e.g. tangrams). Through such activities, students should learn to disembed simple shapes from complex patterns and arrangements, mentally compose, decompose and rearrange figures and objects, and visualise the effect on figures and objects of moving them (or the viewer) in particular ways. Thus, students might visualise and reproduce the folds and cuts needed to produce a 'snowflake' design from a square or a frieze from a strip of paper. Students should carry out simple changes to the shape, size or position of objects and observe the effects of these changes (e.g. *When I turned the square piece over it fitted into the same space.*)

Later, in conjunction with their developing understanding of the effect of particular transformations (see Key Understanding 2), students should predict the effect of such transformations on the shape, size or position of figures and objects and check by experimenting. For example, they may say: *I decided that if I moved this box along the desk in a straight line, all its corners would move the* *same distance. I was right.* Or they may say: *I thought if I used this 2-centimetre grid, the house would be twice as big. Every line was twice as long but the area was four times as much.*

Progressing Through Key Understanding 1

Initially, students predict whether shapes will fit into specified spaces, although this may largely be based on memory of previous attempts. A students continue to progress they know that things look different from different positions and sections but may have difficulty in imagining how they will look. Next, however, they are able to do this quite well, ordering photographs taken as a photographer moved around an arrangement without having to move themselves, for example, and predicting the shape of cross sections.

As students progress further they visualise and reproduce the folds and cuts used to make a complex symmetrical pattern such as on a frieze or 'snowflake'. Later, they predict the effect of specified transformations on the position and orientation of figures and objects, imagining, for example, what a shape will look like if rotated though a 90° turn clockwise.

SAMPLE LEARNING ACTIVITIES

Beginning ✔✔✔

Tangrams 1

Provide each student with a picture that others have made from tangrams and a matching set of correctly sized tangrams. Invite students to select, one at a time, a piece they will need to fill a particular spot. Encourage them to test by putting the tangram piece on the picture. (Link to Represent Shape, Key Understanding 1.)

Tangrams 2

Repeat 'Tangrams 1', but when students select a piece, encourage them to describe how they will need to move it to make it fit. Say: Point to the piece that you will need to fill that space. How will you need to move it to make it fit? Again have students test by putting the tangram piece on the picture. (Link to Represent Shape, Key Understanding 1.)

Fill in Puzzles

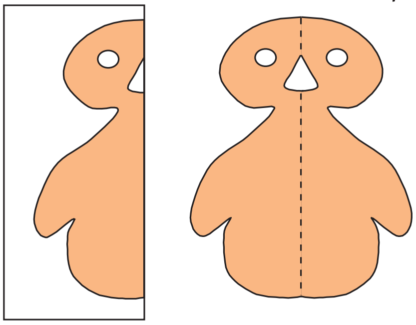
Provide outline puzzles and drawings of familiar objects made from 2D shapes. Invite students to make the 'picture' by moving pieces into place. As they do so, ask: Which piece would fit in this part? How can you tell? (Link to Represent Shape, Key Understanding 1.)

Viewpoint

Have students say how something will look from a different view. For example, invite students to draw a familiar object, such as a cup, a milk carton or a toy. At first, focus the students on representing the parts that they can see. Then ask them to imagine and make another drawing of what they think is on the other side. Ask: Is there something on the other side that you know is there but you can't see from here? What side will the handle be on if you draw it from the other side? How do you know?

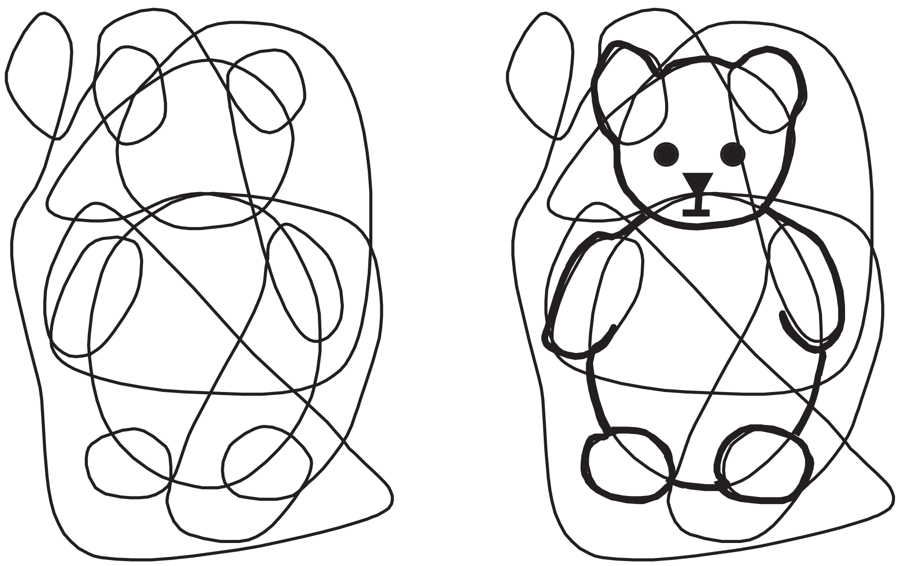
Paper Folding

Have students imagine the shapes produced by opening out a fold line. For example, invite them to fold a piece of paper in half, cut a shape out of the folded side then open out the paper. Before they open out their paper, ask: What do you notice about the holes you have made? Where will they be when it is opened out? Say: Fold the paper in half again and cut out another piece. Draw what you think it will look like when it is opened out. Repeat the activity by having students fold a *new* piece of paper into quarters.



Where Is It?

Have students disembed simple figures from complex patterns. For example, provide students with scribble patterns that have line drawings of objects from familiar stories embedded in the scribble. Invite them to identify the object or character and colour in the figure or the background. Ask: What was it that helped you to find the object?



Familiar Objects

Have students imagine how a familiar thing will look from a different view. For example, show them a picture or a photograph of a familiar object (e.g. car, spaceship, house) made from construction blocks. Invite them to suggest which blocks they will need to construct the object. Ask: What do you think the other side of the model looks like? Encourage each student to build the 3D model using blocks. Ask: How many wheels are on your car? How many are on the picture of the car? How did you know to use that many wheels? (Link to Represent Shape, Key Understanding 3.)

Fruit Bowl

Extend 'Familiar Objects' by using real objects. Have students sit in groups around a bowl of fruit, draw line drawings of what they see, and leave their drawings in a pile on the table. Invite them to exchange tables with another group and walk around the display to see what it looks like from different views. Ask them to choose a seat and deal out the drawings. Without moving from their chair, encourage them to say what viewpoint their drawing shows and how they know. Invite them to pass the drawing to the student sitting in that position to verify. Ask: Which seat would you be sitting in if you could see what is shown in this drawing? How do you know that drawing comes from the position on your right? (Link to Represent Shape, Key Understanding 3.)

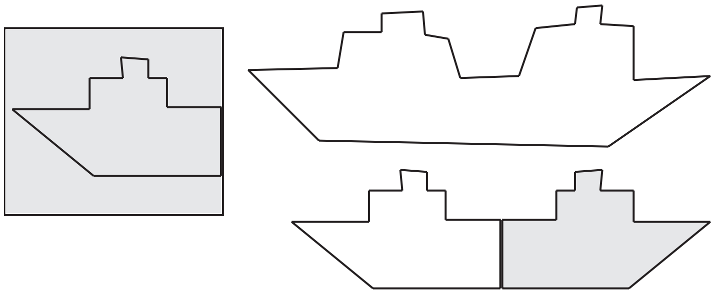
Beginning ✔✔✔

Actions

Have students say how they think shapes will change during a sequence of actions. For example, invite students to fold a circle in half and cut in from the fold to make a shape. Ask: What will your shape look like after you have cut it out? Have students open the shape out. Ask: What will it look like in the mirror? Draw what you think you will see. Invite them to reflect the shape in a mirror. Provide students with inflated balloons and invite them to draw on them. Ask: What do you think will happen to your drawing if you let the air out of the balloon? Have them deflate the balloons. Encourage students to predict how they think shapes will change through incidental activities throughout the day. For example, ask: What do you think this dough will look like after we have cut out nine biscuits with our round biscuit cutter?

Patterns

Have students visualise the effect of reflecting figures. For example, invite them to draw half of a square (cat, boat) on card so that the edge of the card forms the mid-line of the drawing. Have them cut the half figure out, predict and draw what the whole figure will look like. Encourage them to check by reflecting and tracing to make the whole figure. Repeat this to create a border or sequence of objects for a picture. Which figures are reflections of the original shapes? How can you tell?



SAMPLE LEARNING ACTIVITIES

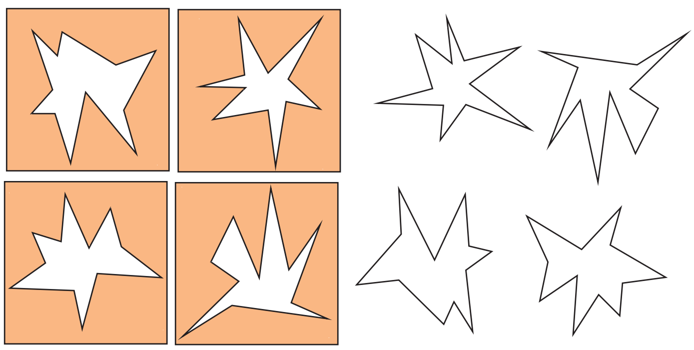
Middle ✔✔✔

Jigsaws

Have students construct double-sided jigsaws by gluing black and white pictures to either side of card and then cutting them into small rectangles and triangles. Invite them to complete someone else's jigsaw. Ask: What clues did you use to help you fit the pieces together?

Broken Windows

Show drawings of broken windows and provide a choice of different-shaped pieces to fit in the hole. Explain that filling the hole with the right piece of glass can repair the window. Encourage students to predict which piece will be required to fix each window and explain why. Ask: How would you move that piece so it would fit in the hole? Invite students to check they chose the correct piece by placing it over the hole.



Coloured Cubes

Provide pairs of students with a line drawing of a cube and a 3D cube with each face a different colour. Invite students to colour in the faces of the cube in the diagram to match those on the cube. Ask: Which colours can't be seen on your picture? Are the colours on your picture different from the colours on your partner's picture? Why? Encourage students to swap their pictures. Ask: What colour are the parts of the cube in the diagram that you know are there but can't see? How do you know? Encourage students to use the actual cube to test their prediction. (See Represent Shape, Key Understanding 3.)

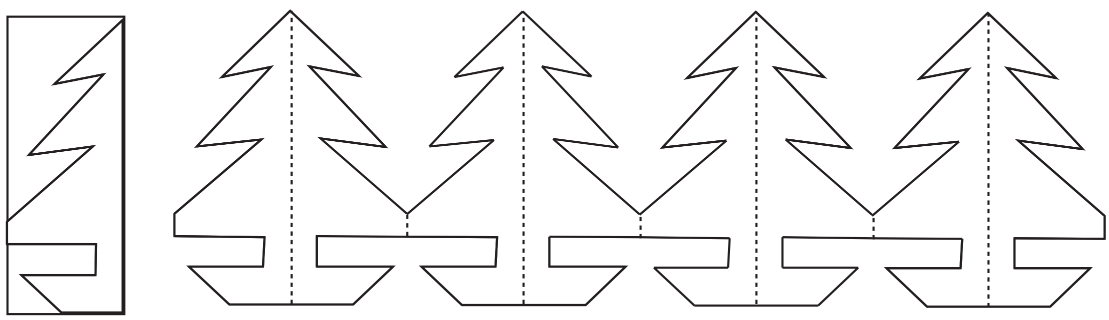
Cross Sections

Invite students to imagine the shape of cross sections of a variety of common objects such as carrots or potatoes, and choose a shape to create a print pattern. Encourage them to say how they will slice their object to make that shape. After printing, ask: Is the shape of the sliced object the same as the shape you had in your head? How is it different? How would you make that shape? (Link to Represent Shape, Key Understanding 3.)

Middle ✔✔✔

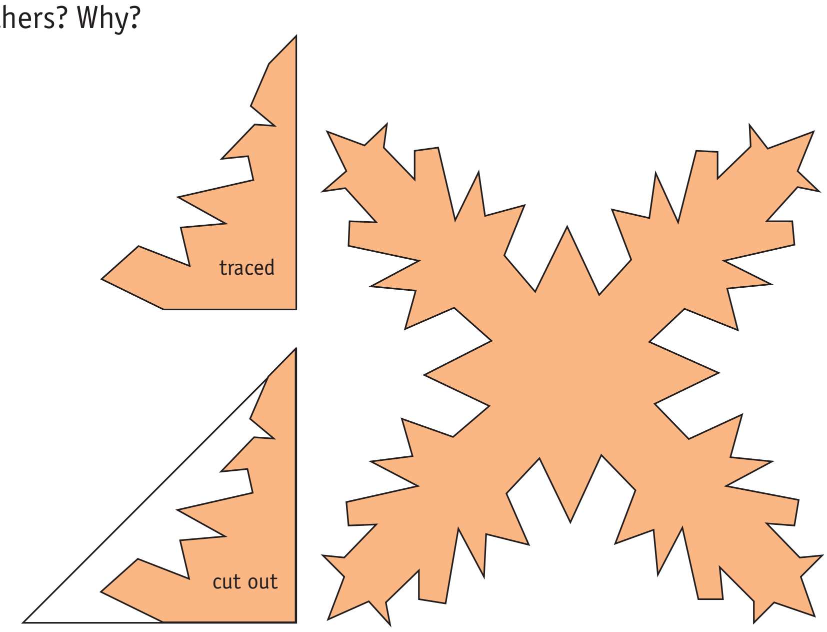
Paper Folding 1

Have students fold paper concertina-style, cut out a shape on one folded edge and open it out to see what frieze they have produced. Invite them to then decide and sketch what new frieze they want to produce, plan their cuts and make their frieze. Ask: Which part of your frieze do you want to connect? How will you cut your folded paper to make that happen? How will you cut the paper if you want to give your 'person' some eyes? Encourage them to assess how well they visualised where their cuts should be and what mistakes they made.



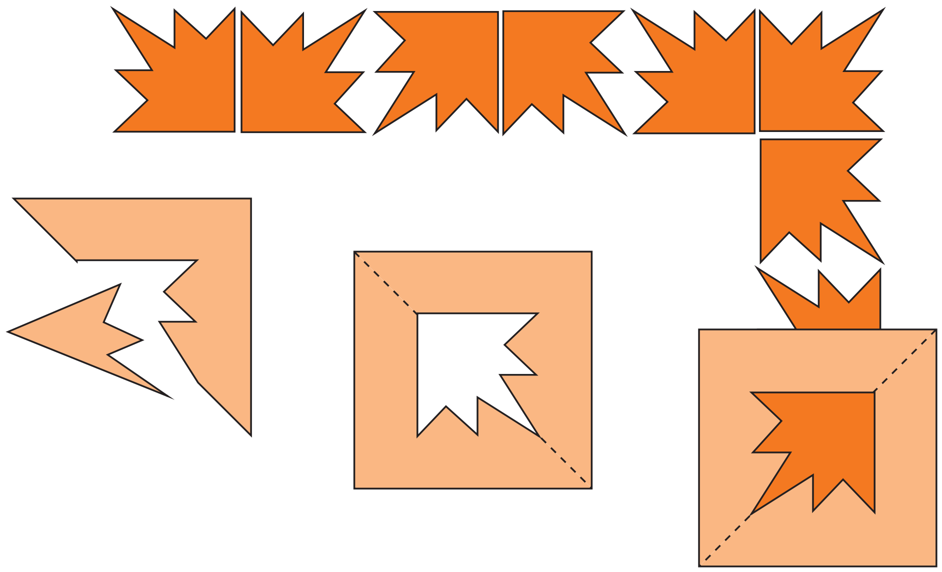
Snowflakes

Ask students to fold paper diagonally, first in quarters and then in eighths, and cut a design into the folded edge. Have them carefully draw a copy of the cut-out piece. Invite them to open out their snowflakes and pin them up. Encourage other students to inspect the traced outline and decide which cut-out shape goes with which snowflake. Have students justify their choice. Ask: What is it about the cuts in the folded paper that match that snowflake? What other clues did you use? Were there some that were easier to match than others? Why?



Paper Folding 2

Have students fold and cut paper to create different templates for stencilling and decide which designs will be more suitable for borders or corners than others. Invite students to test their pattern or arrangement (perhaps using grid paper) before completing the final product. Ask: What would your design look like if you put the stencil together in different ways (turning it sideways, turning it over)? How can you move the shape to make it fit into the corners better? Encourage students to predict and draw other designs.



Changing Shapes

Have students view and sketch what they think an object (e.g. cup, shoebox, saucepan, chair) would look like from above, below and from different sides. For example, students might describe how the rim of the cup appears to change from a circle to an ellipse as they change their viewpoint. Encourage students to compare drawings and say why they used particular stencil. Ask: What did you see in your mind's eye that helped you decide which shapes to use?

Viewpoint

Show students photographs of different locations around the school or classroom and invite them to put them in the order of the pathway taken by the photographer. Without going to the location, encourage students to imagine the exact place each photograph was taken and then identify the parts of the photograph that match the location. Ask: Where do you think the photographer was standing to take this photo? Why do you think it was there?

SAMPLE LEARNING ACTIVITIES

Later ✔✔✔

Tessellate 1

Invite students to select any triangle and visualise and predict whether it will tessellate. Encourage students to give reasons for their predictions and describe the transformations required. Ask: Do you think your triangle would work as a tile? Could you cover your page with that shape and not have any gaps or overlaps? What is it about your shape that makes you think it will or won't tessellate? Have them test by drawing the tessellation. Ask: Do you think all triangles will tessellate? Why? Repeat with other shapes. (Link to Key Understanding 2.)

Tessellate 2

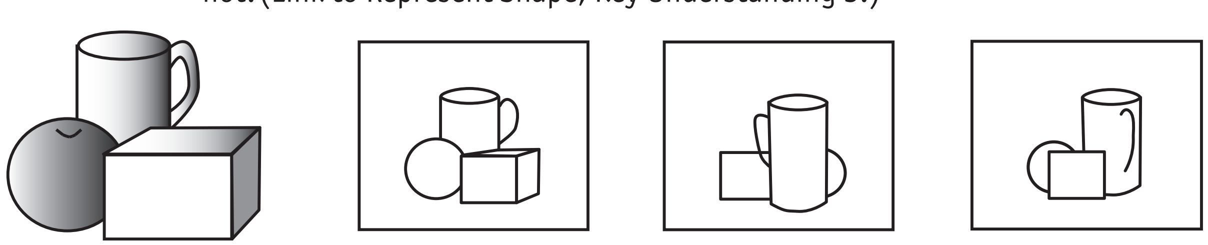
Project an image of a pattern based on tessellating triangles. Number the triangles from 1 to 9. Invite students to visualise the transformation required to move a triangle from one position to another. Ask: How would you move the triangle from position 1 to position 9?

Potato Cut

Give students a potato each and ask them to imagine the minimum cuts they would need to make to create various prisms and pyramids, such as a hexagonal prism. Have them test their predictions and compare their results with others. Ask: How many cuts did you make to create the hexagonal prism? Why is it that you have different numbers of cuts?

Viewpoint

Have groups of four students sit around a table and draw their view of several objects placed in the centre of the table. Without moving, have them draw what they imagine the arrangement would look like from each of the other three positions. Encourage students to compare their drawings of the same viewpoints. Next have students visualise and describe what is the same (different) about the view they would get by rotating the group of objects a half turn (180º). Then have them visualise and describe what is the same (different) about the view they would get by moving themselves 180º around the object. Ask: Would it be the same view, or would it be different? Why? Why not?(Link to Represent Shape, Key Understanding 3.)



Cross Sections 1

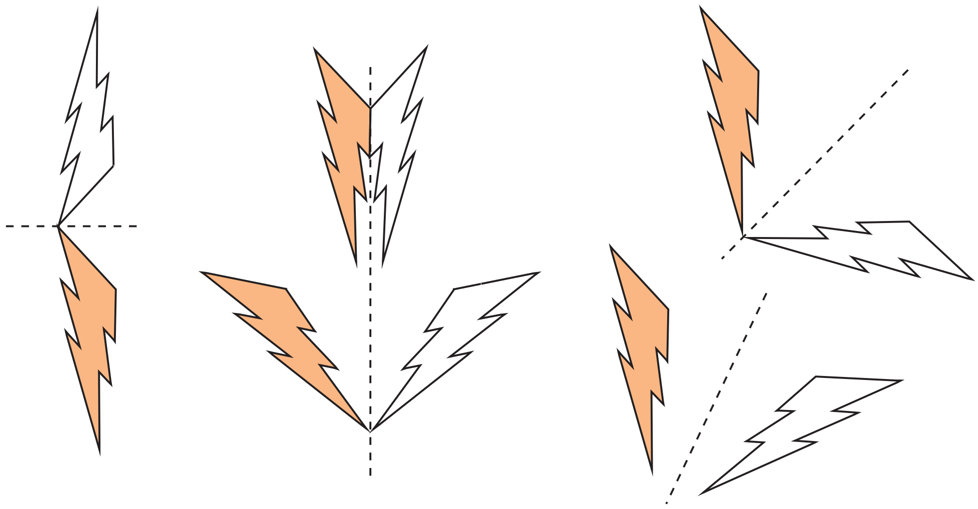
Display a clear plastic 1-litre cubic box containing about 4 centimetres of coloured water. Invite students to examine the cross section of the cube made by the top surface of the water. Ask: What figure can you see? Encourage them to predict the figures created by tilting the box into different positions. Ask: How would you make a square (rectangle, trapezoid, isosceles triangle, equilateral triangle, pentagon, hexagon, octagon)? Which figures cannot be made? Why? Would this change if the top was sealed and you used more water?

Cross Sections 2

Invite students to make cross sections of cones, cylinders, prisms, pyramids and spheres using modelling clay or play dough. Encourage them to first predict what figure or object they could produce and where they would need to cut to create that figure or object. Invite them to test their prediction by cutting. Ask: Did your cut produce the figure that you predicted? How should you have cut your object to produce that figure? Is it possible to get that figure from the object you chose? What object would produce that figure? How could you have made a more accurate cross section? (Link to Represent Shape, Key Understanding 3.)

Reflection

Ask students to visualise and sketch the reflection of a shape about a given line of reflection before carrying out the actual reflection itself. Begin with a shape with a horizontal or vertical reflection. Ask: What would your shape look like if it was flipped horizontally? What would it look like if it was flipped vertically? Later extend to shapes on a slope and then to a reflection line on a slope. Ask: How can you check that a reflected shape over a diagonal line is positioned correctly? Draw out that measuring the angle between one of the sides of the shape and the reflection line and then using that same angle to position the flipped shape will ensure the shape is positioned correctly. If there is a space between the reflection line and the shape, then students will also need to work out the distance from the reflection line.



Later ✔✔✔

Paper Folding 1

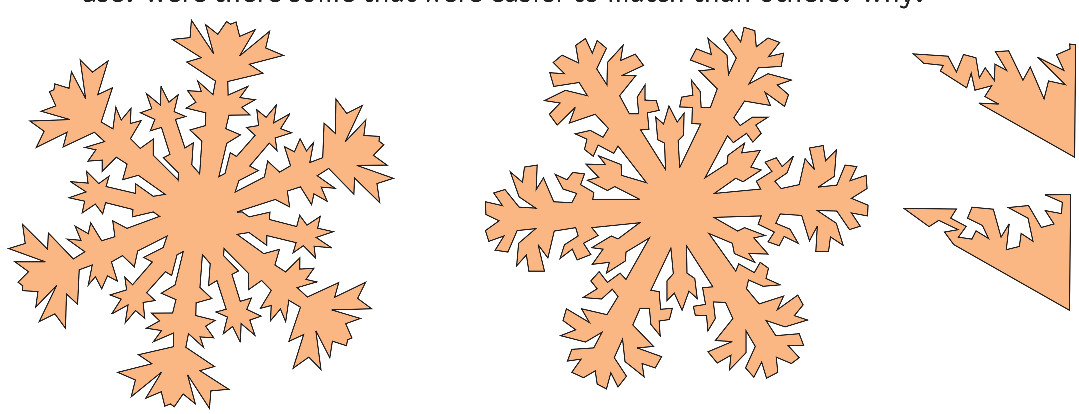
Invite students to think about how they would have to fold and cut paper to produce four or eight people holding hands in a circle. Ask: How would you fold the paper to make the people come together in a circle? How many folds for two people? How many for eight? If you folded again, how many people would you get? Encourage students to test their prediction by folding and cutting. Ask: Why are your people in a straight line and not in a circle? What if you were to fold it in a different way, what do you think you would get?

Paper Folding 2

Organise students into pairs and have them fold paper and draw a figure or design on the front to be cut out from the folded edge. Before cutting the paper, have students swap designs with their partners and sketch how they think the pattern will look once it is cut and opened out. Invite them to cut and unfold the pattern to see how close they were. Ask: Why didn't your design turn out as you predicted? What will you need to do to your drawing on the folded paper to produce the design you want?

Snowflakes

Extend Middle Sample Learning Activity (p. 108) 'Snowflakes' by having students use more complex designs. Give them a set of drawings of 'snowflakes' and matching cut outs. Invite them to imagine the cut out unfolded and match each one to the right snowflake. Ask: What is it about the cuts in the folded paper that match that snowflake? What other clues did you use? Were there some that were easier to match than others? Why?

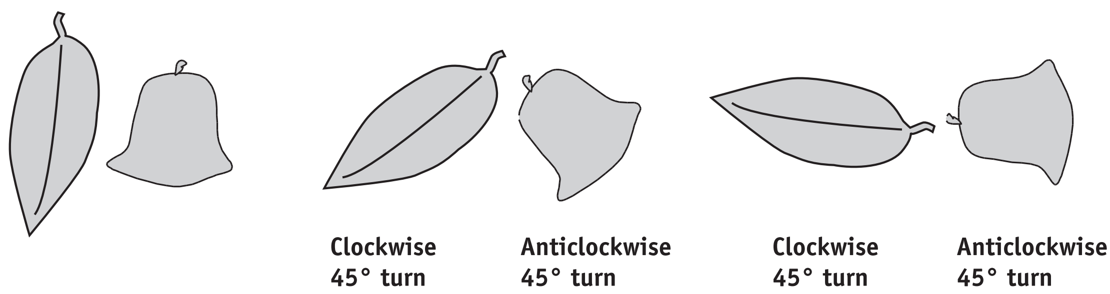


Actions

Have students imagine how a shape will look after part of it has been moved. For example, give students a 2D figure and ask them to visualise and draw the results of slicing off a section of the figure and rotating, reflecting or translating it before reattaching it to a different part of the original figure. Ask: What shape do you end up with if you cut off the right side of the parallelogram and translate it to the left end and reattach it? How would you cut and rearrange a triangle in order to make a rectangle?

Template

Give students two templates (e.g. templates of a leaf and a nut) and invite them to visualise creating a pattern by moving one of the templates through a 45° turn clockwise and then the other through a 45° turn anticlockwise. Ask them to sketch their visualisation. Ask: How do you think the leaf will change through your pattern? How will the nut change through the pattern? Encourage them to visualise and sketch other rotations and transformations using the same template.



KEY UNDERSTANDING 2

We can move things around in space by reflecting, translating and rotating. These do not change size or shape.

We often think of the transformations of translation, reflection and rotation in an informal way as motions that move points from one position to another. When a transformation is applied, some properties of the whole figure or object will be changed; other properties will remain unchanged. These three transformations are grouped together because they each leave the shape and size of the object unchanged so that the original object and the transformed object are congruent.

- *Translation:* To translate a point means to move it in a straight line. When a figure or object is translated, the whole thing 'slides' a specified distance in a specified direction. Objects passing by on a straight conveyor belt give a good model of a translation.
- *Rotation:* To rotate a point means to move it as though around the circumference of a circle. When a figure is rotated, the whole thing turns around a specified point by a specified amount, and when an object is rotated, the whole thing turns around a specified line by a specified amount. A windmill or objects on a pottery wheel give good models of a rotation.
- *Reflection:* To reflect a point means to move it as if it were seen in a mirror. When a figure is reflected, the whole thing flips over a line so that every point of the image is as far from the line as was the matching point of the original figure. When an object is reflected, however, the idea of 'flipping' does not really work. A shoe, for example, looks different in a mirror compared to what would happen to the shoe if you 'flipped' it over a line. A left and right shoe give a good model of a reflection (but you could not get a right shoe from a left shoe by flipping).

Young students should note and describe their body movements in dance, drama and play, and the movements of objects around them, leading to an informal understanding of different transformations. As they progress, they should learn to recognise and describe translations, rotations and reflections of shapes embedded in designs and arrangements. This might involve reproducing a design or picture by identifying and matching component shapes and turning or reorienting them to fit. Students should be encouraged to notice the balances, repetitions and movements in figures, objects and arrangements and to talk about what they see. We should use the correct language in context, helping them to refine their descriptions of what they see, though not expecting a high level of precision initially.

The study of tessellations helps students learn about the properties of shapes and transformations. The question is this: can this shape be used repeatedly to cover a plane without gaps or overlaps; that is, would the shape work as a tile? Initially, students use multiple copies of figures so they can begin by trial and error. They should over time become more systematic in producing the tiling pattern, focusing on how the shapes are moved relative to each other in order to generate the tiling. Later, they should draw around a single copy of the shape. This is more demanding, requiring students to visualise and test the effect of moving the figure and translate, rotate and reflect it systematically to create the tiling pattern. Simply finding out by trial and error that a figure will or will not tessellate is insufficient. Students should analyse what it is about the sides and angles of a particular shape that convinces them that it cannot ever tessellate and make conjectures about which shapes must tessellate.

Progressing Through Key Understanding 2

Initially, students will fit figures and objects together based on shape or orientation. As students continue to progress they use multiple copies of a figure or object to create patterns and arrangements and can describe simple movement rules for generating patterns; for example, *We started with a diamond and each time we flipped it over and moved along one.*

Next, students use multiple copies of figures to create patterns based on systematic movements of shapes (e.g. make a border from two different shapes) and informally describe the movement used. As students progress further they can use systematic movements of a shape to create a pattern, identify the transformations used to produce a sequence and describe how one shape can be superimposed on another (e.g. rotate it at right angles around the centre and slide or translate it to the left).

Later, students can describe the effect of a translation, rotation or reflection on the position and orientation of a figure and use transformations to modify tessellating shapes to produce other tessellating shapes and informally explain why this works.

SAMPLE LEARNING ACTIVITIES

Beginning ✔✔✔

Jigsaw Puzzles

Draw attention to the movements needed to solve problems. For example, while students are doing jigsaw puzzles, ask: Does the piece need to be turned over or turned around in order to make it fit?

Pattern Blocks

Invite students to use pattern blocks to make a given design (e.g. a train, a house, a duck). Ask: How do you need to move that piece to get it into the same position as the piece in the design?

Shadow Puppets

When students are performing shadow puppet plays projected from the overhead projector, invite them to describe the movements of the characters. For example, ask: What did you do so that the wolf didn't walk backwards? How did you move the puppets so that the characters stayed in focus?

Tiles

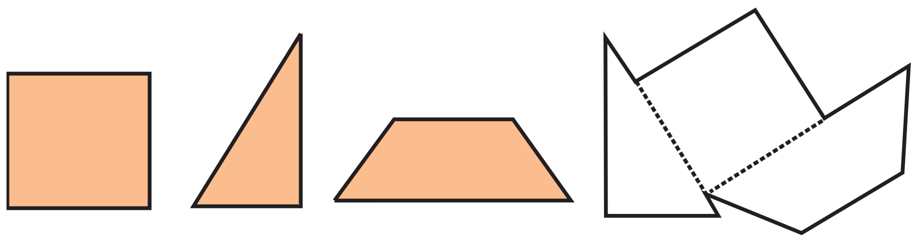
Invite students to use coloured paper cut into small squares and triangles to cover areas in their artwork such as house roofs, roads, clothing and trees. Talk about the need to slide the pieces together so the sides and ends touch and check for gaps and overlaps. Ask: Do the squares (triangles) fit together without any gaps or overlaps? How could you turn the triangles to make no gaps?

Prints

Involve pairs of students in using prints (e.g. stamps, potato prints) and stencils (e.g. templates, paper cut outs) to make patterns. Invite them to make multiple copies of the one figure, cut them out and use the figures to make a pattern. Encourage them to describe how they transformed their figures to make their pattern. Ask: How did you move your figures to make the pattern look right? How did your partner make their pattern?

Rotation and Reflection

Have students rotate and reflect figures (e.g. tangrams, pattern blocks, Polydrons™) to fit them in a given outline. Ask: What will you need to do to fit this blue block into that space? Will this piece fit in here if I slide it across? If you turned the figure over, would it still fit? What would be different?



Tangrams

Invite students to combine tangram pieces to create a given figure such as a rabbit. Display the pictures and ask: Where are the large triangles in each of these pictures? Why do these triangles look different? How could they be made to look the same? Would it change the picture? (See Sample Lesson 1, page 127.)

Wrapping Paper

Invite students to identify patterns on different pieces of wrapping paper. Ask: How is this shape moved to create the pattern?

Sorting

Extend 'Wrapping Paper' by having students cut out sections of patterns and add them to a class poster. Encourage students to say how each group can be sorted. Ask: Are they all sideways flips or are some flipped up (down)? Sort them according to the type of transformation.

Friezes

Invite students to use rotations or translations to create a decorative border for a notice board using vegetable prints on a paper strip. Ask: What information would you need to include if you wanted another class to use your vegetables to make a frieze exactly the same? What information would you need to tell them if you used translations? What information would you need to tell them if you used rotations? Did you rotate it a full turn or a half a turn?

Tessellate

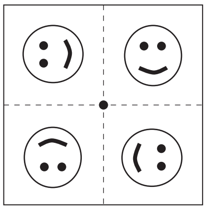
Invite students to each choose a regular-shaped tile, block or object (e.g. pattern blocks, cubes) to cover a given area of paper. When the paper is covered, ask: Did the block you chose fit with no gaps? Which shapes left gaps? Encourage students to look at each other's work and make a class list of the shapes they have found that will and won't tessellate. Ask: What did you learn about shapes that tile? What shapes won't tile? Why do you think they won't tile?

Stacks

Invite students to think of ways to stack the class stock of tissue boxes so that there are no gaps. Encourage them to take turns to add a box to the stack. Ask each student to predict where their box will fit. Ask: Are there different places it could go? How could the boxes be packed to create a strong stack? Which arrangement makes the tallest stack? Which one makes the smallest stack?

Rotating Designs

Have students each fold a piece of paper in quarters, open it out and pin it to a board through the centre. Invite them to print in each square by stamping in the same position while rotating the paper. Ask: What else can you see around the room that matches this type of pattern making? How is the fan the same as the pattern? How is it different? (See Key Understanding 4.)



Beginning ✔✔✔

Potato Prints

Ask students to cut a simple stamp from a potato and use it to make a pattern by rotating and translating the stamp. Have them look at the prints one at a time and say how the print maker moved the potato stamp to create the pattern. Encourage students to use their own language to describe the direction and the amount of turn. Ask: How did Jan move his stamp from here to here to make this pattern? Did he rotate the stamp? Did he move it in a straight line across the page?

Rubbings

Take students on a tessellation hunt around the school to make rubbings of tessellated areas (e.g. brick walls, paved and tiled surfaces, security screens, grates, floorboards). Display the shapes and invite students to decide which are more likely to tessellate and how they could tell by 'just looking'. Ask: Which shapes seem to be the most common in our rubbings? Encourage students to decide on a general statement about the types of shapes that tessellate. For example, *We found lots of examples of squares and rectangles tessellating, but not many examples of triangles tessellating*.

Leaves and Coins

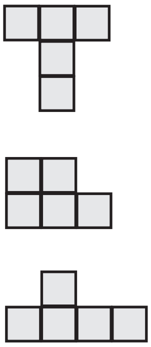
Invite students to create rubbings of flat objects (e.g. leaves, coins) and compare the object to the picture. Ask: How is the picture the same or different from the original? Encourage students to create a pattern from their rubbings using a series of translations. Ask: How did you need to move your picture to create the pattern? Could you have used a different movement to create the same pattern?

SAMPLE LEARNING ACTIVITIES

Middle ✔✔✔

Pentominoes

Ask students to explore how many different pentominoes (figures with five squares joined along the edges) they can make using grid paper or a drawing program on a computer. Invite them to compare with a partner to see if they have different ones. Encourage them to check by turning and flipping the shapes to make sure that they are not a transformation of another shape. Ask: Is this pentomino the same as this one if you turn it over? How can you be sure it is not the same arrangement of squares?



Shadow Puppets

Extend Beginning Sample Learning Activity (p. 116) ‘Shadow Puppets’ by listing suggestions students give for directions and movements of a puppet character. For example:

1 Place the hen in the basket under the table.

2 Slide her past the leg of the table then …

3 Flip her sideways to show she has gone the other way.

4 Slide the hen up then rotate her to show her looking for an egg.

Border Pattern

Have students create and explain movements required to produce a border pattern around a picture frame. For example, invite students to choose from a selection of small objects (e.g. pattern blocks, cubes, tiles) to use as templates. Have them use their objects to make a border pattern around the edge of a frame. Encourage students to show and describe how they made their pattern. Ask: How did you need to move your pattern block to get this part of your border pattern?

Guessing Game

Extend ‘Border Pattern’ by having a partner examine the completed pattern and say which transformation or transformations were used to create the pattern. Ask: How did your partner move their figure to create this pattern? Is it possible to use a different movement to get the same pattern?

Rotation

Have students pin a template of a shape (e.g. a book character, a hammer, a frog) by its top and trace around it. Invite them to rotate the shape a quarter turn and trace it again and repeat until the paper is filled. Ask: Does the shape change size? Does it change shape? What was it that changed? How has the location of the shape been decided? Where is the point of rotation? Encourage students to identify the point of rotation in the pictures of others.

Middle ✔✔✔

Pattern Units

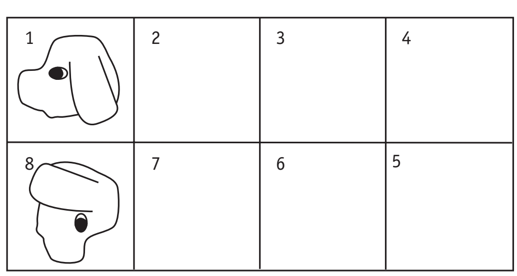
Ask students to collect a class set of border patterns from greeting cards, computer graphic programs, blackline masters, certificates or book covers. Invite them to examine each pattern for the 'pattern unit' and describe how that unit is transformed around the entire frame. Ask: Which part of the pattern is used to make the whole pattern? How is this part moved?

Strip Pattern

Invite students to use pattern blocks to design and record a strip pattern using one shape in a series of transformations. Have them take turns to describe their pattern for a partner to create the same pattern. Encourage students to compare the two patterns and discuss how different words could have been used to explain the design. List the words that are helpful in describing the movement and location of shapes. (Link to Represent Location, Key Understanding 1.)

Transformation Puzzle

Give students a 2 x 4 grid with each cell numbered in sequence and a template of an irregular figure (e.g. animal, toy). Explain that they are going to create a transformation puzzle. Ask them to trace around the template in cell number one, then move the template from cell to cell in turn, either translating, rotating or reflecting it in some way until cell number eight, where it is traced around in its final position. Have them record each transformation on a separate piece of paper. Invite students to swap grids and try to work out and list the sequence of movements used to get the template from the first to the final position. Compare their records of the transformations with the original and see how they are the same and how they are different. Ask: What different ways could you move the template to its new position?



2 Quarter turn to right.
3 Reflect right.
4 Rotate quarter turn left.
5 Translate down.
6 Reflect down.
7 Rotate quarter turn left.
8 Reflect left.

Triangle Puzzles

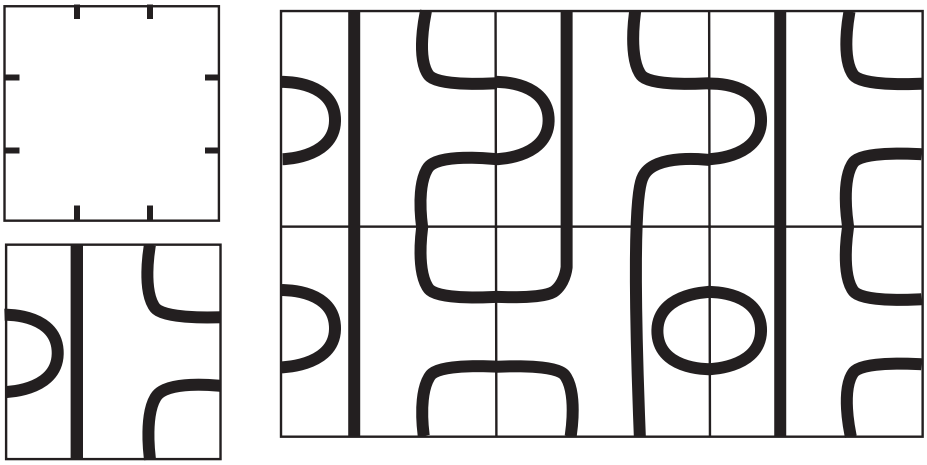
Have students make six coloured copies of an equilateral triangle and arrange them to create a pattern. Invite them to glue them onto card for their partner to copy. Have their partner trace around the outline of one of the triangles in the pattern and try to recreate the pattern by moving and tracing around this one triangle. Ask: How did you move the triangle to draw the second (third, fourth, fifth, sixth) triangle in the pattern?

Paving Tiles

Invite students to design a new paving brick by cutting a four-sided shape that is not a square or rectangle. Have them use this shape to create a tile pattern to cover a given area and explain the transformations they used to create the pattern. Ask: How do you need to move your four-sided figure to make it tessellate? Is the way you have moved your figure the same as anyone else's pattern?

Tile Pattern

Have students construct a pattern on a tile measuring 9 centimetres by 9 centimetres by marking dots 3 centimetres apart on each edge and then joining the dots in some way. Then have them make 5 more tiles exactly the same. Invite students to arrange them in a 2 x 3 array to make a pattern. Ask: Are the squares a translation, rotation or reflection of the original? (See Key Understanding 4.)



Tessellation

Have students investigate tessellation in small commercial biscuits, such as Tiny Teddies™ or Shapes™. Invite students to say how they need to transform the biscuits to fit as many biscuits inside a 9 centimetre by 9 centimetre square as possible. Ask: Does it help fit more in if you tessellate the biscuits? Can you take one biscuit and describe how you would have to transform it to make them tessellate? How would tessellating the biscuits help the manufacturer?

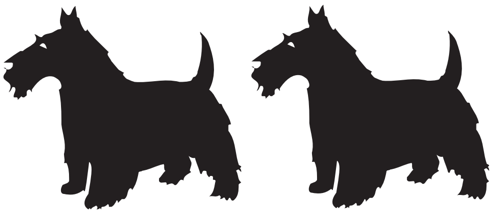
Amusement Rides

Show students videos of amusement rides (e.g. merry-go-round, rollercoaster) and ask them to identify the sequence of transformations of the cars (seats, horses) during the rides. Invite students to describe the movements and repetitions of movements that make up the whole ride. Ask: How do the horses on the merry-go-round move? Do the cars on the merry-go-round move in the same way? (See Reason Geometrically, Key Understanding 1.)

Middle ✔✔✔

Scotty Dogs

Invite students to create border patterns, first by translating a square-shaped template cut out of a piece of card and then an irregular-shaped template, such as the dogs below. Ask: What is different about translating the two shapes? Why is it easier to make a pattern with the square-shaped template? What would you need to say about translating the second shape if you wanted someone to produce the same pattern? (The first shape has a border that you could butt up against; the second shape doesn't.)



Rotational Border

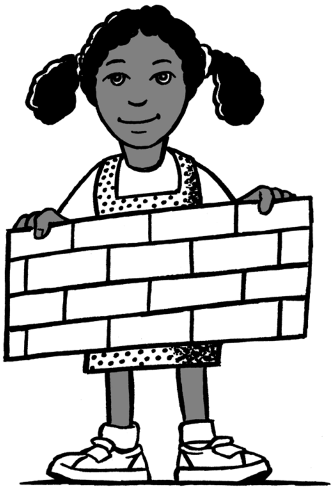
Extend 'Scotty Dogs' by using the two templates to produce a rotational border pattern. Ask: What are the effects of quarter turns or half turns on each of the templates? What are some other turns you could use? What are the different ways you can describe the amount of these turns?

Rotate and Reflect on Computer

Have students use rotate and reflect tools on computer drawing programs such as MS Word™ or Fine Artist™ to create and colour a design based on a chosen sticker or picture. As a class, ask students to identify the transformations used in each other's designs.

Patterns in the World

Ask students to investigate patterns and arrangements in the world around them and identify the repeating component part (e.g. reinforcing beams in the roof of the undercover area, honeycomb cells in beehives, bricks in the wall, pipes stacked on the back of a truck). Encourage students to draw their favourite example and present it to the class, explaining how the smaller shapes are used to construct the larger one. Ask: Is the repeating part translated, rotated or reflected?



SAMPLE LEARNING ACTIVITIES

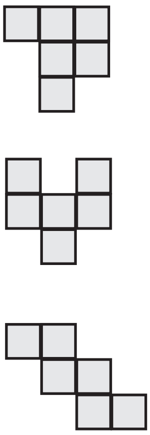
Later ✔✔✔

Transformation Chart

Have students build up a class chart that categorises human and machinery movements as transformations (e.g. a crane movement taking a container from a ship to the wharf, using a can opener, using a hammer). Ask: Which movements are used? Why? What part of the machine moves? What part doesn't move? Are there different types of movements within the one machine? (e.g. the can opener uses rotation and translation)

Hexomino Pairs

This is a game for two players. Have students use grid paper to draw and cut out a collection of pentominoes (figures with five squares joined along the edges) or hexominoes (figures with six squares joined along the edges). Have one student in each pair describe the transformations needed to superimpose one of their pentominoes (or hexominoes) onto one of their partner's; for example, *If I rotate my shape 90°, translate it and then reflect it, it will sit on top of yours*. Encourage them to try it out. If it works, that student 'wins' the congruent pair. Ask: How can you make it more difficult for your partner to match your figures?



Border Pattern

Extend Middle Sample Learning Activity (p. 119) 'Border Pattern' by having students use one object to create a border pattern and then explain the movements required to produce their pattern. Ask: How did you need to move the object to create this pattern? Could you have used a series of different movements and created the same pattern? How can different movements create the same pattern?

Shape Shifter

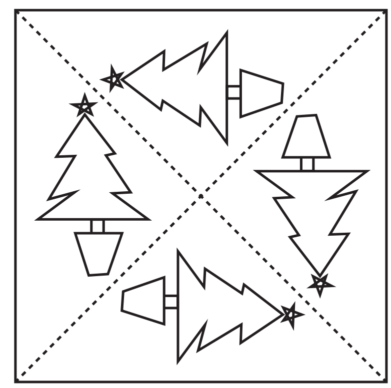
Ask students to trace around a figure (e.g. a parallelogram), transform it several times, recording in words which transformations they have used, and then trace the finishing position. Have them swap with a partner and work out what transformations could have been used to move their partner's figure from its original position to its final position. List the transformations that resulted in the same end position. Ask: Are there any other possible transformations that would give the same ending position? Encourage pairs of students to share what they have found out with the rest of the class; for example, *Tom reflected the rhombus from top to bottom, but Su-Lin rotated the rhombus 90° clockwise, and then rotated it 90° again to superimpose the figure*.

Later ✓✓✓

Centre of Rotation

Have students identify objects that turn (e.g. windmill, doorknob, wing nut, door), and locate the centre of rotation for each. As a class, identify things that have the axis (line) of rotation in the centre of the object and those that have the axis (line) on the edge. Ask: When you compare the axis of rotation of the door with the wing nut, how are they different?

Using the Mira



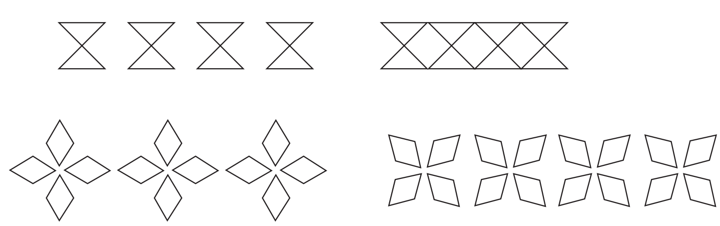
Have students create simple card designs using a Mira on the diagonal. Ask them to rule faint lines across the diagonals and draw a picture such as a Christmas tree into the section on the left-hand side. Then have them place the Mira onto the diagonal line and trace the reflection. Repeat until four images are drawn. Encourage them to compare each of the images. Ask: How are they the same? How are they different? How are the images different from designs where the Mira is placed on the horizontal or vertical?

Categorising Transformations

Have students cut out examples of transformations from newspapers and magazines (e.g. company logos, designs, patterns). Invite them to write a description of the transformations involved and then sort them into categories. Ask: Could the designs in the 'rotation then reflection' category be placed in the same category as those in the 'reflection then rotation' category? Why? Why not?

Re-creating Border Patterns

Invite students to select a pattern from a collection of border patterns, trace the pattern unit or motif and record in words how that unit is transformed.



Ask them to swap with a partner and use the tracing and the description to re-create and draw the border. Encourage partners to compare re-creations with the original borders and explain any differences. Ask: Was there enough information? What other information would be required to re-create the original design? Was it necessary to describe the distance between figures? Did you need to be more specific about the distance between the figures or the amount of turn or both?

Tessellation

Have students create and describe a tessellating pattern. For example, say: Imagine you are a bricklayer and have to describe brick paving patterns to a client. Draw two different patterns you could show them using a brick. Have students draw a rectangle 4 centimetres by 2 centimetres to use as a template or to make multiple copies to make the pattern. Ask: How many different patterns are possible using this one shape? What different transformations are possible? Which different combinations of transformations are possible?

Three Tessellating Tiles

Give students three square tiles. Ask them to cut a section out of one square and translate the cut piece to the other side. Then have them cut out a section from a second square and reflect it to the other side, and with a third square, cut out a section and rotate it to an adjacent side. (See Background Notes, page 152.) Invite students to create three different tessellating patterns by moving each new figure. Ask: Which movements do you need to use for each of the tessellating patterns? Can you use translation or rotation to make the figure you made by reflection tessellate? Why not? What do you notice about the transformations needed to create each figure and the movements needed to tessellate it?

Scalene Triangle

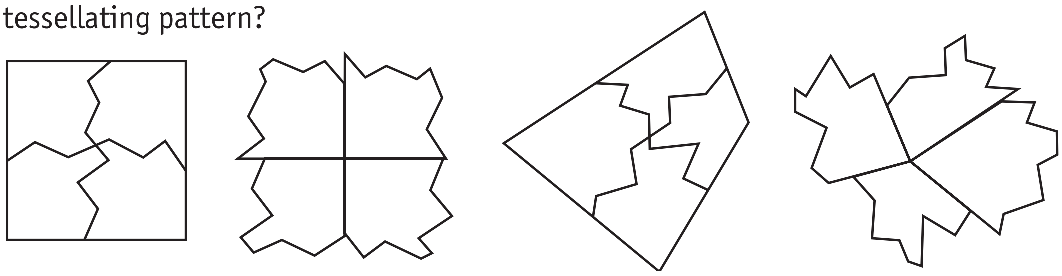
Have students create a template of a single scalene triangle and investigate how it can be used to tessellate. Encourage them to identify and describe the transformations used to tessellate the figure so that someone else can follow their directions. Ask: Why do scalene triangles tessellate?

Quadrilaterals

Ask: Do all quadrilaterals tessellate? Encourage students to investigate and decide whether they do or not and give reasons for their response. If students find a particular quadrilateral is difficult to tessellate, ask: Which transformations have you tried? Have you tried translation (reflection, rotation)?

Paper Square

Extend 'Quadrilaterals' by having students tear a paper square into four parts, ensuring that each part has a corner, then join all of the corners. Ask them to work out the total measurement of the angles in the centre of the shape. Try this with a range of other quadrilaterals. Ask: Do the corners of these quadrilateral shapes fit around a central point in a similar way? Does this help us know whether the shapes we began with can be used to make a tessellating pattern?



Later ✔✔✔

Tangrams 1

After creating a barrier between them and their partner, ask students to create a tangram design and explain to their partner how to create the design. Have their partner move the tangram pieces according to the instructions. Both students should be able to see what is being created, but only the person giving the instructions should see the original design. Ask: Why is the second design so different from the first? What instructions and words could we use to make sure they are exactly the same?

Tangrams 2

Have students describe the movements required to modify one tangram design to make another. For example, invite students to create a tangram design (e.g. a rabbit) and glue this onto one half of a piece of card and then create a modified version and glue this version onto the other side of the card. Have students show their partner the original version and ask their partners to create it with another tangram set. Then have students (while looking at their modified version) give their partner instructions on how to move each piece to make the modified version. Ask: Is your design the same as your partner's? What instructions and words could we use to make sure they are exactly the same? (See Sample Lesson 1, opposite.)

Irregular Figures

Have students use templates of irregular figures (e.g. Christmas tree, a hammer, a vase of flowers) to produce a translating pattern on card. Say: Describe your pattern so that someone else could make it. Ask: Why is their pattern not the same as yours? Did you include information about distance between figures? Did you include information about the direction of the pattern? How do you describe a pattern that is translated on a diagonal, not simply vertical or horizontal translation? Draw out that they would need to describe the angle of the translation as well as the distance.

SAMPLE LESSON 1

Sample Learning Activity: Later—'Tangrams 2', page 126

Key Understanding: We can move things around by reflecting, translating and rotating. These do not change size or shape**.**

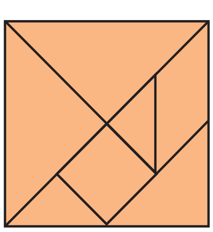
Teaching Purpose

I had noticed that the students in my Year 5 class were quite competent at describing which transformations they had used to create symmetrical patterns but did not use specific language to describe the distance, orientation or position of the shapes. Therefore, I decided to use a barrier game to encourage them to think about using this language.

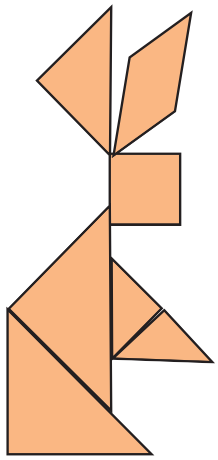
Connecting and Challenge

The students had been using tangrams to make animals in various poses, so I asked them to choose one animal and create two different poses of it using all of their tangram pieces. They then folded a sheet of card in half and glued the two poses on, one on either side of the fold line.

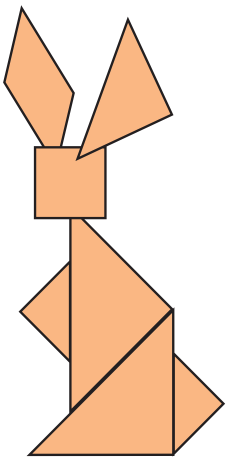
Students then took turns to play a barrier game. One person folded back their second animal pose so that it could not be seen and both students copied the first pose with a tangram set. When this was done, they created a barrier between them, and the first student gave instructions for their partner to change the pieces in the first pose so that they created the second pose.



Tangram set



First pose



Second pose

My Year 5 students had experience using the terms 'slide' (translate), 'flip' (reflect), and 'turn' (rotate) in various other activities and I encouraged them to use these terms when they were talking about what they were doing with the tangram pieces to make and change their shapes. For the two rabbits in the diagram, for example, this is what I heard several students say they did to change the first pose into the second pose.

'I flipped the big triangles and the rabbit is looking the other way.'

'I had to slide the end of his leg and turn it round to make a tail.'

'The ears stayed nearly the same, just turned them a bit.'

'Slide the head over to the left a bit.'

'And the leg just has to turn a bit, too. It has to touch the body properly.'

When the students compared the result, they found that most of the shapes appeared different.

'Why are the shapes so different?' I asked.

I wanted them to realise that they had not been specific enough in their instructions. The students who had followed the instructions were quick to point out to their partners why the shapes were in different positions.

'You only said to slide it across, how was I supposed to know that you meant for it to go there!'

'I turned this ear, but you didn't say how much to turn it.'

I decided to harness these conversations, so asked, 'What detail did you need to know to be able to get the shapes in the right position? Work with your partner to create a list of the sorts of things that would have helped.'

As the pairs worked to create these lists, I noticed that some students did not consider angle, for example, in their discussion so when they thought they were finished I asked the students to get into bigger groups to share their lists.

Drawing Out the Mathematics

Then it was time to do a whole class discussion, so I asked a student from each group to share their final list. As they talked, I wrote key words up onto the board and, after all of the groups had shared their ideas, I drew attention to these words.

Go up by $6\frac{1}{2}$ cm.	Turn it half of 90°.
Slide it up to the left.	Do a right angle turn.
Turn it to the right.	Near the top.
Slide in a straight line.	Turn to left.
Move it 8 cm from the fold.	Middle of top part of the triangle.
To the left of the square.	Flip it on a diagonal.
Turn it 90°.	Slide it down to the right.

'Could we group these words in any way?' I asked. 'For example, I can see "turn it left" and "turn it 90 degrees" here—both of these are about changing the angle of the shape.' We worked systematically through so that students could see the groups of words they might use to describe the distance, position, orientation and angle of the transformations.

It was then time to resume the game, this time with the students using all of the new language. I noticed that when a student gave an instruction such as 'Slide the triangle over and turn it', their partner's response was, 'How much?' This prompted the students to refer to the words on the board and attempt to be more precise.

KEY UNDERSTANDING 3

Some transformations, such as enlargement, change size but leave shape unchanged. Others change shape and size.

There are many types of spatial transformation. As described in Key Understanding 2, some keep both size and shape unchanged. Others keep the shape of figures and objects the same, but change their size. The latter transformations produce a scaled version of the original figure or object. All the angle sizes remain the same, but the length of the sides are scaled either up or down. While there are a number of such transformations, we tend to think about them all under the one heading of e*nlargement* (which also includes reduction). They are the basis of such technology as cameras and projectors, the production of scale drawings and models, and our understanding of the behaviour of certain plants and animals.

Initially, students should investigate enlargements and reductions in a general way; saying that under the magnifying glass the object looks the same shape but is larger, for example, and that their bookmark school photograph is the same but smaller than the regular one. Older students should analyse what changes and what stays the same. They might, for example, compare lengths and angles of matching parts of a drawing and an enlargement of the drawing. During the middle and later primary years, students should also learn to produce enlargements of simple shapes using grids and other strategies.

Other transformations may distort both shape and size. There are many such transformations, each serving its own purposes. There are the transformations that represent 3D space on a 2D surface, for example, as described in Represent Shape, Key Understanding 3, and Represent Location, Key Understanding 2. These include a perspective drawing of an object and the wide variety of different map projections of the earth (e.g. a Mercator projection). Drawing on a rubber sheet and then stretching it produces its own transformation, generally called ‘topological’. Such transformations preserve betweenness and order, but not distance or direction. The networks described in Represent Location, Key Understanding 2, are of this kind. Students can use projected images, shadow shapes, computer graphics and grids to investigate a variety of transformations, thinking about which features are maintained and which are distorted. They should at least understand that there are many ways we transform figures and objects, for a variety of purposes.

Progressing Through Key Understanding 3

Initially, students are able to use a grid to enlarge or reduce a figure (to a whole number and unit fraction scale) and to make distortions (e.g. double widths but not heights), and can enlarge models made with cubes to a small whole number scale.

SAMPLE LEARNING ACTIVITIES

Beginning ✔

Tangrams

Have students copy a tangram picture using a different-sized set of tangram pieces. Ask: How is your picture different from the picture you copied? Are the pieces the same shape as those in the picture? What is different about them?

Sets

Give students sets of objects graduated in size (e.g. mixing bowls, saucepans, dinner set, groceries, stationery, Panda bears) for imitative free play. Focus the discussion on 'exactly the same shape but different sizes'. Ask: How are these the same? How are they different? Help students use comparative adjectives (e.g. tall, taller, tallest, wide, wider, widest) to name the sizes of the bowls. (Link to ***First Steps in Mathematics: Measurement***, Understand Units, Key Understanding 2, Indirect Measure, Key Understanding 2.)

Same and Different

Give students objects that are the same but vary in size (e.g. a baby's jumper, a student's jumper and an adult's jumper; a Year 1 chair, a Year 4 chair and a Year 7 chair). Focus the students' attention on what is the same about these objects (the shape) and what is different (the size). (Link to ***First Steps in Mathematics: Measurement***, Indirect Measure, Key Understanding 2.)

The Three Bears

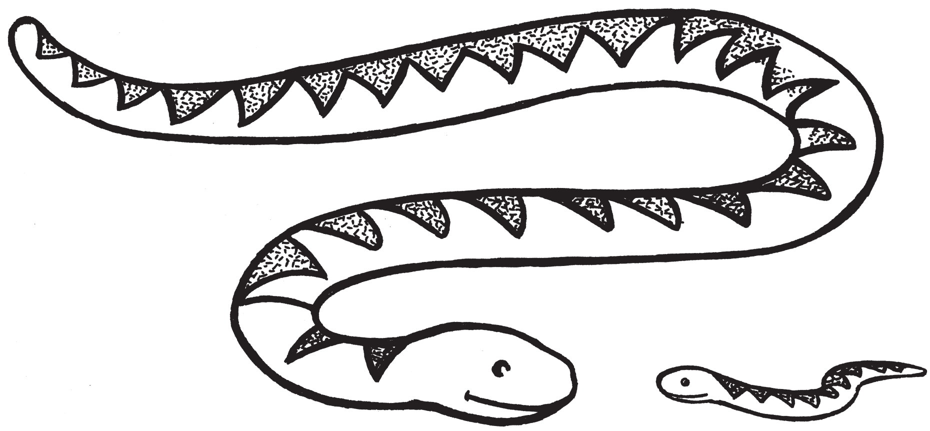
When students are illustrating stories such as The Three Bears, provide them with a teddy bear to copy. Before drawing, focus the students on the shapes they can see in the ears, face, body and legs. Invite them to draw it as Father Bear and see if it looks right. Then ask them to redraw it smaller to be Mother Bear and smaller again to be Baby Bear. Ask: How have you changed the body to make it look like it belongs to Mother Bear (Baby Bear)? What stayed the same? What changed? Can you see which bear it is just by looking at the faces? How? (Link to ***First Steps in Mathematics: Measurement***, Indirect Measure, Key Understanding 2; see Represent Shape, Key Understanding 3.)

Enlarge a Drawing

Choose a student's drawing and use technology to project the image onto paper. Ask the 'artist' to trace around it and invite students to compare the original drawing to the enlargement. Ask: What has changed? Is there anything that has stayed the same? Can you think of another way the drawing could have been made bigger? (Link to ***First Steps in Mathematics: Measurement***, Indirect Measure, Key Understanding 2.)

Mother and Baby

Show students mother and baby animal pictures and encourage them to describe the differences between them. Ask: How can you tell this baby belongs to this mother? Does the parent look exactly the same as the baby? Which parts of the baby's body will change the most?



Fish

Show students pictures of fish for them to talk about the basic shape. Ask: What makes this a fish shape? Give students directions to follow to make a fish shape on a geoboard. Invite each student to then move the elastic bands to create a different type of fish. Ask: How is this fish different from the original? Encourage students to draw diagrams of their distortions and display them with photographs and pictures of fish.

Stretching

Invite students to use modelling clay or play dough to make models of 3D objects (e.g. cylinders, cubes, cones), then stretch the model or parts of the model by gently squeezing, patting, pulling or rolling. Encourage each student to display their model and say how they changed the shape. Ask: Is your object the same size? Is it the same shape? How has it changed?

Tracing

Provide students with a selection of pictures (e.g. from magazines, books) and ask them to locate shapes that they know to be square, rectangular and circular in real life. Invite them to place a transparency over each and trace around the shape. Say: This wheel is an oval in the picture. Is it an oval in real life? How has the shape of the wheel changed? (Link to Represent Shape, Key Understanding 3.)

SAMPLE LEARNING ACTIVITIES

Middle ✔✔

Photographs

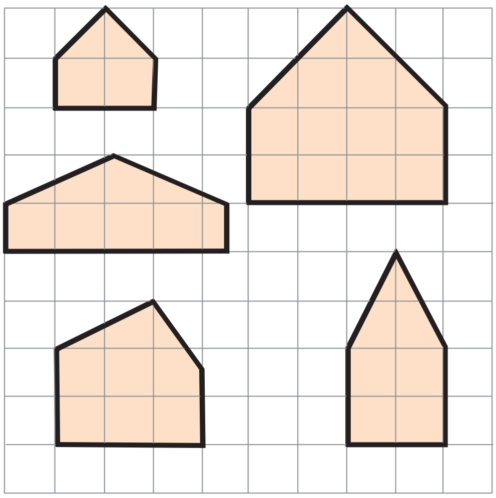
When designing a newspaper page on the computer, encourage students to reduce or enlarge scanned photographs. Ask: How is the scanned image different from the original? What has remained the same? (Link to ***First Steps in Mathematics: Measurement***, Indirect Measure, Key Understanding 2.)

Print

Have students investigate the page setup and icons on the computer to reduce and enlarge print. Invite them to key in their name and address and print several different-sized copies to use when deciding on a print size for publication. Ask: Which tool is required to view the whole page? What effect does this have on the size of the print? What effect does it have on the size of the page? How can you enlarge the print so it is larger when actually printed? (Link to ***First Steps in Mathematics: Measurement***, Indirect Measure, Key Understanding 2.)

Figures on a Grid

Provide students with figures on a grid that are enlargements or distortions of each other. Ask them to circle the one that is the same shape, but bigger (the enlargement). Ask: Why have you chosen that shape? How do you know it is the same shape as the smaller one? Encourage them to discuss their choices; for example, That one is not the same shape, it has been stretched. That one has been squashed. Continue with a variety of simple, everyday shapes, encouraging students to decide which are true scaled versions. (See ***First Steps in Mathematics: Measurement***, Indirect Measure, Key Understanding 2.)



Shadows

Invite students to draw around the shadows of regular geometric 3D shapes and everyday objects (e.g. books, balls, plastic cricket wickets) to investigate changes in the shape and size of the shadow. Ask: What happens to shadows of rectangular and circular shapes when the object is tilted? How are the shadow shapes different from the original shape? How can you change the size of the shadow?

Enlarging a Design

Ask students to use straight lines to draw a design onto a 4 x 5 grid using 1-centimetre squares. Have them then make a copy on another 4 x 5 grid using 2-centimetre squares. Ask: What has changed? What has stayed the same? Encourage students to compare the lengths (including diagonals), areas and angles. (See ***First Steps in Mathematics: Measurement***, Indirect Measure, Key Understanding 2.)

Balloons

Invite students to draw a series of regular 2D figures (e.g. square, rectangle, pentagon, hexagon) onto a balloon and then blow it up. Ask: What part of the shape changed the most? How do the shapes change? What happens to the sides of the shapes? Invite them to draw the shapes onto an inflated balloon and then deflate it. Ask: What happens to the shapes when the air is let out?

Shadows Over Time

Have students investigate the distortions of shadows over a period of time. For example, ask students to draw around the outside of their shadow onto the bitumen in the morning, and repeat the activity at noon and again later in the day. Ask: How has your shadow changed during the day? How is the size different? How is the shape different?

Model Square

Have students construct a square using straws or popsticks and trace around the inside. Invite them to distort the model by making it lean to one side and trace around each new shape. Ask: How did the distortion change the shape? What parts of the shape have stayed the same? Investigate other 2D figures and 3D objects in a similar way.

SAMPLE LEARNING ACTIVITIES

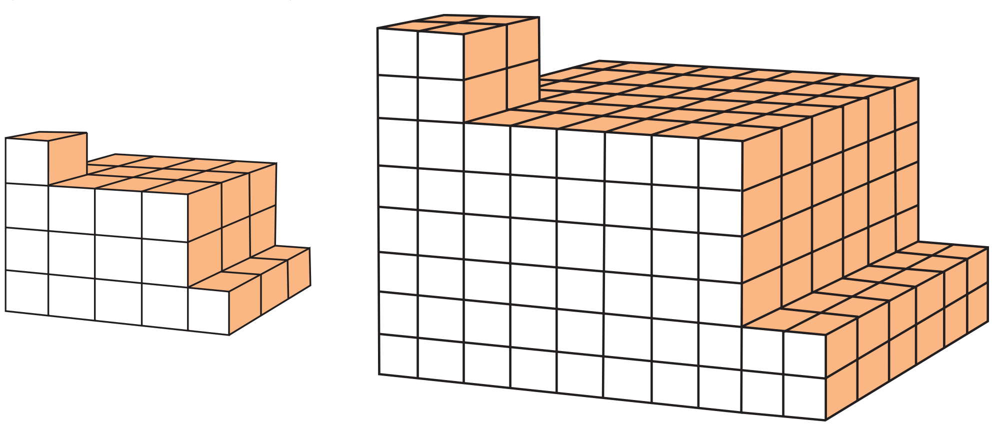
Later ✔✔

Area

Have students enlarge a picture photocopied onto square grid paper. Ask: If you double the length of each side, what happens to the area? (It quadruples.) What happens to the area when you triple the length of the sides? How would the area change if you enlarged the dimensions four times? Draw out that the lengths and areas change in a predictable way but the angles stay the same. (See ***First Steps in Mathematics: Measurement***, Indirect Measure, Key Understanding 3.)

3D Objects

Have students enlarge or reduce simple 3D objects made from cubes, ensuring the 'shape' of the object remains the same. Ask: What did you have to do to make sure the shape remained the same? How can you enlarge the object so that it is two (three, ten) times its original size? What happens to the length (width, depth, volume) of the structure during any enlargement?



Reductions

Guide students in making a series of reductions of a shape. For example, ask them to draw a 6-centimetre square, draw the diagonals across the square, mark the diagonals 1 centimetre (3 centimetres) from the midpoint and join the points. Ask: How is the new shape different from the original? Does the same thing happen to other regular shapes (e.g. regular triangles or hexagons)?

Predicting Dimensions

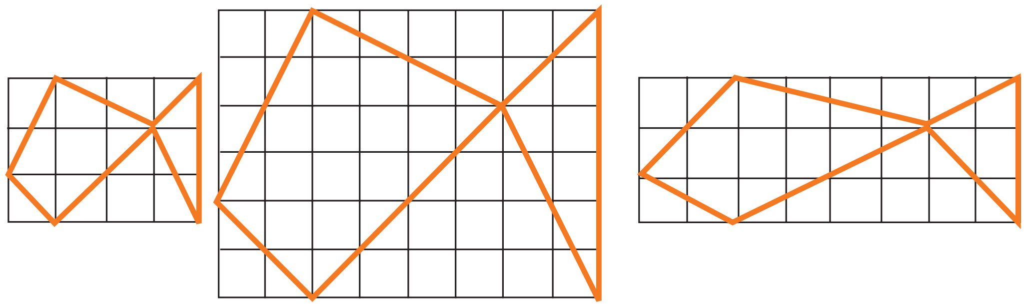
Ask students to predict the dimensions of a simple figure after it has been photocopied at 50 per cent, 100 per cent or 200 per cent of the original size, and test their predictions using the photocopier. Ask: What would you key in if you wanted your picture to be a quarter of the original size (three times the original size)?

Images

Have students use a graphic program to create simple figures and drawings. Invite them to predict how the image will change when it is dragged from a corner or from the horizontal or vertical edge. Ask: Is this an enlargement or a distortion? How do you know? What is the difference? Encourage them to use height and width measurements to tell. Ask: Have both the length and width been enlarged by the same proportion (e.g. doubled or enlarged by half as much)? (enlargement) Has just either the length or the width been enlarged? (distortion) (Link to ***First Steps in Mathematics: Measurement***, Indirect Measure, Key Understanding 4.)

Changing Shape

Have students say what happens to the angles, lengths and areas of shapes when the two linear dimensions are changed by different amounts. For example, ask students to make a straight line sketch of a fish on grid paper. Then have them make the fish twice as long and twice as high. Encourage them to measure various lengths on the two fish. Ask: How do matching lengths compare? Work out the area of the body of each. How do they compare? Now have students make another fish the same height as the original but twice as long. Ask: How is the last fish the same or different to the original fish? How do matching lengths compare? What happens to the area? (See ***First Steps in Mathematics: Measurement***, Indirect Measure, Key Understandings 2 and 3.)



Flags

Have students draw a simple geometric design for a flag on a sheet of paper, then use technology to project it and trace a copy. Invite them to compare the original with the copy. Ask: What is the same? What is different? Consider angles, lengths and area. Now, angle the image so that it is distorted. Ask: What is the same? What is different? Again compare angles, lengths and areas. (See ***First Steps in Mathematics: Measurement***, Indirect Measure, Key Understanding 2.)

Cartoon Character

Have students enlarge their favourite cartoon character by using different-sized grids as an enlargement tool. Ask: What size grid will you place over the original picture? How will you decide on the grid size for the enlargement?

Later ✔✔

Grid Picture

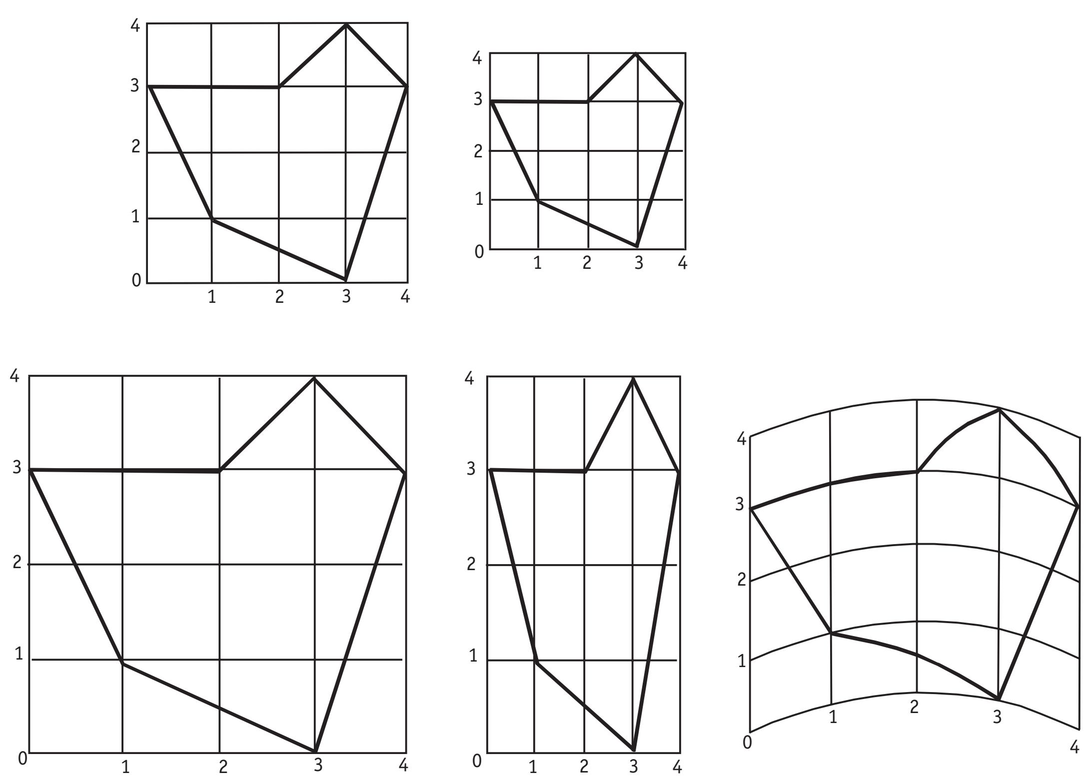
Invite students to draw small 2D figures (e.g. squares, pentagons, octagons) on a grid and use coordinate points to label the vertices of the shape. Have them double the coordinate numbers at each vertex and use the new points to redraw the shape. Ask: How many times bigger is the length of the sides (width, height, area)? How many copies of the original shape will fit into the new shape? Has anything else changed? (Link to Represent Location, Key Understanding 1.)

Elevation Plans

Extend Later Sample Learning Activity 'Elevation Plans' (Represent Shape, Key Understanding 3) by having students use cubes to construct a building that doubles the dimensions of another student's plan. Ask: Which attributes have changed? Shape? Angle? Volume? Area?

Grid Variations

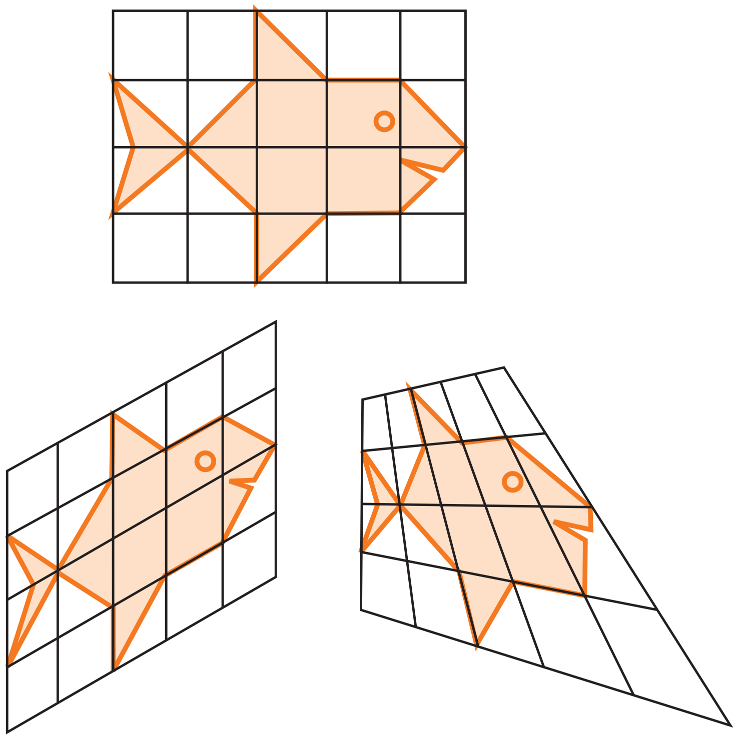
Have students produce variations of a grid with numbered x and y axes (e.g. a 4 × 4 grid enlarged, reduced, width halved but not the height, one or both of the axes curved). Invite them to use the grids to compare the changes to a simple picture when drawn on each grid using the same coordinate points. Ask: Which drawings are distortions of the original? Which are enlargements or reductions of the original? How did you tell the difference?



(1,1), (0,3), (2,3), (3,4), (4,3), (3,0), (1,1)

More Grid Variations

Extend 'Grid Variations' by experimenting with grids where the angle of the axes is less than or more than 90°. Ask: How is this distorted figure different from the original?



Toy Car

Say: Sam bought a toy car that had the scale written on the packet as 1:64. 'That doesn't sound right', he said. Ask: How can Sam check that the scale is correct? What should the length of the real car be? What would the dimensions of the real windscreen be? (Link to ***First Steps in Mathematics: Measurement***, Indirect Measure, Key Understandings 3 and 4.)

KEY UNDERSTANDING 4

Symmetrical things have component parts which can be matched by rotating, reflecting or translating.

People appear to have a strong intuitive sense of symmetry, recognising and responding positively to it. Certainly it is the basis of a great deal of design. Although we often associate the word 'symmetry' with mirror symmetry, there is a form of symmetry associated with each of the transformations described in Key Understanding 2. Thus, we have translational (or slide) symmetry, reflectional (or mirror) symmetry and rotational (or turn) symmetry. Each of the transformations maintains shape and size, and it is this property that makes symmetry possible. All symmetrical things have congruent or identical units that can be matched in some way. We say that a figure or object is symmetrical if a transformation exists that moves its individual points or parts into a different position but leaves the whole thing looking the same. Students should develop the following understandings.

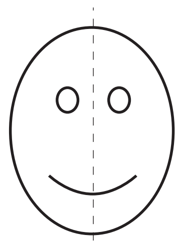


Figure 1

Figure 1 has mirror symmetry because 'flipping' it around the mirror line would move each point to the other side of the mirror but leave the overall picture looking the same. That is, two 'parts' can be matched by folding along the mirror line.

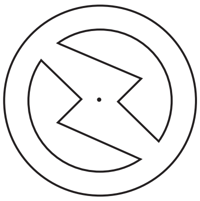


Figure 2

Figure 2 has rotational symmetry. You could rotate the picture a half turn, which would move each point (except the centre) to another position, and yet the whole picture would look the same. That is, parts can be matched by turning, or rotating about the centre.

Figure 3

Figure 3 has translational symmetry, if we imagine it as a non-ending frieze. You could translate each part to another position and yet the whole frieze would look the same. That is, parts can be matched by translating along a line.

Students need structured experiences that progressively build up their understanding of symmetry. For mirror symmetry, these might include:

- observing examples of things that are symmetrical in some way and things that are not
- folding, measuring and looking for symmetry
- discovering and explaining that matching parts of figures are the same distance away from the mirror line
- investigating the symmetry properties of some regular shapes, such as squares and isosceles triangles, and then the regular pentagon and hexagon and other regular polygons
- constructing a concept map summarising the symmetry properties of regular shapes.

Progressing Through Key Understanding 4

Initially, students can use a fold line to produce a picture with reflection symmetry. As students continue to progress they can use a variety of means, including cut-out figures and flipping and drawing around templates, to create symmetrical pictures. Next, students are able to identify component parts that are symmetrical and to justify this by showing the movement required to place one part onto the other.

As students progress further they can reproduce the folds and cuts needed to make complex symmetrical patterns, such as a frieze or a snowflake. They can also decide which transformations are involved in producing a particular symmetrical pattern or arrangement (e.g. explaining that a logo type will be repeated each third turn and so it has rotational symmetry).

Later, students can use an appropriate grid to produce a specified symmetrical shape (e.g. using circular grid paper to make a figure that has rotational symmetry).

SAMPLE LEARNING ACTIVITIES

Beginning ✔✔

Butterflies

Invite students to make butterfly pictures by placing blobs of different coloured paint on one half of a sheet of paper, folding it and then unfolding it. Ask: Are the sides the same or different? Do the blobs of blue on this side match the blobs of blue on the other side? Are they in the same place on each side? Are the green blobs the same distance away from the fold?

Rotating Designs

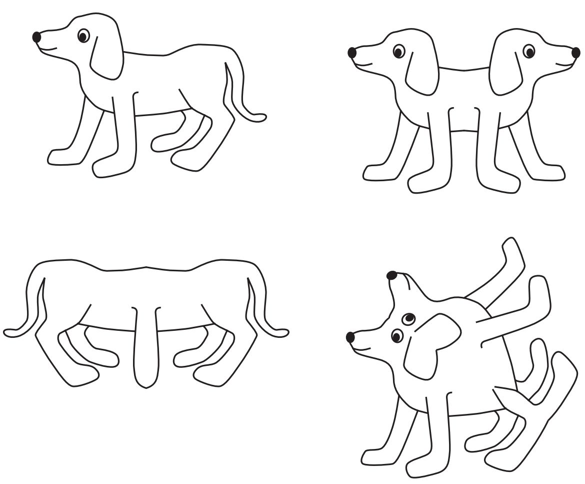
Have students fold a piece of paper in quarters, open it out and pin it to a board through the centre. Invite them to print in each square by stamping in the same position while rotating the paper. Ask: What else can you see around the room that matches this type of pattern making? How is the fan the same as the pattern? How is it different? What would we start with if we wanted to make a fan pattern in the same way we made our stamp pattern? How would we move the blade to make that pattern? How far do we have to turn the blade before we stop and trace around it? Encourage students to collect other simple examples of rotational objects or pictures and experiment with pinning a cut out of the component part to paper and then turning and tracing around the part to produce the design. (See Key Understanding 2.)

Mini Beasts

Give students a collection of class drawings of mini beasts glued to card to investigate the effects of reflection by using a mirror placed in different positions on or near the mini beast. Say: Use the mirror to make the worm grow (a friend for the ant, a two-headed snail). Ask: What happened each time? How does the picture in the mirror match the drawing? Which part of the drawing doubles?

Nonsense Animals

Have students use a Mira or mirror to make nonsense animals. Place the mirror in different positions to create a variety of different animals. Encourage students to describe to a partner how to create their animal, saying how each part of the animal is made.



Reflection Symmetry

Invite students to use a Mira or a mirror to find which capital letters, numbers or signs have reflection symmetry. Have them write the numbers and letters on some clear plastic and fold each to match the sides to show if they are symmetrical and how they are symmetrical. Encourage students to compare their decisions with their partner. Ask: Were there any letters that you could not agree about? Why couldn't you agree? Which letters had more than one kind of symmetry? How do you know? (See Key Understanding 2.)

Body Symmetry

Have students investigate body symmetry by comparing one side of their body to the other, using a mirror, photographs, handprints or footprints. For example, encourage students to find ways to place their hands to show different forms of symmetry. Ask: What happens when you place one hand next to the other? How would you move your hands to make that pattern? How are your hands different? How are they the same? Could you use a mirror to make that same arrangement?

Matching

Invite students to collect and investigate a variety of objects (e.g. leaves, nuts, seeds) to see if one side of the object mirrors the other side. Encourage them to describe to a partner how the objects match and suggest ways to sort them. Ask: Are there any objects that are the same on both sides except facing the other way? Look at your leaf. Is it the same shape on both sides of the vein going down the middle? How can you show that the sides match?

Folded Paper

Have students fold a piece of paper in half, cut a shape out from the folded edge, then open out the paper. Ask: What do you notice about the hole you have made? Look at the fold line. Is the hole on this side of this line the same as the hole on that side? Encourage them to refold and cut out another piece. Ask: What do you think it will look like when it is opened out? How far away from the first fold line is this hole? What about its matching hole on the other side? What would it look like if the paper was folded into quarters?

Turning Around

Invite students to think of ways they could turn figures (e.g. numbers, letters) or objects (e.g. duster, chair) around so that their partner can't tell it has been moved. Ask: Why does this one need a full turn? Why does this one only need a half turn?

SAMPLE LEARNING ACTIVITIES

Middle ✔✔

Boxes and Lids

Give students a collection of boxes with lids to find out how a lid can be turned to fit in a number of ways. Ask them to sort the boxes into groups according to the number of times the lid can be turned. Ask: Which lids can only be placed two ways to fit? What about a cylindrical box?

Reflection Symmetry

Extend Beginning Sample Learning Activity (p. 143) 'Reflection Symmetry' by having students create words and number sentences using only symmetrical letters numbers and signs. Ask: How are the letters in your words symmetrical? Are there any letters that have more than one kind of symmetry? Which are they? What kinds of symmetry do they have?

Butterfly

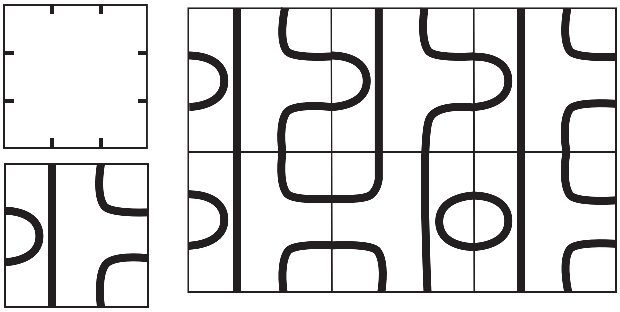
Give students an outline of a butterfly on paper and invite them to create a design on one wing, then swap the picture with a partner to complete as a reflection. Ask: How could you check that the reflection is the same as the original image? (Perhaps by folding and holding the paper up to the light.)

Tissue Paper

Have students write their name on tissue paper or tracing paper and experiment with a Mira to make a variety of different reflections of it. Ask: How does the placement of the Mira change the image? What happens if you place it at the bottom next to the words (at the bottom away from the words, halfway across the words)? Have students copy one of the reflected images and swap with a partner to find where the line of symmetry has to be. Encourage students to fold to verify. (See Key Understanding 2.)

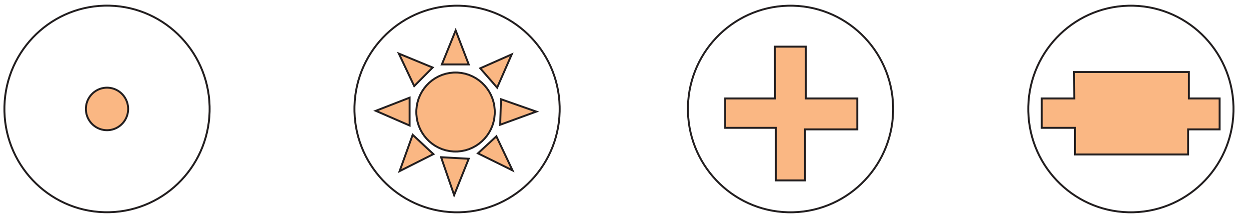
Tile Pattern

Have students construct a pattern on a tile measuring 9 centimetres by 9 centimetres by marking dots 3 centimetres apart on each edge and then joining the dots in some way. Then have them make five more tiles exactly the same and arrange all six in a 2 x 3 array to make a pattern. Encourage students to rotate one tile 180°, and identify which has been rotated in each other's pattern. Ask: Why is it that some rotations can be easily identified, while others cannot? When you turn the tile, does it look the same, or different? (See Key Understanding 2.)



Turning Things

Ask students to list the things they have turned during the day (e.g. door handle, tap). Then have them draw a diagram of one of the things they have turned, mark its point of rotation and make a template. Invite them to place the template over the original object and turn it around the point of symmetry. Ask: In how many places in a full turn can this object be matched? How do you know? Draw out that for some things there is an infinite number of matching positions of symmetry and for others there is not.



Matches at infinite places in a full turn (matches throughout the turn).

Matches at eight places in a full turn (matches every eighth of a turn).

Matches at four places in a full turn (matches every quarter of a turn).

Matches at two places in a full turn (matches every half of a turn).

Plane of Symmetry

Have students construct a 3D object using blocks and swap with a partner to construct a reflection of the shape. Encourage them to check using a Mira or mirror. Have students place the constructions 10 centimetres apart. Ask: Can you find the plane of symmetry between the two structures? Is it possible to find other planes of symmetry in the two structures?

Pattern Blocks

Have students create an arrangement of pattern blocks on their desks and mark a line of symmetry along one edge for someone else to complete the reflection. Encourage them to check the position and orientation by measuring the distance of vertices and lines from the line of symmetry.

Pattern Units

Ask students to identify the pattern unit in fabrics, pictures or Escher designs and decide which transformations or lines of symmetry were used to create the pattern. Ask: Where are the lines or points of symmetry? Invite students to use a different basic unit but use the same transformations to create a similar pattern or arrangement.

Reflection Symmetry

Invite students to use a Mira in different positions and copy the reflections of their name. Ask: How has the placement of the Mira changed the position of the reflection? How could you move the reflection of your name further away from the original? Which letters do not change when they are reflected? Encourage students to create a design with their name and different reflections, including some on the diagonal.

SAMPLE LEARNING ACTIVITIES

Later ✔✔

Border Patterns

Have students use grid paper to construct two different border patterns from the one figure, one using translation and one using reflection. Ask: How are the patterns different from each other? Which figures would result in the same pattern? Why? (The figures themselves have reflectional symmetry; e.g. square, circle.)

Quadrilaterals

Ask students to fold a range of different quadrilaterals and sort them onto a chart showing their lines of symmetry: no lines, 1 line, 2 lines, 3 lines, 4 lines, more than 4 lines. Encourage them to say how they decided where each figure belonged.

Logos and Crests

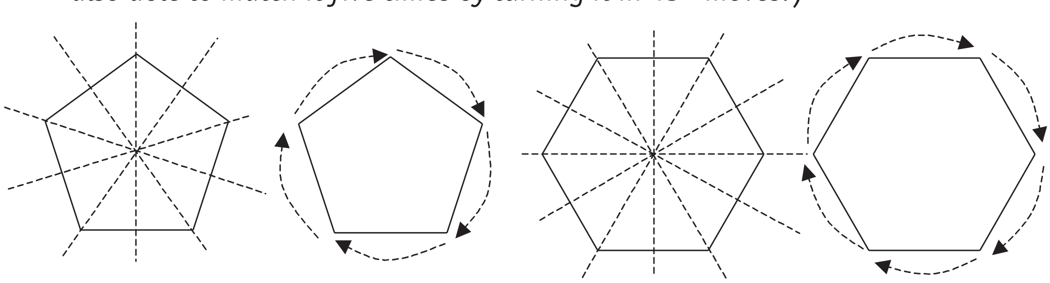
Invite students to look through newspapers and magazines to find symmetrical company logos and crests and investigate which transformations produced each. Devise a way to check. Ask: Which ones have reflectional symmetry? Which ones have rotational symmetry? Are there designs that have both? Did you find any that had translational symmetry?

Design Brief

Have students produce specified symmetrical designs from a given figure. For example, ask them to design a reflecting floral border no wider than 3 centimetres around a piece of paper. Invite them to choose from using folded paper, circular and square grid paper, geometry drawing template or computer graphics. Encourage students to say what movements they used to produce their particular design.

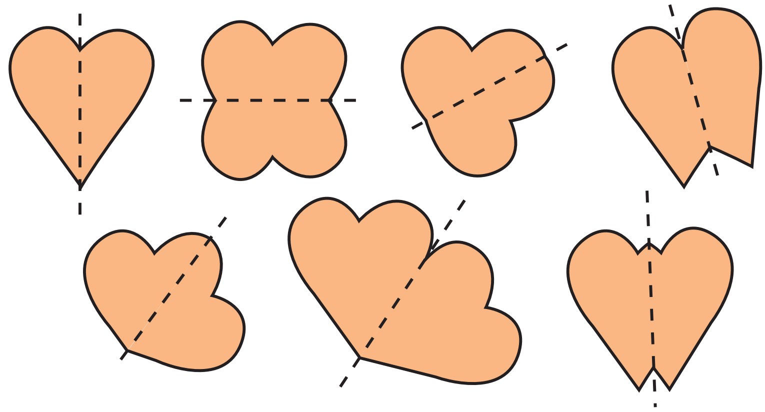
Regular Polygons

Give students a collection of regular polygons and ask them to choose one and work out how many lines of symmetry it has. Then ask them to explore the links between the number of lines of symmetry and the number of matching positions in a full turn around the centre point; for example, *For a regular pentagon, I was able to find five lines of symmetry by folding and I was also able to match it five times by turning it in 45° moves.*)



Card Designs

Invite students to create simple card designs using a Mira on the diagonal. Ask: How are the images different from designs where the Mira (the line of symmetry) is placed on the horizontal or vertical? Encourage students to use the Mira to move the line of symmetry into different orientations around the page to create a design or pattern. (See Key Understanding 2.)



Graphics Programs

Invite students to use a graphics program to make symmetrical designs using rotation, reflection and translation. Have a partner make the same pattern using a different motif. Ask: What did you have to find out about the design before you could reproduce it?

Venn Diagram

Have a large Venn diagram drawn on the floor and labelled 'translation', 'reflection' and 'rotation'. Ask students to sort a range of symmetrical objects and patterns according to the type of symmetry. Ask: Where would you place a pattern or a figure that has translation and reflection symmetry, but not rotational symmetry? Where would you place a kite (patterned plate, glass, plain t-shirt)?

Model of a Cube

Provide a collection of cubes made from modelling clay. Invite students to say how a cube is symmetrical and then use fishing line to cut the cube to justify their decision. Ask: How did you decide where to place the cut to produce the symmetry? How many different planes of symmetry can you find in the cube? Repeat the activity with other polyhedrons (e.g. prisms, cylinders, pyramids, cones).

Rotational Designs

Invite students to choose from pattern blocks, circular grid paper and computer graphics to create rotational designs with varying angles of turn. Have them swap their designs with a partner and decide how the motif must be turned in order to recreate the design. Ask: How many degrees did you need to turn the motif to make this design? Are there other designs that use the same type of rotational symmetry as that one? How are they the same?

Later ✔✔

World Art

Invite students to research and collect examples of art and designs from around the world that exhibit reflectional symmetry (e.g. Aboriginal rock art, Maori carvings, Italian sculpture, Escher designs). Encourage them to examine differences between how the figure is reflected over the mirror line. Ask: Are they reflected vertically or horizontally? Is the mirror line on an angle? Is the figure next to or apart from the mirror line? Have students make a display with written descriptions about the symmetry used in each.

Rotational Symmetry

Invite students to examine objects (e.g. patterned bowl, compact disc), patterns (e.g. snowflake patterns on circular paper, borders of plates and bowls, patterns in art) and arrangements (e.g. students sitting in a circle, block pentomino made to look like a cross) that have rotational symmetry. Ask: How do you know they have rotational symmetry? Draw out that they have rotational symmetry if they can be turned and still look the same. Invite students to describe the symmetry so someone else could sketch the placement of the repeating figures. Ask: What information do you need to include if the person is to produce the correct number of repeats? Draw out that the angle or amount of turn needs to be provided. For example, *My arrangement starts with a cube in the middle and then has a cube joined to the centre cube at every 45° turn.*

Did You Know?

Often it is suggested that a picture (or a figure or arrangement) is symmetrical if 'it is the same on both sides'. This is not a helpful explanation of mirror symmetry, since the two sides are usually different, being reflections of each other. In Figure 1, the two parts are the same, but the total picture does not have mirror symmetry; in Figure 2, the two parts are different (being reflections of each other) but the whole picture is symmetrical.

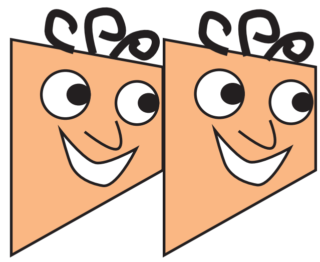


Figure 1

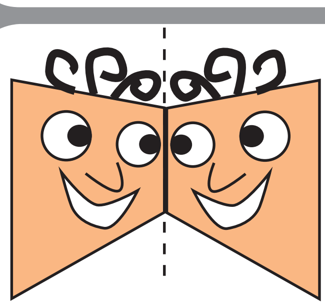


Figure 2

SAMPLE LESSON 1

Sample Learning Activity: Beginning—'Nonsense Animals', page 142

Key Understanding 4: Symmetrical things have component parts which can be matched by rotating, reflecting or translating.

Teaching Purpose

My Year 2 class had been working on nonsense poems in language so I thought it a good time to do some learning about symmetry by trying to create some 'nonsense' animals. They seemed to use symmetry in their pictures and pattern making, but I had not found the opportunity to talk about the mathematics of this up till now.

Action to Develop Familiarity with the Materials

I gave each student a mirror and some pictures of animals from magazines and colouring books so they could explore reflectional symmetry. After they'd had a little time to experiment on their own, I showed them how to change the image by sliding the mirror's edge back and forth on the page. Once the students mastered this basic process, they became very interested in the images they were producing and tried to outdo each other in finding humorous and strange animals.

'Look, I made a dog with a head each end.'

'I've got a baby monkey with no head and four legs and four arms.'

Opportunity to Focus on the Mathematical Idea

I asked students to take particular notice of where they put the mirror on the page to get each of their funny images, then to find their favourite image and mark the line that they used to make it. I began using the terms 'mirror line' and 'reflection line' to refer to the line where the mirror had been placed and encouraged students to use these terms to describe the position of the mirror. After they had marked in their line, I asked them to share their images with a partner, explaining what their image should look like.

The terms 'mirror line' or 'reflection line' are a more accurate way to say where the mirror should be placed.

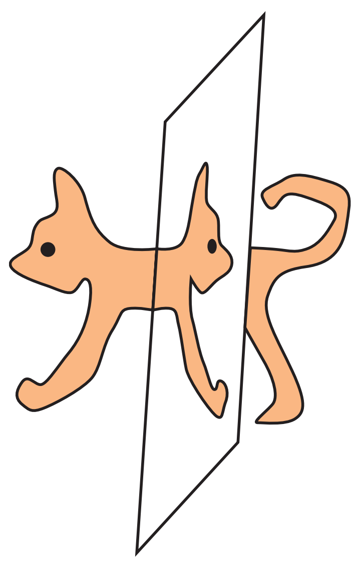
The term 'line of symmetry' is used in reference to the total image; that is, it only becomes a 'line of symmetry' when the mirror is in place and produces a symmetrical image.

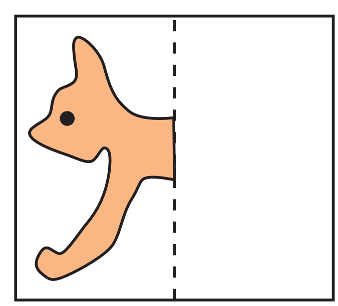
The students I listened to were talking about the whole image rather than how the two parts of the image were the same, so I moved around to notice whether the other conversations in the room were similar. This gave me enough information to know that I needed to talk with the students about how the new shape had two parts which were the same except facing in opposite directions. I wanted to make sure that they knew that the image they had created was symmetrical.

'Tell your partner how your new image is different from the picture without the mirror on it,' I suggested.

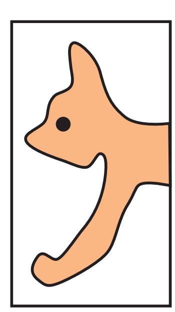
This prompted more discussion, but again the conversations were about things like, 'Well, this one is much fatter than without the mirror' or 'My new animal has four legs and the old one only had two legs.'

I decided that the students needed to make the second part of their image, to see it flat rather than in the mirror, so I asked them to cut their picture along the mirror line and glue it onto a piece of card. The next day, when the glue was dry, I showed the students how to fold the piece of card along the mirror line and to cut out the shape. After the students had done this, and flattened out their nonsense animal, I asked them to look closely at the two parts and say what was the same.

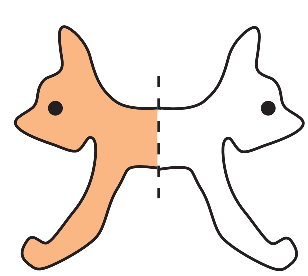




Cut out the picture along the mirror line and glue to a piece of card.



When it is dry, fold on the mirror line and cut out.



Open it up to check your 'nonsense animal'.

'Well, it has two legs on both sides and one head on both sides,' was a typical response.

'What do you need to do to your animal to make it the same on both sides?'

'Colour it in,' was the response.

Drawing Out the Mathematical Idea

'You have made an animal that is symmetrical,' I said, writing the word on the board. 'That means that both sides of the shape are the same but facing the opposite way. So far our animal has the same shape on both sides, but it needs to have the same colours on both sides. We call this line in the middle a line of symmetry.'

'Lots of things around us are symmetrical,' I went on. 'If I drew an imaginary line down the middle of my body, I would be symmetrical.'

I showed where the line would be with my hand. 'See, one eye on each side, half a mouth on each side, one arm and one leg on each side. Can you see anything else in this room that is symmetrical, has two parts that are exactly the same but facing the opposite way?'

The students immediately started identifying things around them such as the window, their chair, the teddy bear in the corner, and so on. It seemed that most students had the basic idea. We had an art activity to get on with— colouring in the other side of our nonsense animals— so I shifted the focus of the activity back to this. I knew I would be able to discuss symmetry further while they coloured. I also knew that their nonsense animal would no longer look symmetrical when they had finished, but this would provide another opportunity to talk about what made something symmetrical.

Activities such as this one help students to develop a basic understanding of symmetry and how it can be created by reflecting shapes across a line. Later, students will come to understand that the reflected image is the result of points on either side of the line of symmetry being equidistant.
I planned to set up a symmetry table, with initially, pictures of animals for students to make more nonsense animals, but later to include challenges such as …

Use your mirror to make:
1. *The longest worm you can make.*
2. *The shortest worm you can make.*
3. *The fattest worm you can make.*
4. *The thinnest worm you can make.*
5. *A worm going round a corner.*
6. *Two worms.*

BACKGROUND NOTES

Shapes that will tessellate, such as rectangles, squares, triangles and hexagons, can be made into irregular shapes that also tessellate. Cut a section out of one side of the shape and use either translation, rotation or reflection to move the shape across to one of the other sides. Figure 1 uses translation to create the shape and then translation to create the tessellation.

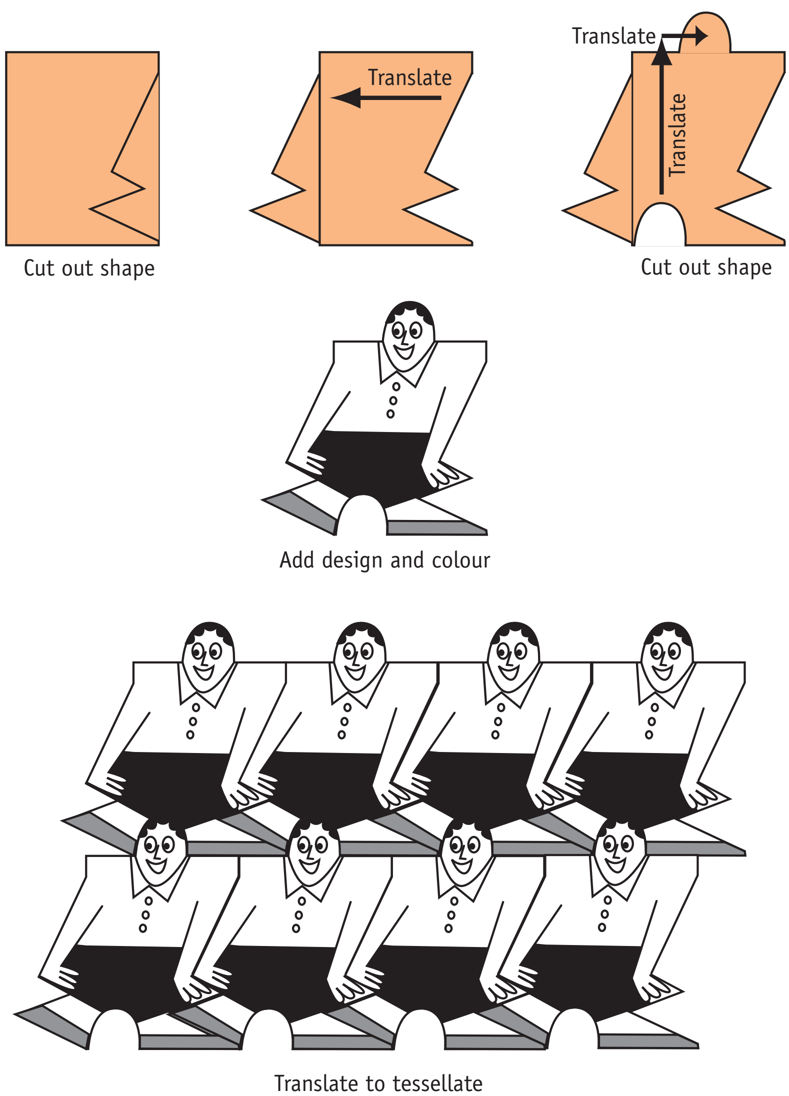


Figure 1

Tessellating shapes can also be created using rotation and reflection, or a combination of different transformations. The shape in Figure 2 was created using translation and reflection.

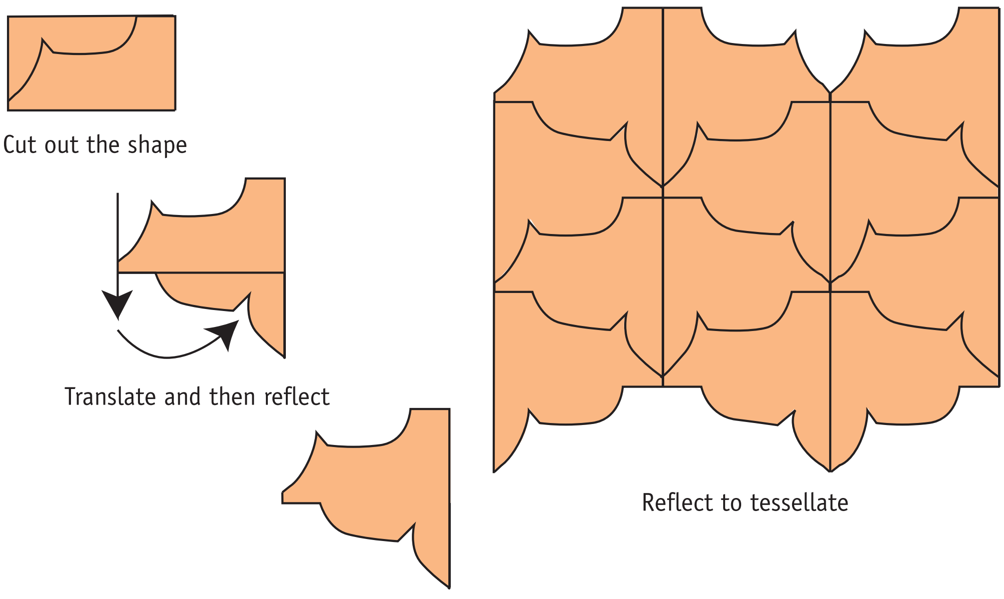


Figure 2

Students need to consider which transformation or series of transformations needs to be used to make the tessellating design. They will discover that the same transformations they used to create the beginning shape are also used to create the tessellating design. The tessellation in Figure 2, for example, is created by reflecting and translating the shape.

Students will find that some shapes will not tessellate with some transformations. The shape and the tessellating design in Figure 3, for example, were created using rotation and a ‘hole’ has appeared that cannot be filled with the shape.

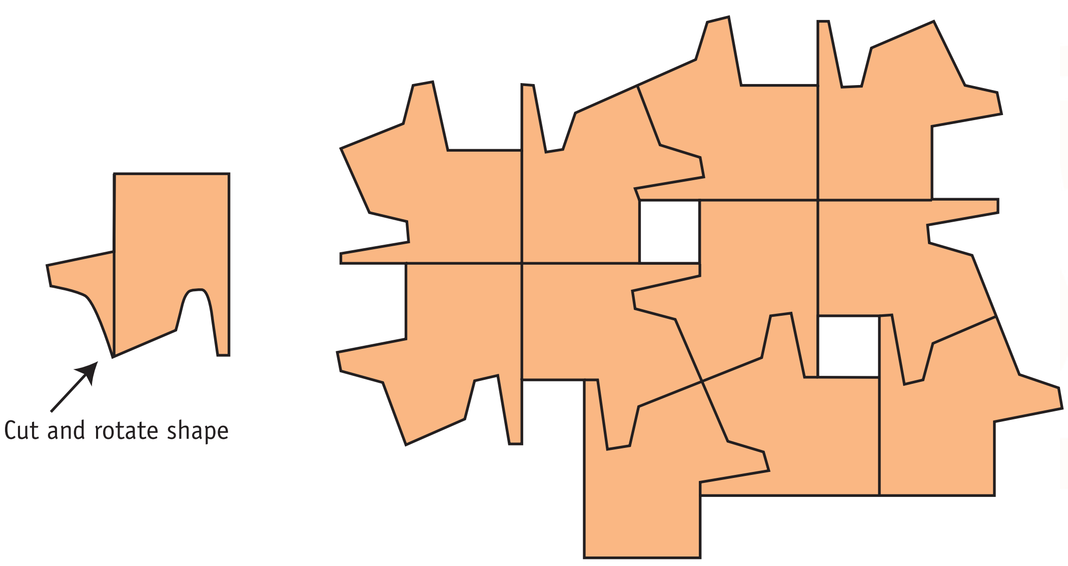


Figure 3

When the beginning shape (the rectangle) is rotated in the same fashion, a hole, similarly, appears (see Figure 4). This is because rectangles cannot be made to tessellate using rotation. The same transformation on a square, however would tessellate (see Figure 5).

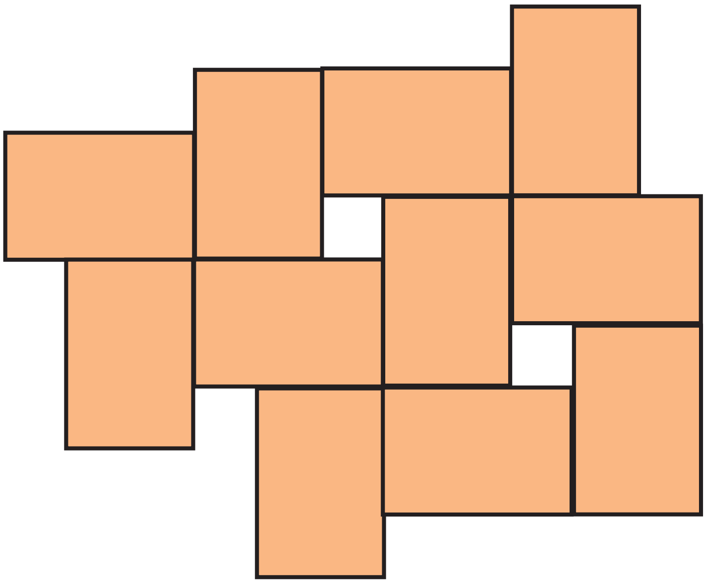
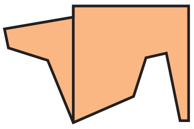


Figure 4



Start with a square, and the same cut out and rotation tessellates without holes.

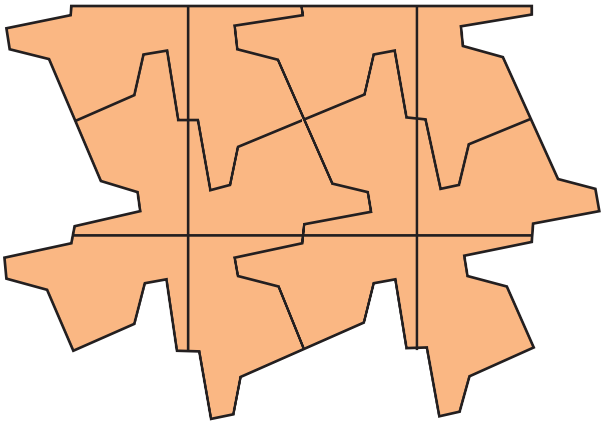


Figure 5

CHAPTER 6

Reason Geometrically

This chapter will support teachers in developing teaching and learning programs that relate to this outcome:

Reason about shapes, transformations and arrangements to solve problems and justify solutions.

Overall Description

Students use conventional names (square, cylinder) and criteria (curved, parallel), to describe and analyse two-dimensional and three-dimensional shapes. They make general statements about the properties of shapes and the relationships between them, including similarity and congruence. They know, for example, that for a shape to be a square it must have four equal angles as well as four equal sides, that if the diagonals of a four-sided polygon bisect each other it must be a rectangle or a rhombus, and that prisms always have two opposite faces that are congruent (that is, the faces have the same shape and size). Further, they apply this knowledge to link the shape and structure of objects to their function and hence to the problems of design. For example, they explain why some shapes are not widely used as floor tiles, why milk cartons are the shape they are, why some shapes are suitable for building purposes and others not, and how the properties of the diagonals of a rectangle enable builders to ensure that the corners of their structures are right angled.

Students also describe the effect of transformations on shape, size, orientation and arrangement. Thus, they recognise the symmetries associated with reflection, rotation and translation in a wide range of contexts, such as patchwork, designing clothes, crystal growth and map projections. They also enlarge and reduce shapes to plan and understand how the similarity properties of enlargement and reduction enable us to produce such technology as overhead projectors and photocopying machines and mathematical techniques such as those of trigonometry.

Markers of Progress	Pointers Progress will be evident when students:
Students talk about likenesses and differences between things that they can see or handle and begin to connect shape, movement and function.	• interpret the words 'like' and 'different', describing how two things they can see and handle are alike (e.g. *these are the same colour*) and how they are different (e.g. *but this is pointy and this isn't*) • interpret and begin to use language such as 'flat', 'straight', 'curved', 'side', 'round', 'corner'; e.g. select tree trunks, posts and pencils to show 'long but round' • interpret and begin to use spatial language, such as 'rolls', 'slides' and 'stacks', to describe an object's function; e.g. *This could be used for the wheel because it rolls.* • begin to connect shape and function in some familiar things; e.g. choose a triangular shape and reject a rectangular shape to rest a board on for a seesaw • classify objects by a familiar attribute relating to shape, movement or function; e.g. good or not good for holding a drink • recognise shapes that are (close to) rectangles, squares, triangles and circles in everyday things; e.g. on boxes and bicycles and cubes
Students sort things according to everyday spatial criteria and, prompted, use their spatial language to describe the shape of things.	• respond to 'Tell me about the shape of this object (place, mathematical model)' using language such as 'flat', 'curved', 'corner', 'side', 'round', 'square', 'edge' • identify figures in faces and cross sections of objects; e.g. draw and (where a commom name is available) name six different figures in a jug • distinguish rectangles (including squares), triangles and circles from other figures that are quite like them; e.g. describe some of their slices of salami as circular, but others as not circles because they are 'longer' one way • identify and name cubes, rectangular boxes, cylinders, cones and spheres in common things; e.g. *If I remove the handle from this mug it looks like a cylinder.* • distinguish between a 3D object and its face; e.g. know a cube is not a square and a box is not a rectangle • sort things according to simple spatial criteria and describe and represent classifications including in two-way tables; e.g. classify objects by how many faces they have and the shape of the faces
Students interpret common spatial language and use it for themselves to describe and compare features of things.	• integrate conventional names of shapes and component parts of shapes into their descriptions of things; e.g. side, face, edge, vertex, base, surface, curved, triangular, circular • identify the spatial features of things and link shape to functions such as stability, strength and storage • identify prisms and pyramids (including cones and cylinders) within their environment • describe and compare the spatial features of various mathematical objects that they can see and handle, including by the number of edges and vertices and the number, shape and position of faces • visualise and describe cross sections of familiar objects; e.g. *We thought the carrot slices would be circles but they weren't all. Some were like long circles and we found they are called ellipses.* • select an object from a collection given a description of its spatial features; e.g. *I have two flat faces and one curved face. I also have two edges. What am I?*
Students select, describe and compare figures and objects on the basis of spatial features, using conventional geometric criteria.	• choose geometric language with care in order to describe things clearly; e.g. use 'face' and 'edge' rather than 'side' • provide a description of a diagram or shape so that a peer could reproduce or recognise it; e.g. describe a logo 'over the telephone' • make figures and objects that fulfil criteria related to sides, faces, angles and edges • visualise and select figures and objects that fulfil geometric criteria; e.g. *I am made from two triangles that are the same shape and size.* • link features of structures such as flexible or rigid, fragile or strong to their shape • explain why they think certain shapes will predominate in situations that they have not yet investigated • conjecture about whether certain types of shapes will or will not tessellate and explain their thinking; e.g. *We think that all parallelograms will tile. This is because you can always slide them together to make long strips and then push the strips together.*
Students analyse, describe and apply distinguishing features of common classes of mathematical figures and objects, including by using the concepts of parallel and perpendicular.	• describe features that distinguish one common class of shapes from another; e.g. *Prisms have two parallel faces that are exactly the same but pyramids don't.* • generate and classify shapes that satisfy a given condition; e.g, use straws to produce shapes that have two equal diagonals. 'What do they have in common? What if we add the constraint that the diagonals cross at their midpoints?' • apply the distinguishing features of common classes of quadrilaterals to determining 'inclusive' relationships between them; e.g. show parallelograms, rectangles and squares in a Venn diagram and say *All squares are rhombuses but the reverse isn't true because …* • identify and name parallel and perpendicular lines and planes in figures and objects; e.g. in a triangular prism • identify inadequacies in descriptions of shapes; e.g. *Saying the shape has triangular faces isn't enough to describe triangular pyramids. This other shape is also made up of triangles but it isn't a pyramid.*

Key Understandings

Teachers will need to plan learning experiences that include and develop the following Key Understandings (KU), which underpin achievement of the outcome. The learning experiences should connect to students' current knowledge and understandings rather than to their year level.

Key Understanding	Stage of Primary Schooling—Major Emphasis	KU Description	Sample Learning Activities
KU1 Things can be the same in some ways and different in other ways. When we classify, we sort things into groups that are the same in specified ways.	Beginning ✔✔ Middle ✔✔✔ Later ✔✔✔	page 158	Beginning, page 160 Middle, page 163 Later, page 166
KU2 Thinking about shape can help us to understand the way things work and fit together.	Beginning ✔✔ Middle ✔✔✔ Later ✔✔✔	page 172	Beginning, page 174 Middle, page 176 Later, page 179
KU3 There are special words, phrases and symbols that help us to think about and describe the shape and structure of things	Beginning ✔✔ Middle ✔✔ Later ✔✔	page 182	Beginning, page 184 Middle, page 187 Later, page 190
KU4 People have developed useful ways to classify shapes. Knowing that a shape is one of the standard types can tell us a lot about it.	Beginning ✔✔ Middle ✔✔✔ Later ✔✔✔	page 196	Beginning, page 198 Middle, page 201 Later, page 204

Key

✔✔✔ The development of this Key Understanding is a major focus of planned activities.

✔✔ The development of this Key Understanding is an important focus of planned activities.

✔ Some activities may be planned to introduce this Key Understanding, to consolidate it or to extend its application. The idea may also arise incidentally in conversations or routines that occur in the classroom.

KEY UNDERSTANDING 1

Things can be the same in some ways and different in other ways. When we classify, we sort things into groups that are the same in specified ways.

Classification underlies almost all aspects of mathematics at some level. We can classify numbers as odd or even. We group problems by the processes involved in their solution, such as addition or subtraction, and classification of data is a major outcome of the Chance and Data strand. It is included here because of its particular significance for reasoning geometrically. A number of ideas underlie classification:

- Two things may be alike in some respects and yet different in other respects.
- We often wish to focus on the likenesses while ignoring the differences.
- We can sort according to whether things do or do not have a particular characteristic.
- The same collection of things can be sorted in different ways.
- Whether we think of things as belonging together will depend upon the context and our purpose.

To develop increasingly sophisticated classification skills during their primary years, students need to be engaged in sorting according to criteria specified by others and in developing their own criteria.

Classifying according to criteria specified by others

Students should learn to sort all of the items in the collection where it is possible and use each of the criteria consistently and correctly. Sorting by one criterion is likely to be easier than sorting by two or three or more. The familiarity of the criteria, however, may be as significant an influence on difficulty as the number of criteria, and familiarity relates to background experiences including culture, language and location. Students who understand the idea of classification may be unable to classify by criteria that they do not fully understand. Others with a good understanding of particular criteria may not consistently stick to the rules in sorting things.

Developing criteria for classification

Students should learn to think about whether their sorting criteria are comprehensive enough to enable all the items to be put into one group or another and whether they are clear (unambiguous) enough to enable a decision to be made about where particular items go. This is not always as easy as it appears and should receive continued attention as students proceed through the primary years. We usually sort things for a reason and students should learn to think about whether their chosen criteria achieve their purposes (link to ***First Steps in Mathematics: Chance and Data***, Collect and Organise Data, Key Understanding 3).

Activities should be provided that encourage and enable students to focus on spatial characteristics. Classifications based on shape (how things look and their properties), transformation (how things move or change) and locations (where they are) should be included. Older students should make general statements about what is common to the shapes or movements in a collection. At times this will draw out the properties of conventional classes of geometric shapes such as circles or cylinders, linking to Key Understanding 4. At other times, this may not lead to common classifications but may still be sensible (e.g. *All these shapes make good tiles. All these objects have the same area base and the same height*).

Progressing Through Key Understanding 1

Initially, students are able to talk about how two things they can see and handle are 'alike' and how they are 'different'. As students continue to progress they classify using one or two familiar and unambiguous spatial and other criteria. With prompting, they will also sort according to things that *don't* have a familiar attribute (e.g. *These don't have points*) and things that do (e.g. *These have points*).

Next, students will describe and compare objects, saying how they are alike or different, and what characteristics they do and do not have. They accept that the same collection can be sorted in different ways.

As students progress further they are aware that classification is purposeful, and that sorting by different criteria may tell us different things. Later, students infer common features that distinguish one class of things from another and generate things that satisfy a particular set of criteria.

SAMPLE LEARNING ACTIVITIES

Beginning ✔✔

Alike and Different

Have students show a 3D object (e.g. a leaf, a nut, a rock) to a partner and ask them to find another object like it. Encourage them to together describe all of the ways the two objects are alike and how they are different.

Comparing Shapes

Ask students to describe the things that are the same about two different shapes (e.g. square and rectangular blocks, two different types of flowers, toys, pillows). Ask: How is this flower the same as that one? Is there anything else about the two flowers that is the same?

Sorting Containers

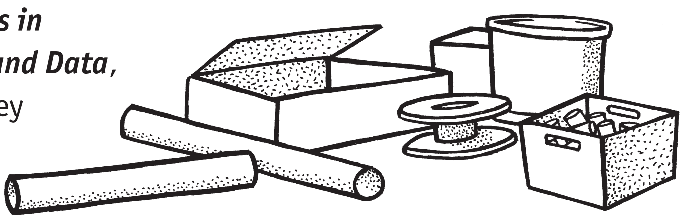
Provide students with a collection of boxes and containers (e.g. toothpaste, salt, cereal, cake mix, soup cans) and ask them to sort them into 'those that roll' and 'those that don't roll' ('those that stack' and 'those that don't stack', 'those that pack' and 'those that don't pack') and say what it is about the boxes that mean they can be sorted this way. Ask: What label would you give to this group of boxes (containers)? Why have you included all of these boxes under this label?

Shape Families

Invite students to sort a variety of objects (e.g. marbles, buttons, shells) into groups and give each group a 'family' name such as 'two holes' or 'three holes'. Ask: Why is this one in this family? How is it like the others in the group? Why is this one in a group of its own? How come these can fit into both groups at the same time? Which one belongs with these, but not with these? Make a family group called 'round buttons'. What other groups could there be?

Building

Ask students to sort a collection of recycled materials into those that are useful for building and those that aren't. Encourage students to discuss reasons for grouping them and then have them construct a house and say whether they have changed their minds about the grouping. Ask: Why did you change your mind about the cardboard cylinder? Which group does it belong to now? (See ***First Steps in Mathematics: Chance and Data***, Collect and Organise, Key Understanding 3.)



Storing Containers

Invite students to decide on ways to store empty packaging for 3D construction by sorting a pile of boxes, cylinders and plastic containers into like groupings. Encourage them to suggest which things belong together and why. Ask: Why do these two things belong in the same group? Could this box go in the same group as that box? What other packages might come tomorrow that will belong to this group? Help students to regroup the materials based on different criteria (e.g. how things move, how things stack, how many bases they have, whether they are 'inside' objects or 'outside' objects).

What's Different?

Invite students to sort a collection of shells (leaves, seed pods, toys). Encourage them to make general statements about how they are different and why. Focus on the shapes, symmetrical and non-symmetrical parts and where they are located. Ask: Why have you grouped your shells in this way? What part of the shells were you looking at in order to decide on your groups? How is this group different from that group? How have you grouped your seed pods? How is each group different from the others? Could we classify them in a different way (e.g. where they are found on the plant)? (Link to Represent Transformation, Key Understanding 4.)

Shoeboxes

Give each student ten different things in a shoebox. Ask them to sort the things according to criteria given on different days (e.g. sort by shape on one day, colour the next, size the next, whether manufactured or natural the next). Use the students' suggestions for groupings and include a group of things that 'are not'. Ask: How did your groupings change from yesterday? Why have the things in the groups changed?

Attribute Blocks

Give students attribute blocks to make an attribute track. Have one student place a block down and the next student add a block with one attribute different from the previous block. Blocks may be placed at either end of the track, or branching off from the main track. The winner is the student who places all blocks first. Ask: Why is it that you can place the blue, thin, large triangle next to the blue, thin, small triangle? What is the same about these two shapes? What is different?

Beginning ✔✔

Sort by Location

Ask students to look around the classroom and list all the things that are above the teacher's table, behind the door, lower than the blackboard, inside, outside, left of the lunch basket, right of the lunch basket. Ask: Could we classify things around the classroom by where they are located? Have students write a list of the items that can be classified as being lower than the blackboard or higher than the blackboard. Ask: Why is it helpful to classify things by where they are located? How might we use this classification? Draw out that this might be helpful when giving or receiving directions on where to find things.

Object Templates

Invite students to choose everyday objects (e.g. blocks, toys, crockery, packets of food, their hands) to use as templates to make patterns or drawings. Ask: Which kinds of objects are easy to draw around and which are not? Which different objects make the same shape?

Collage

Have students make collage pictures of an object by combining parts of other objects that are the same shape (e.g. a picture of a banana for hair, a collection of hand pictures to make a tree canopy, pictures of pencils to make a fence). Ask: What is it about the pencil that means you can use it to make a fence? What part is the same and what part is different?

Sorting Oddments

Organise students into small groups and ask them to sort a collection of 'oddments' (e.g. left-over pieces of toys and games, leaves, shells) according to which figure they most look like (e.g. square, oval, diamond) and name each category. Ask: What else can you see around the room that belongs in one of your groups? Could it belong in another group? Why? (See Key Understanding 3.)

SAMPLE LEARNING ACTIVITIES

Middle ✔✔✔

Classrooms

Have students sort the classrooms according to their location. Ask: Whose class will have the most time on the play equipment? Which class will get to the cafeteria line first if all students leave their rooms at the same time?

Animals

Have students sort a collection of plastic animals, or pictures of animals, saying what each group has in common (e.g. they live on farms, they come from Africa, they are mammals, they have four legs). Ask, for example: Why are cats and dogs in different groups? How are cats the same as dogs? How are cats different from dogs?

More Animals

Extend 'Animals' by having students consider how their groups might be combined to make larger groups. Ask: Could all of these animals go together to be a group of reptiles? What name would you give your groups now? Encourage students to compare their groups with standard scientific classifications of animals. Ask: Could this group named 'animals with fur or hair' be renamed mammals? What do all primates have in common?

Food Groups

When creating a table of the different food groups (fruit and vegetables, cereals, meat and protein, fats and oils, and dairy products) invite students to decide how the foods in each group are alike. Ask: Why do the eggs belong in the same group as meat? Why are fruit and vegetables in the same group rather than in two different groups? Encourage them to add any foods they have eaten to the table. Ask: What foods would we add to the fruit and vegetable group? What foods would we add to the dairy group? Where would we put chocolate cake? (See ***First Steps in Mathematics: Chance and Data***, Collect and Organise, Key Understanding 3.)

Middle ✔✔✔

Guess My Shape

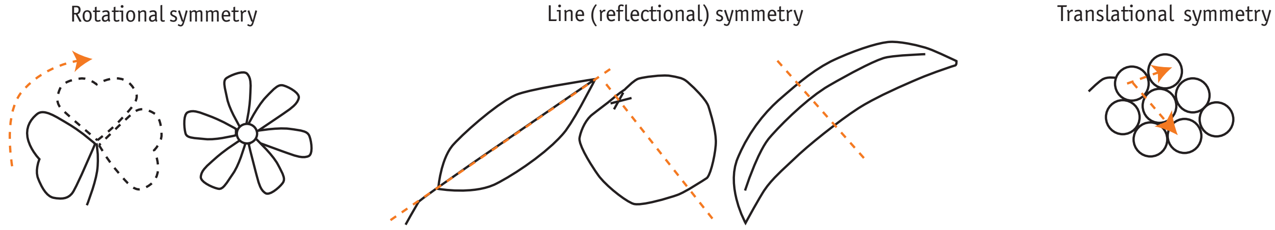
Have pairs of students make figure cards by drawing and naming 20 different 2D or 3D figures between them. Invite them to share the cards evenly and spread the cards out face up in front of them. Ask each student to choose a figure, write its name down and hide the piece of paper. Then have them take turns to ask a question about the figure; for example, *Has it got four sides?* If the answer is yes, then all figures without four sides are turned over. If the answer is no, then all figures with four sides are turned over. Encourage them to take turns asking questions until one student is able to identify the correct shape by looking at the remaining card. Ask: Which clues were more helpful? Why?

Classifying Boxes and Cylinders

Invite students to classify a collection of boxes and cylinders. Have them list all of the things that each group has in common and name the group. Encourage students to compare their group names and listed characteristics. Make a class list of the categories that have been used and invite students to see if they can reclassify their collection based on spatial characteristics (e.g. boxes that are rectangular and those that are not, boxes that have one face that is a square, boxes that are symmetrical). Ask: How can a box belong to more than one group?

Leaves, Flowers and Fruit

Invite students to classify a variety of leaves, flowers and fruit according to their type of symmetry (line, rotation, translation). Explore whether the leaves, flowers and fruit of a plant belong to the same group; that is, if the leaves have reflection symmetry, do the flowers and the fruit? Encourage students then to reclassify them according to other obvious characteristics of shape (e.g. long, thin shapes, round, fat shapes). Ask: Did you have to move things after reclassifying? Why? How was the final arrangement different from the first one?



Recycled Material

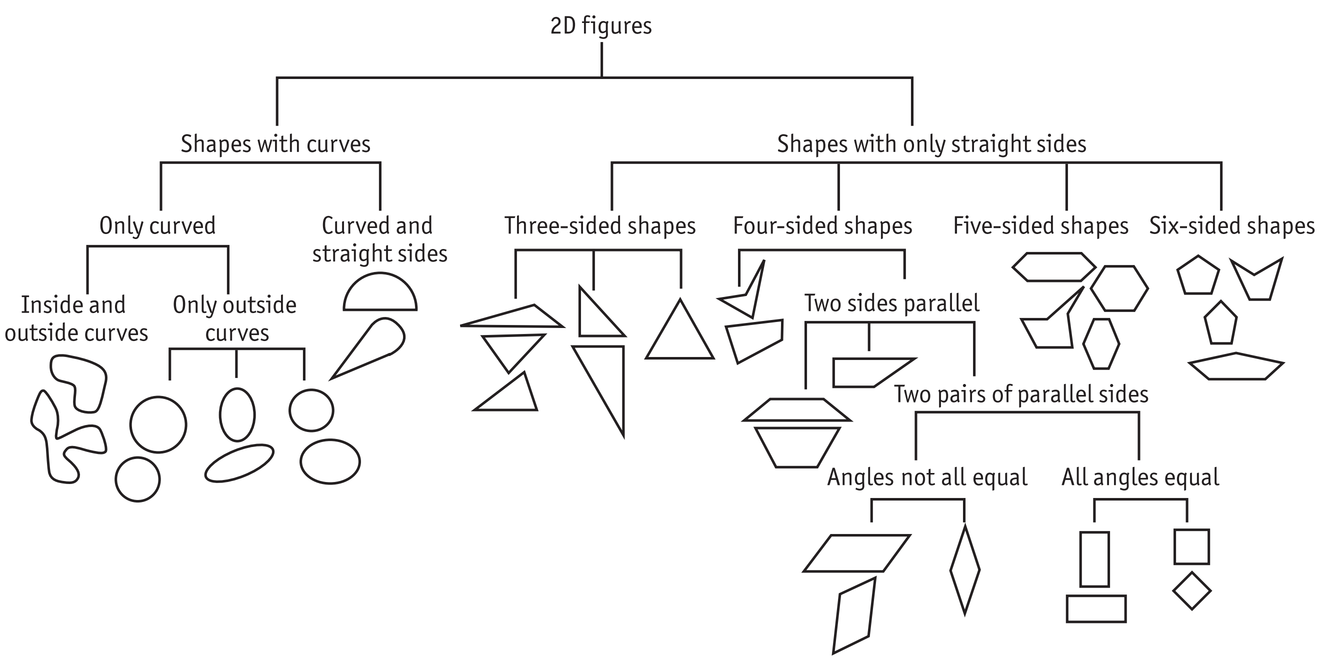
Invite students to suggest a way of sorting a collection of recycled material (e.g. cans, boxes, plastics, bottles) and then say whether they can place all of their materials into the suggested groups. Ask: Do all paper-based products belong in the same pile, or should we have one for newspaper and one for cardboard?

Sorting Objects

Ask students to sort a collection of 3D objects (e.g. cans, boxes, plastics, bottles) into two groups, then resort them into two groups and then resort into two more groups. Ask: How can you sort this collection into just two groups? Could you use a group that is 'not plastic'? Draw out that to sort into 'having a certain characteristic' and '*not* having that characteristic' enables you to sort any set of objects into two groups. (See Sample Lesson 1, page 169.)

Sorting Figures

Extend 'Sorting Objects' by giving students a collection of 2D figures (e.g. squares, rectangles, quadrilaterals, triangles, hexagons, ovals, circles). Ask them to construct a tree diagram to show how figures can belong to different groups. Invite students to draw a picture of each group and write an explanation of what the figures in each group have in common. Encourage them to refer to a mathematics dictionary to find a name for each of their groups (e.g. polygon, quadrilateral, rectangle, square). (Link to Key Understanding 4.)



Sorting by Tessellation

Have students sort a collection of figures (e.g. squares, rectangles, quadrilaterals, triangles, hexagons, ovals, circles) according to whether they will or will not tessellate. Ask: What do you need to check for to see if a shape will tessellate? Just by looking at them, can you sort shapes into three groups, shapes that *must* tessellate, shapes that *might* tessellate, shapes that *cannot* tessellate? What is different about the shapes that cannot tessellate? (Link to Represent Transformation, Key Understanding 2.)

SAMPLE LEARNING ACTIVITIES

Later ✔✔✔

Question Box

Invite students to sort questions from their class question box into categories according to whether they can answer them from their own experience, an information source like a book or a website, or by gathering and producing their own data. Encourage students to write the questions on cards, place the cards onto a Venn diagram of the three options and add to it over time. Ask: Can some questions be answered in more than one way? How do you show that on your diagram? (See ***First Steps in Mathematics: Chance and Data***, Collect and Organise, Key Understanding 3.)

Food Groups

Extend Middle Sample Learning Activity (p. 163) 'Food Groups' by having each student decide on their own food groups and define what things will have in common within each group. Invite them to list everything they eat and drink over a day and sort their list into the groups to categorise what they ate and drank. Ask them to write a definition for their categories and see if a partner can use the definitions to sort the same list in the same way. Ask: Is it easy to sort the foods into these groups, or do you need to define another group? How are these groups similar to (different from) the standard food groups? Which way of grouping is the most helpful in designing a healthy eating plan? (See ***First Steps in Mathematics: Chance and Data***, Collect and Organise, Key Understanding 3.)

Classifying Data

Have students classify data in order to see a pattern or relationship. For example, have each student write on a sticky label their age, gender and how they get to school. Invite them to examine the raw data. Ask: What are the ways you could sort the data? If you sorted according to how you get to school, could you re-sort your data into different groups (e.g. by age)? How? Can you group how you get to school to get a better feel for the data? Do different ways of sorting the data show different things? (Link to ***First Steps in Mathematics: Chance and Data***, Collect and Organise, Key Understanding 3.)

Sorting Cars

Ask students to sort pictures of similar products or objects (e.g. cars, bicycles, chairs, shoes, native animals) and use language to describe the spatial likenesses and differences between them. Ask: What is the difference in shape between a Volkswagen and a Mercedes (a mountain bike and a BMX bike, a kitchen chair and a lounge chair, running shoes and basketball shoes, a kangaroo and a wallaby)? (See Key Understanding 3; ***First Steps in Mathematics: Chance and Data***, Collect and Organise, Key Understanding 3.)

Trapezoids and Triangles

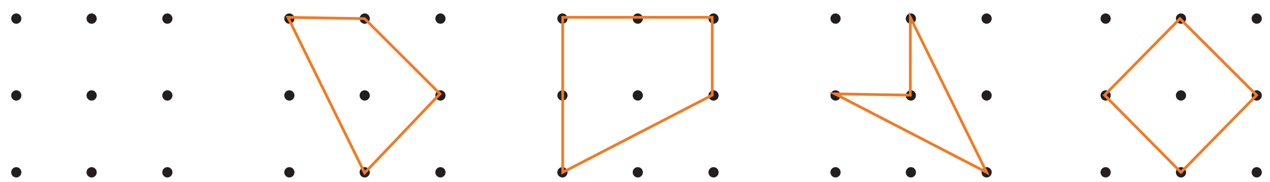
Have students choose a figure or solid and give examples of it and examples that are not it (e.g. figures that are trapezoids, figures that are not trapezoids, objects that are cubes, objects that are not cubes). Invite them to swap with a partner and give more examples in their partner's categories (e.g. figures that are triangles and figures that are not triangles). Ask: What is a trapezoid? What is a triangle?

Venn Diagram

Have students sort patterns according to the transformations involved. For example, draw a large Venn diagram labelled 'translation', 'reflection' and 'rotation' on the floor. Invite students to choose from a class collection of patterned objects (e.g. patchwork, china, wrapping paper, fabrics). Encourage them to identify the basic pattern motif and the transformations used to make the pattern and then sort them accordingly. Ask: Where would you place a pattern or a figure that has translation and reflection symmetry but not rotational symmetry? What do all of the patterns in each section have in common? (Link to Represent Transformation, Key Understanding 4.)

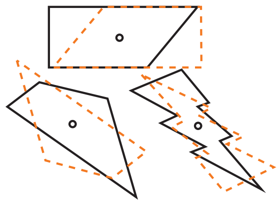
Quadrilaterals

Invite students to draw non-congruent (not the same shape or size) quadrilaterals on 3 x 3 arrays of dots (16 are possible). Ask: What makes them different? How many different squares can you draw? How are they the same? (They all have four sides and four corners.) How are they different? (They vary in size, shape, position within the array and orientation.) Invite students to cut out the figures and sort them according to their own rule. Ask: Do your groups make sorting easy? Do you need to make another group so that all items can be easily sorted?

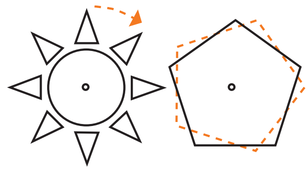


Later ✔✔✔

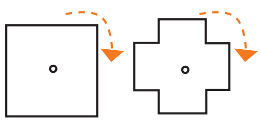
Sorting by Rotational Symmetry



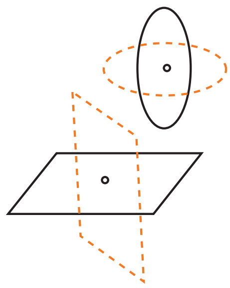
No rotational symmetry



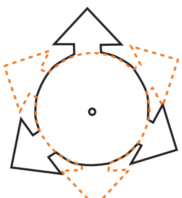
Less than a quarter turn



Quarter, half and three-quarter turn



Half turn only



Between a quarter and a half turn (one-third turn)

Have students sort figures (e.g. squares, rectangles, quadrilaterals, triangles, hexagons, ovals, circles), and, later, objects (e.g. blocks, toys, crockery, tools) into those that have rotational symmetry and those that do not. With the figures that have rotational symmetry, invite students to reclassify them according to how much turn each needs to go through in order to show that it is symmetrical. To find this out, have students trace the shape onto a transparent sheet, put it on top, place a pin or pen in the centre and turn it a quarter turn, a half turn and a three-quarter turn. Ask: Does the symmetry occur at the quarter turn, the half turn, the three-quarter turn, or at more than one position? Do figures within the groups have other characteristics the same? Which groups do all regular shapes fit into? Why? (Link to Represent Transformation, Key Understanding 2.)

Amusement Rides

Show students videos of amusement rides (e.g. merry-go-round, rollercoaster) and identify the sequence of transformations of the cars (seats, horses) during the rides. Invite them to classify the rides according to the transformation in each of the rides. Ask: If you get dizzy by going round and round, which rides could you go on? (See Middle Activity, Represent Transformation, Key Understanding 2.)

Grouping Triangles

Give students a geoboard to make a triangle, then make one that is different in some way and then one that is different to the first two, and so on, using dot paper to record as they go. Ask: How many different triangles can you make? How are they different? Have students make a template of each triangle and decide how they might sort them into groups (e.g. by the number of congruent angles). Have them place the triangles into their groups, compare their groups with others and redefine their groups if necessary to make the sorting easier. Ask them to write a description of what the triangles in each group have in common. Ask: What do all of these triangles in this group have in common? Do you have any triangles that have three different-sized angles? Do you have any triangles that have two angles the same size? Could you use this to group your triangles?

SAMPLE LESSON 1

Sample Learning Activity: Middle—'Sorting Objects', page 165

Key Understanding 1: Things can be the same in some ways and different in other ways. When we classify we sort things into groups that are the same in specified ways.

Teaching Purpose

My Year 3/4 class had recently investigated favourite lunches and had grappled with the issues of organising their data into categories. To develop their classification skills, I planned an activity that would allow them to develop their own sorting criteria based on the spatial characteristics of objects.

Action

I gave about thirty objects to each group of four students and asked them to think about how they were the same and different.

The 3D objects included a variety of shapes and sizes of prisms, pyramids, spheres, cones and cylinders—commercial plastic shapes, cardboard boxes and cylinders, ping pong ball, tennis ball, cans, cake tins and timber off-cuts.

I continued, 'If you had to sort all your objects into two boxes, what labels could you put on the boxes to say, in just a few words, what was in each box?'

'We could have "plastic things" in one box and "cardboard things" in the other,' suggested Toni.

'That wouldn't work,' commented Justin, 'you'd need another box for the wooden things.'

'Everything has to go into just two boxes,' I said, 'so we might have to sort them in a different way.'

'You could say "plastic things" on one box and "wooden and cardboard things" on the other,' suggested Justin.

'But you'd have nowhere to put the metal cans,' I said, 'they won't belong in either box.'

'We could make it "plastic and metal" and "wood and cardboard",' said Phil.

'I know,' said Bree, 'You could have "plastic things" in one box and "not plastic things" in the other, then everything that's not plastic has a box.'

I knew the idea of sorting by *not* having a characteristic was particularly difficult for young students, so I had deliberately limited the sort to two categories to make it more likely that students would need to deal with this idea.

Students in their groups of four sorted their objects into two categories, choosing their own sorting rule and category labels.

After the groups described their various sorting rules to the whole class, I asked each group to split into two pairs. Each pair took one of the two sets of previously sorted objects and separated them into a further two categories, keeping their sorting rule a secret. When they'd finished, they had to try and guess, just by looking at the piles, the sorting rule (or category labels) the other pair had chosen.

I then challenged each pair to carry out a further sort on each of their two categories, so that each pair of students then had four piles. Again their partner pair had to try and guess the sorting rules of the other pair.

Drawing Out the Mathematics

This process focused the students on the likenesses and differences of the objects and created a need for them to notice and talk about various spatial features, particularly in the final sort because the students had used more obvious distinguishing features such as colour and type of material from which the objects were made in their earlier sorts.

While they were working on the task, I was able to talk to individuals and groups to help focus their attention on finer distinctions among objects that share some common spatial characteristics.

'We can't sort this group, they're all triangles,' said Emma.

'What do you mean, "all triangles"?' I asked, to encourage Emma to refine her language.

'Well, that one has got triangle sides and so has that one,' she explained, as she pointed to a triangular prism and then to a square pyramid.

'Can you tell me how many triangular faces each object has?' I asked.

Emma and her partner decided the prism had two and the pyramid had four triangular faces.

'Can you see another difference between the objects?' I prompted.

'This one has a pointy bit sticking up and the other one doesn't,' noticed Emma.

'That's right,' I added, 'the four triangular faces of the pyramid all meet at a point at the top. Do the triangular faces of the prism meet at a point too?'

'No, it's got one triangle at this end and the other triangle at that end. It hasn't got a pointy bit at all, so they are different.'

'So, look at these,' said Vivian, pointing to an ice-cream cone and a can of soup. 'They're different 'cause the cone has one circle and the can has two circles.'

'Yes,' I agreed, 'when we think about shapes we can think about what they look like. But we can also think about what they can do.'

'Like the can of soup will roll but the ice-cream cone won't?' queried Vivian.

'Yes, it will roll,' argued Emma, 'but just around in a circle.'

'Well, that is what we'll say,' decided Vivian, 'We'll have one that rolls in a line and one that rolls in a circle.'

Reflection

I asked the students to put their 3D objects back into one large collection again and do another sort, this time using different categories. I was pleased to observe that more used spatial characteristics in this sort and were attempting to use spatial language to describe features they were now noticing.

KEY UNDERSTANDING 2

Thinking about shape can help us to understand the way things work and fit together

Both living and non-living things have characteristic shapes and structures that often relate to their function or use. Furthermore, whether for practical, religious or decorative reasons, people use shapes and transformations of shapes as the basis for design. During the primary years of schooling, the emphasis should be on exploration, both free and structured, of the students' local environment and objects within it. The essence of this Key Understanding is that students, building on this exploration and investigation should begin to *reason* about and explain the function of shape and transformation in their environment.

Most young students come to school with a practical understanding of shapes and their transformations. They stack the plates, play on swings, seesaws and slides and build with construction material. They should learn to focus upon the shape of things and why and how they are used in the way they are. Students may consider, for example, what it is about the sphere that makes it good for ball games, and why plates are usually round, although they need not be, and why people may find certain shapes pleasing and others not. In making such generalisations, students should link shape to function by considering how the shapes in natural and made objects relate to the way the objects were formed or are used (e.g. *Why not use squares for wheels? How can we make this stronger?*).

Physical structures, whether a body, a plant, a machine or a building, are complex systems that are, at the one time, both a whole and a composition of parts arranged together in some way with each part playing its role within the whole. Students should learn to analyse structures into their component parts, seeing how the parts function to support the whole. They should think about why animals and plants have evolved into their characteristic shapes and how simple machines are composed. They should use mathematics to assist them in cross-curriculum activities that require them to design practical and aesthetic objects, which may range from patchwork quilts to toys to stage lighting.

Progressing Through Key Understanding 2

Initially, students start to see a relationship between the shape of things and the purpose to which they are put. For example, they will choose 'round' (that is, circular) things for wheels because the wheels they have seen are that shape, but they also will reject shapes that have corners as not useful for a wheel and try to explain why, perhaps by talking about it 'being bumpy'.

As students continue to progress they are able to make things that meet simple requirements relating to shape and structure. For example, they can select one type of building material and use it repeatedly to build a wall that will remain standing when they remove and replace a 'door' piece.

Next, students can link the shape and structure of familiar things to their production and uses. For example, they might compare milk bottles and milk cartons and conjecture that differences in their shapes relate to the materials used to make them. As students progress further they continue to do this but their spatial knowledge has developed. They link features of structures such as flexible or rigid, fragile or strong to their shape and explain why they think certain shapes will predominate in situations that they have not yet investigated, such as a storage locker used by the gardener.

SAMPLE LEARNING ACTIVITIES

Beginning ✔✔

Ice-Cream Cones

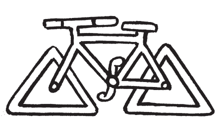
Have students discuss the features of everyday objects that make them useful. For example, show them an ice-cream cone and ask them to think about what makes it useful for holding ice-cream. Invite them to use a ball and a cone and several different prisms (e.g. cube, rectangular prism, cylinder) to work out why the cone is a useful shape. Ask: Why are some ice-cream cones made with a flat bottom?

Rolling

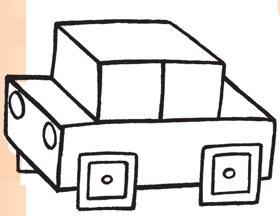
Have students use flexible materials (e.g. paper, plastic straws, pipe cleaners) and non-flexible materials (e.g. popsticks, toothpicks, heavy card) to construct an object, given specified criteria. For example, ask them to make something that will roll. Encourage each student to select suitable material and construct a shape they think will roll. Ask: Why does your shape do that? What other shapes could do that also? What other materials could you use to make that shape?

Containers and Lids

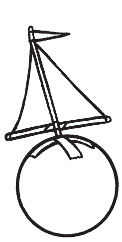
Invite one student to choose a container from a collection, keep it hidden and describe the shape of the lid it needs. Encourage another student to choose an appropriate lid from the collection and test to see if it fits. Ask: Why did you choose that lid? Ask students to describe the movement needed to fasten the lid (e.g. turn around, slide on). Ask: What do you need to do to the lid to get it off the container? How do you move it?



Mixed-Up World



Tell students stories about a mixed-up world where everything is made of unusual shapes. Describe square-wheeled cars that nobody wants, tissue boxes and boats shaped like balls, triangular bicycle wheels, and so on. Invite the students to contribute and weave their responses and suggestions into the stories. Encourage them to make a model of one of these things made of unusual shapes and explain why the shape is not functional. Ask: Which parts of the shape make it not useful for that purpose?



Embedded Shapes

Show students large pictures of vehicles or buildings and name the shapes they see embedded in the picture. Place a large sheet of clear plastic over the picture for students to trace over parts of the vehicle or building to highlight the shape (e.g. a circle over a wheel, a rectangle over a door). Ask: What is the shape? Why has that shaped part been used? (Link to Represent Shape, Key Understanding 1.)

This Reminds Me of ...

Invite students to choose a figure from a collection including ovals, crescents and different types of triangles and rectangles and ask them to say what it reminds them of. Model this for students. Say: This triangle reminds me of part of the roof of a house. This crescent reminds me of the moon. Ask: If you hold it pointing down, what does it remind you of? Why does this figure remind you of part of a train? Have each student glue the shape to a blank page and incorporate it into a drawing.

Friezes

Show students geometrically designed friezes from different architectural styles and ask them to talk about which they think look good and why. Focus on the different shapes they can see in the designs. Invite them to design their own friezes, choosing the shapes they particularly like. Ask: What do you think it is about that shape or pattern of shapes that makes it look good to you?

Models

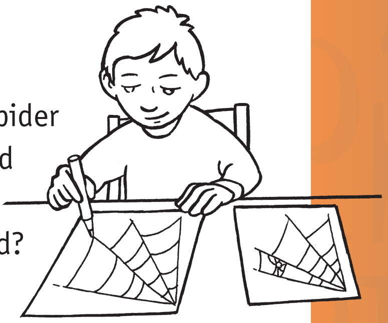
Invite students to select the objects they will need to construct various models (e.g. several different-shaped pieces of wood off-cuts to make a train). Ask them about the reasons for their choices. For example, ask: What made you decide to choose the lids for your wheels? Where will they go on your model?

Model-Making Categories

Invite students to sort through containers and separate the objects into various model-making categories. Encourage them to label the groups according to their uses; for example, *These things make good wheels, these make good flowerpots, these make good towers, these make good walls*. Ask: What sort of shapes would make good windows? Would these shapes also make good walls? What could we call this group of things?

Photographs

Have students make a collection of photographs of natural objects (e.g. spider webs, honeycombs, shells, leaves, flowers). Invite them to make a detailed drawing of one of them. Ask: What shapes did you notice? Why do you think that shape has been used? Where else have you seen that shape used?



Dining Tables

Invite students to draw the shape of their dining or kitchen table at home. Ask: Why do some people choose to use a round table? Discuss the shape of different tables and the best uses for the different shaped tables. Ask: Does anyone have a round table in a round room at home? Why not? How do you fit visitors around your table? Does the shape of the table make a difference to how many people can fit around it? What else affects how many people can sit around a table?

SAMPLE LEARNING ACTIVITIES

Middle ✔✔✔

Explaining Shapes

Supply students with objects and invite them to suggest why they are shaped the way they are (e.g. a baby's dummy is moulded to fit the mouth; a plate is created round on the potter's wheel; a leaf shape protects the roots or directs water to the roots). Ask: What is the object used for? Where is it used? How is it used? What other shape could it be?

Circles

Have students find examples of manufactured circles and construct a chart to show this shape in its environment. For example, one picture may show a car, with the tyre, hub cap, steering wheel and headlights highlighted. Encourage students to think about the function of each part and decide why a circle is the best shape. Ask: Why is a circle the best shape to use for all of these things? Would an oval work instead? Why? Why not?

Cylindrical Shapes

Have students investigate cylindrical shapes to find why they are commonly used for packaging liquids, such as soft-drinks. Ask: Why are cylindrical-shaped bottles sometimes used for milk? Why is milk also packaged in cartons, and some fruit drinks in rounded rectangular bottles? Draw out that these rectangular shapes are for easy packing on supermarket shelves but cylinders are stronger and use less material for the amount they contain.

Triangles 1

Invite students to use straws or similar construction materials to make a triangle, a square, a pentagon and a hexagon. Encourage them to compare each of the shapes for rigidity by holding one of the sides and pushing one of the corners. Ask: Which shape is the most rigid? Why? How can the other shapes be made rigid? Where have triangles been used in the local environment? (bridges, sheds, roofs, towers) Was each triangle necessary to provide rigidity to the structure? Invite students to build a bridge or tower using triangles and test for rigidity.

Triangles 2

Extend Triangles 1 above. Have students investigate structures through history that have been used to support weight. What shapes have been used? Why?

Space Shapes

Have students draw the shape of the space needed for a variety of everyday movements (e.g. swinging on a swing, riding a seesaw, skipping with a skipping rope, swinging a t-ball bat or a tennis racquet, the movement of a windscreen wiper, the amount of grass a tethered goat could graze). Ask: What if the goat was tied to the corner of a building? What if the batter was not allowed to move his or her feet? What if the skipping rope was longer? What is it that all of these shapes have in common? (curved or circular shape) How is it helpful to know the amount of space needed for everyday movements? Draw out that considering the amount of space needed can help us to allow space for these things to move safely.

Sports Fields

Invite students to draw the layout of different sports fields (e.g. baseball, football, tennis, basketball). Ask: Why are these shapes used? Why do basketball and netball courts both have a semicircle around the goal? (The points around the goal are all equidistant.) Why not a square or any other shape? Do any other games use a semicircle around the goal? Are any other shapes used around the goal? Why isn't a baseball track round? (The shortest distance between two points is a straight line.)

Packaging

Invite students to design and make alternative forms of packaging for a breakfast cereal that will appeal to consumers and be easy to stack on supermarket shelves. Ask: What type of shapes would make it easy to stack? Which shapes would be more difficult to stack? What shapes are most commonly used for breakfast cereal? Could you modify this shape to make it more interesting but still easy to stack? (Link to Represent Transformation, Key Understanding 2.)

Bricks

Give students rectangular blocks and invite them to explore the different ways that bricks can be used to construct walls. Ask: How does the pattern affect the stability of the wall? Give them cubes to construct a wall. Ask: Is the cube wall as strong as the rectangular block wall? Why? Why not?

Homes

Help students find pictures of past and present homes from different countries and cultures. Discuss the reasons for the choice of the basic shape of the structure and what the advantages of that structure might be. Ask: Why might they use very steep roofs in alpine countries? Why are terrace houses common in the United Kingdom? Why are verandahs common in outback Australia?

Middle ✔✔✔

Decorations and Designs

Give students a range of geometrically based traditional decorations or designs from various cultures or religions (e.g. Roman, Islamic, Indigenous Australian, Celtic, Christian) and look for recurring shapes within and between the various groups. Talk about why particular motifs might be chosen and which particular shapes and arrangements are pleasing to the eye. Ask: Which shapes are used most often in each of these designs? If you were to make a design to look like a traditional Islamic design, what figures would you use and how might you arrange them? Why?

Container with Lid

Invite students to design and make a container with a lid for a purpose of their choice (e.g. pencil container, box to hold a gift). Encourage them to then draw a plan that enables another to make the same container. Ask: What shapes are shown on the plan? How would changing the shapes on the plan change the shape of the container?

Patchwork Quilt

Invite students to choose three different 2D figures from a range of cardboard templates and use the figures to design a patchwork quilt. Ask: Will your shapes fit together to make a design or will they need to be altered to fit? What changes will you need to make to get them to fit together? Are they still the same three shapes? Is the triangle still a triangle (the square still a square)? (Link to Represent Transformation, Key Understanding 2.)

Mosaics

Give students examples of tiling patterns used on bathroom floors or in mosaics. Ask: What is it about the shapes that means they can fit together? Compare designs that use one shape with designs that use more than one shape. Ask: Why do you think the designer uses different shapes? (Link to Represent Transformation, Key Understanding 2.)

Animals and Plants

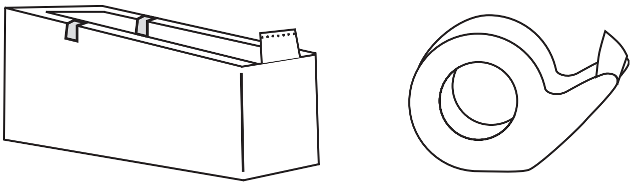
Invite students to find examples of animals and plants where shape plays an important role in their adaptation to the environment or performs a special function. For example, compare the surface area of leaves from desert plants and plants that grow in temperate regions; the beaks of eagles and honeyeaters; the feet of different types of birds; the teeth of carnivores and the teeth of herbivores. Ask: What is the difference in the shape of the feet of different species of birds? Why might water birds have webbed feet?

SAMPLE LEARNING ACTIVITIES

Later ✔✔✔

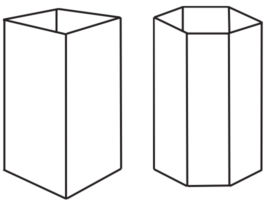
Sticky Tape Dispensers

Have students consider the shape of a variety of different sticky tape dispensers. Ask: How are they the same? How are they different? What parts of the dispenser are essential? Which are not? Which parts could you change without changing its function? Invite students to design a new sticky tape dispenser and develop a marketing plan that tells why the new shape is better than the old one.



Honeycomb

Invite students to identify the shapes within the structure of honeycomb. Ask them to make a hexagonal prism and then a square prism from the same sized rectangle of card. Encourage them to use rice to compare the volume of the two containers. Ask: Which container holds the greatest volume? Which container is the most rigid? Which was the easiest to make? What other examples of hexagons in nature can you think of? (e.g. cracked mud, snowflakes, crystals) What property of the hexagon makes it useful in each situation? (They fit together, or tessellate, and they have a smaller perimeter than other shapes such as triangles.)

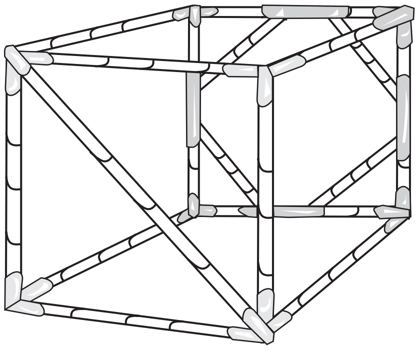


Bee Debate

Extend 'Honeycomb' by having students carry out a debate on behalf of three bees. One bee wants to stay with hexagon-shaped cells, one wants square cells and one wants circular cells. Have them research their arguments for each case and present as a debate. Ask: What is 'your' honeycomb going to be used for? What characteristics does good honeycomb need to have? How well does your shape suit the purpose of honeycomb?

Skeleton Cube

Have students use rolled newspaper and masking tape to build a skeleton cube the height of a desk. Ask: Would this support your weight? How can you modify the cube to make it stronger? (Add struts to form triangles.) What it is about the new structure that makes it stronger?



Later ✔✔✔

Everyday Objects

Invite students to give reasons for the shapes of everyday objects (e.g. playing cards, sink plugs, margarine containers, dinner plates). Ask: What would happen if plugs were square? What if playing cards were round? Why do you think particular shapes are chosen? What else would have to change if the shape of a plug was square?

Why This Shape?

Extend 'Everyday Objects' by exploring the effect of having a neck on a container of liquid? Invite students to investigate a range of containers for pouring (e.g. teapot, milk carton spout, soft drink bottle, jug, eyedropper) and consider how the shape affects its pouring function. Ask: Why is the spout of a teapot much smaller than the neck of a juice bottle? How does the function of the container affect the choice of the opening?

Sports Fields

Invite students to use their knowledge of properties of shapes to solve problems. For example, ask: What would be the effect of playing basketball on a triangular court? (A rectangular court suits having a goal at each end; a triangular court doesn't.) How does the shape of a tennis court affect the play? What about the shape of a baseball diamond?

Puppet Theatre

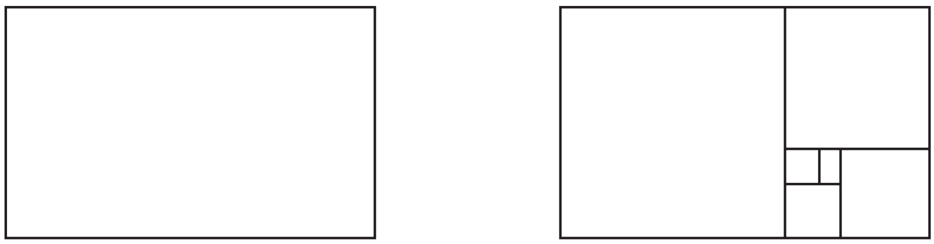
Invite students to design practical and recreational objects such as a milk carton raft or a puppet theatre and puppets from recycled materials (e.g. paper, cans, boxes, plastics, scraps of felt, scraps of fabric). Ask: Why did you use felt for the trees? Why did you use that box for the bed? How did you try to get perspective into your stage? In what way did your recycled material determine the shape and structure of the theatre? How did the design of your theatre affect your choice of puppets and props?

Architecture

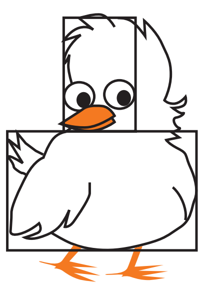
Compare architectural designs from different historic periods (e.g. the Louvre and the pyramid outside the Louvre; Federation houses and Victorian terraces) and decide which shapes relate to structure and which to visual appeal. Explore the idea that architects sometimes emphasise structural shapes in ways to add to visual appeal (e.g. the Sydney Opera House). At other times they cover up the shapes with decorations of different shapes (e.g. the cladding on skyscrapers). Give students reference material or access to the internet and encourage them to find examples of both approaches in buildings.

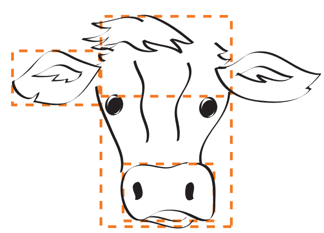
Golden Rectangle

Invite students to investigate the history and origins of the 'golden rectangle', using reference sources such as the internet, encyclopedias or texts such as *The Joy of Mathematics* by Theoni Pappas. Identify golden rectangles in built environments (e.g. the Parthenon) and natural environments (e.g. cow) and in art (e.g. *Bathers* by George Seurat). Discuss why this shape is so pleasing. Encourage students to look for other things in their environment that look good to them and see if there is a golden rectangle within the object or picture. Ask: What are you looking for within this? To check, divide the length by the width. If the answer is 1.618 (or close) it is a golden rectangle. (See ***First Steps in Mathematics: Number***, Reason About Number Patterns, Key Understanding 2.)



The length of a golden rectangle is approximately 1.618 times as long as its width. If you cut a square from one end, you get another golden rectangle. You can keep doing this and you will always end up with a golden rectangle.





Imaginary World

Invite students to imagine being in an imaginary world where there are no parallel lines or parallel planes. Encourage them to make models of everyday objects from that land. Ask: What problems might you face in this world?

Groups of People

Invite students to look for shapes in famous paintings of groups of people (e.g. Rembrandt, Rubens, Da Vinci). Ask: Which paintings feature triangular shapes? Can you find rectangular or circular groupings? Why do you think triangular groupings might be used? Invite students to experiment with drawings or photography to set up different-shaped arrangements. Ask: What is it about the shape that makes some groupings look more attractive or interesting ?

Tessellating Patterns

Invite students to design tessellating patterns using combinations of different geometrically shaped templates. Have students cut sufficient shapes from rolled-out clay to make a trivet or picture using this design, and then, after baking and glazing, have them assemble it. Ask: Which combinations of figures were most commonly used? What ways could they be used? Why are some figures impossible to combine?

KEY UNDERSTANDING 3

There are special words, phrases and symbols that help us to think about and describe the shape and structure of things.

In order for students to learn to reason about shape and structure and hence to solve spatial problems, they need to learn to 'notice' shape and think of it as something significant (as developed in Key Understanding 1 and Key Understanding 2) and have appropriate vocabulary and notations for describing it. Noticing and describing interact with and support each other. Having ways of talking about features of things often helps us to focus on them and remember them, but noticing things also causes us to seek ways of talking about them. As students analyse likenesses and differences (Key Understanding 1) and investigate shapes and structures (Key Understanding 2) they should develop the special words, phrases and symbols needed to support their understanding. The focus of this Key Understanding is on the use of spatial language in a descriptive way. The names of classes of standard shapes (triangles, squares, polygons, prisms) are often used in a descriptive way also and, in practical terms, will be developed alongside and with other spatial language. Key Understanding 4, however, addresses these classes more explicitly.

Students should learn to use a wide range of spatial words in their own descriptions and explanations—'round', 'bent', 'straight', 'curved', 'pointy', 'flat', 'intersection', 'perpendicular', 'circular', 'triangular', and so on. Young students may use an informal description for a shape, e.g. 'round' for a circle or 'the pointy one' for a triangle, showing they are focused on the shape. Alternatively, they may use their local community language. If they consistently use the same word to describe a particular shape or spatial property, then we can assume they are noticing or attending to that shape or property. The shift to conventional terms should occur as they hear them used in context by others. If students do not have a name for a particular shape or property, then it may be that they do not think of that shape or property as an important feature of the things they observe. It would then be helpful for teachers to draw students' attention to these features by using appropriate language in context.

Students should then learn to focus on the spatial features of shapes and describe likenesses and differences between them. They need many opportunities to observe, handle and manipulate figures and objects, including when they are embedded in more complex objects and arrangements. Without being unduly technical, teachers should use correct spatial language repeatedly in contexts that make the meanings clear. Students should then be assisted and increasingly expected to use the language for themselves, practising describing the spatial features of objects and surroundings in situations where there is some purpose in doing so. In the later primary years, some of the conventional geometric symbols for representing spatial ideas (such as for angles) should be introduced. The difficulty students have in learning to use the words, phrases and symbols of shape should not be underestimated. Many who are able to respond to their teachers' use of spatial language, cannot use it for themselves, even with help.

Progressing Through Key Understanding 3

Initially, students interpret and begin to use the everyday spatial language of their communities. Their informal descriptions show they are focused on features of shapes, but they may use words like 'round' for a circle or 'the pointy one' for a triange. As students continue to progress they can talk about shape as well as colour or size, when asked to describe the shape of something such as a flower, animal or building.

Next, students use common spatial language, both oral and written, to describe and compare things they can see and handle. For example, they may inspect, then describe and compare typical houses from different climates (using appropriate spatial language in reporting on the suitability of the shape for its context), or a triangular pyramid and prism (reporting on the number, shape and position of the faces).

As students progress further they can identify or make geometric shapes given a description that uses conventional spatial language and they can describe figures so that peers can select or make them. Later, they describe geometric features of a collection of shapes, incorporating terms such as 'perpendicular' and 'parallel' in their descriptions, saying, for example, *On each of these prisms, the number of edges was three times the number on one of the parallel faces.*

SAMPLE LEARNING ACTIVITIES

Beginning ✔✔

Constructing a Vehicle

Share your thinking with students as you construct a vehicle using 3D objects (e.g. packets, boxes, cardboard cylinders). Describe the type of parts needed as you go; for example, say: I need something to be the cabin of the car. It needs to be a box shape, big enough for two people to sit in. What could I use? Now I need something on the back to carry the hay. It needs to be flat. What would be best to use? I need something round for the wheels so that it can roll along the road.

Trains

Invite students to take turns to line up objects (e.g. packets, boxes, cardboard cylinders) to form a train. Have them match each object to a part of the previous object and describe the part of the object they have chosen to match. For example, *I can match that flat, round bit with this round part of my container. I can match your tall, wide box with my short, wide box.* Encourage them to continue the process until all of the objects are in line. Ask students to recall why two objects were placed together. Ask: Why was this cylinder placed next to this paper plate? (See Key Understanding 1.)

Models

Give students objects to handle (e.g. model vehicles, houses, cages, animals, telephones) and ask them to say what shaped packages and containers could be used to make a model of the object. Focus students on parts like the neck of a giraffe and ask: What sort of shape would a package need to be to make this part of the giraffe? What do you call that shape? What else could you call it? (Link to Represent Shape, Key Understanding 1.)

Spatial Language

Have students follow classroom instructions and answer questions that include spatial language. For example, say: Take the materials from the container shaped like a cylinder. Use a long pathway to return to your seat. Which container do you mean? The one with the square base or the base shaped like a circle? Who owns this lunch box with rectangles on the sides?

The Clue Game

Invite students to take turns to describe the position of a toy that is part of an arrangement on a large tray, by giving one clue at a time. For example, *My toy is near the tractor and the cow and it is above the shed.* Encourage others to work out which toy is being described. Ask: Is it between the sheep and the shed? Is it behind or in front of the shed?

Writing Directions

Ask students to draw a map to show where they played at recess and then write directions to say how to reach their spot, beginning at the classroom. Support them in using terms showing direction (e.g. turn, straight ahead, past, left, right, east, west). Ask: How could someone reach that spot starting at the office?

Moving Parts

Invite students to use construction materials (e.g. Lego®) to create objects with moving parts (e.g. the wheels on a car, a snow mobile with skis, a boat with oars). Encourage students to describe how each part moves (the transformation) (e.g. the ski slides, the wheels turn, the oars turn) before demonstrating how it moves. Ask: How does that part move? Does it turn? Does it slide along or does it flip over each time?

Over the Telephone

Ask students to pretend to be on the telephone and describe an object (e.g. a piece of furniture, a part on a car, an egg cup) to the person they are speaking to. For example, *Can you get this out of the drawer in the kitchen? It is silver. It is shaped like a half of a small hollow ball and it sits on a cone shape.* Ask: What shapes can you see in your object? How are they joined together? What is your object used for?

What Shape Am I?

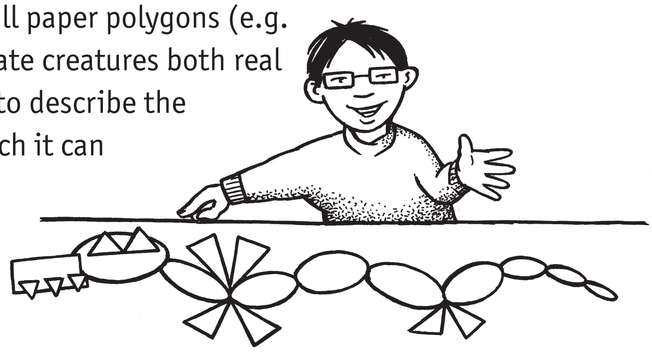
Have students view large pictures of vehicles or buildings and name the shapes that they see embedded in the picture. Place a large sheet of clear plastic over the picture and ask them to trace over the shapes to show what the shape is (e.g. a circle over a wheel, a rectangle over a door) and name it. Invite them to play 'What shape am I?', taking turns to describe the features of the shape for others to identify in the picture. (Link to Represent Shape, Key Understanding 1.)

Shapes in the Environment

Encourage students to look around their environment and observe and describe different shapes. Ask: What shape is the lamppost? What other shape could a lamppost be? What shapes are the playing fields?

Polygon Creatures

Have students arrange a collection of small paper polygons (e.g. ovals, circles, rhombuses, squares) to create creatures both real and imaginary. Invite them to take turns to describe the creature, using spatial language from which it can be identified by others. For example, *My creature has two circle heads and a triangle body with six long thin rectangles for arms.*



Beginning ✔✔

Pick an Object

Invite students to describe an object out of a collection spread on the floor in order for another student to identify it. Encourage them to use shape words, or location words, but not the colour or purpose of the object. Ask: There is more than one box shape; can you say where it is to help us decide which box it is?

Feely Bag

Invite students to describe the shape of an everyday object (e.g. box, foam cube, cylinder, cone, packet of noodles) placed into a feely bag for others to guess what it is. List all the words used and encourage students to decide which of these describe shape and which don't. Support students to use more mathematical language. Ask: Is there another word we could use instead of round? (circle) Can you tell us more about the soft spongy shape?

Riddles

Invite students to write 'What Am I?' riddles using everyday words to describe shape. For example, *I am long and thin, with a pointy part at one end. I am used in the classroom every day. What am I?* (pencil) (See Middle Sample Learning Activity (p. 189) 'Riddles'.)

SAMPLE LEARNING ACTIVITIES

Middle ✔✔

Guess My Shape 1

Have one student select a 2D figure or 3D object and answer only yes or no to questions asked by others about its characteristics. For example, *Does it have a circle? Has it got three corners? Is it 2D?* Gradually reduce the number of questions that can be asked before the answer is given. List all the questions that were asked and ask: Which ones really helped you pick out the figure or object? Which questions didn't really help? Which questions do you think you would start with next time?

Guess My Shape 2

Extend 'Guess My Shape 1' by having students describe a 2D figure or 3D object, having felt it but not seen it. Encourage others to draw a diagram of the figure or object using the information provided until one student is able to identify and name it. Check in the bag to see if they are correct. Ask: Does feeling the shape help? Why? How?

Guess My Shape 3

Extend 'Guess My Shape 2' by organising students into pairs and having one student place a 3D object inside a cardboard box. Invite their partner to shake the box and ask questions to work out what the shape is. Record the questions asked and the responses given. Ask: Which shapes were easy to guess by using the sound? What was it about the shape that made different sounds? Which questions were the most useful? Why?

Barrier Game

Invite students to use tangrams or other 2D figures to make a picture or design and give instructions to their partner to make the design using the same pieces. Encourage them to compare the new picture with the original to see if they are the same. Have students share the instructions that were helpful and those that were confusing. (Link to Represent Transformation, Key Understanding 2.)

Back to Back

Have pairs of students sit back to back. Invite one student to describe a line drawing or design for the other to draw without saying the name of the objects in it. When the drawing is complete, ask them to compare the two pictures and decide which instructions were most helpful. Ask: How can you make sure that the person knows how big to make each shape? How can you accurately describe where the shape should be placed?

Middle ✔✔

Favourite Vase

Show students a picture of a vase and say: You broke your grandmother's favourite vase and need to replace it. Have students take turns to describe the vase to a shop assistant over the telephone to see if the shop has one exactly the same. Write a class list of all of the descriptive words used. Ask: Which words describe the shape rather than its colour or pattern?

Describe a Pattern

Provide students with a drawing of a simple object or arrangement (e.g. a nut and leaf arrangement) and a variety of transformations of it. Invite them to describe to each other what has been done to the original and try to recreate the transformation. Ask: How did you have to move the drawing to make that pattern? Did you just slide it along? Did you flip the pattern or did you turn it in some way? Was there more than one way you could move the drawing to make the pattern? (Link to Represent Transformation, Key Understandings 2 and 4.)

Blindfold

Organise students into pairs and have them give their blindfolded partners directions for moving from their desk to a given place in the room. Invite them to reverse roles. Ask: Which parts of the instructions were the most help?

What Shape Am I?

Show students large complex pictures (e.g. a city scene, a view of the main street of a small town, a photo of a classroom taken from a corner) and ask them to name the shapes that they see embedded in the picture. Place a sheet of clear plastic over the picture or photo and have them trace the shapes. Invite students to play 'What shape am I?' by describing the features of the shape for others to identify in the picture. Draw out that some shapes may be distorted and not appear as they would if you were looking at them front on (e.g. the door may look like a squashed rectangle). Ask: How would you describe a desk from the photo? How do you know that it is a rectangular prism in the photo when it doesn't look like one? (Link to Represent Shape, Key Understanding 3.)

Riddles

Extend Beginning Sample Learning Activity (p. 186) 'Riddles' by having students write 'What Am I?' riddles for 2D figures and 3D objects. For example, *I have four sides but not all of them point straight up. People think I'm squashed but I'm not. What am I?* (parallelogram) (Link to Key Understanding 4.)

Border Patterns

Invite students to create a collection of border patterns using a computer program and then choose one of them to describe to a friend. Ask: Which shape words did you need to use? Which transformation words did you need to use?

Feely Bag

Hide an unfamiliar object like a funnel, hose joiner, drill bit or a wrench in a feely bag and have one student feel and describe the spatial characteristics of the object. For example, *My thing is like two small thin cylinders that make a cross*. Invite students to sketch the object from the description and then compare finished sketches to the object. Ask: Which one most closely matches the object? Which words were the most helpful?

Same Purpose

Invite students to examine a collection of objects that are used for the same purpose (e.g. a milk carton and a milk bottle, collection of different staplers, collection of jugs) and say how the shape of each varies. Ask: Which part of the objects has to stay the same?

SAMPLE LEARNING ACTIVITIES

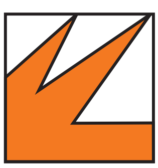
Later ✔✔

Barrier Game

Organise students into pairs and have one student make a construction or pattern behind a screen then give step-by-step instructions to their partner to make the same construction. Invite them to compare the finished models and say how they are the same and different. Encourage them to reverse roles and repeat the process. Support students who are following the instructions to ask questions about the meaning of the instructions. Ask: What instruction can you give rather than 'next to' so that the person knows where to place the object? How can you tell your partner how close one object is to another? How can you explain when an object is to be turned slightly?

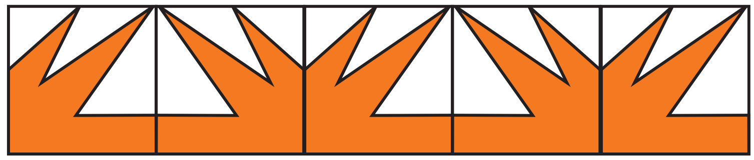
Motif

Have students choose a motif, a simple design or a picture. Invite them to give verbal transformation directions to a partner to follow to sketch or glue copies of the starting shape onto a long piece of card to make a design. For example, *Rotate it 180°, translate it to the right and then trace. Do this four more times.* Ask: Were there some instructions that were confusing? What other information did you need in order to carry out the transformation?



Rotate 180 degrees, translate to the right, repeat four more times.

Reflect sideways and translate to the right, repeat four more times.



Computer Graphics

Have students use computer graphics to print a figure (e.g. polygon) and a variety of enlargements and distortions of the figure. Invite them to examine printouts of others and explain what has been done to the original shape. For example, *This rectangle has been dragged from a corner so the angles have changed size and it looks like a kite.* Ask: Which words have you used that describe the shape? (Link to Represent Transformation, Key Understandings 2 and 4.)

Guess My Shape 1

Extend Middle Sample Learning Activity (p. 187) 'Guess My Shape 1' by having one student select an object and then inviting other students to ask questions requiring a yes or no answer. For example, *Are the sides equal in length? Are the equal sides next to each other? Are they opposite one another? Are any of the sides parallel? Are the diagonals equal?* Encourage students to continue to ask questions until they have the correct shape. List all of the questions that were asked and then ask: Which ones really helped you identify the shape or object? Which questions didn't really help? Which questions do you think you would start with next time? Why?

Lines and Planes

Invite students to find and sketch examples from the environment of parallel, perpendicular and horizontal lines and planes. Encourage them to justify their choice of examples for each category through questions. Ask: What is it about the louvres, and the sides of the drawer that makes them parallel planes? What is it about the edges of a cupboard that makes the planes perpendicular?

Glossary

Have students build up personal and class glossaries of shape and space words. Encourage them to extend their glossaries as they ask questions about geometric figures and objects. Ask, for example: What do you call a squashed square? What do you call shapes when they are exactly the same?

Coloured Straws

Have pairs of students use coloured straws to play a barrier game. First have them construct a screen and then invite the first student to arrange their straws and describe their arrangement for their partner to construct. Encourage them to compare arrangements. Ask: Which words were most helpful (unhelpful) in describing the location and arrangement of the straws?

Chart of Words

Over time, have students build up a chart of words that have both a common and a mathematical usage (e.g. negative, figure, solid, cone). Encourage them to use a mathematics dictionary to find the differences between everyday meanings and mathematical meanings.

Sorting Cars

Ask students to sort pictures of similar products or objects (e.g. cars, bicycles, chairs, shoes, native animals) and describe the spatial features of each group so that a partner can select the objects. Ask: Why couldn't your partner choose between the picture of the Volkswagen and the Mercedes (mountain bike and BMX bike, kitchen chair and lounge chair, running shoes and basketball shoes, kangaroo and wallaby)? How can you focus your description on the shapes and other geometric features of the cars (bikes, chairs, shoes, native animals) to make the differences clearer? (See Key Understanding 1.)

Later ✔✔

Mathematical Language

Present students with sketches of shapes (e.g. prisms, cylinders, cones) along with descriptions using non-mathematical language (e.g. box, tent, tube, diamond, squashed rectangle, bowl shape). Invite students to edit the descriptions by substituting more appropriate spatial terms. For example, *I've sketched this box shape* becomes *I've sketched a rectangular prism*.

Building Barrier Game

Organise students into pairs and have one student build a cube building behind a screen and then describe the different views of their cube building for their partner to interpret and build a replica. Invite them to compare the two buildings and explain any differences in the interpretation of the words used in the description. Ask: Which part of your structure did you find difficult to describe? Are there other more helpful words that we can think of that might describe that more clearly? Which part of your partner's description did you find confusing? (Link to Represent Shape, Key Understanding 1.)

Straw Shapes

Have students distinguish between the words 'regular' (all sides same length and all angles equal) and 'equilateral' (all sides same length) and use symbols to show the difference. For example, give students 25 straws each and say: Use these 25 plastic straws to make a set of polygons by threading elastic through them and tying them together. Your set of polygons has to have the largest possible number of polygons, the straws all have to be the same length and all the polygons have to be different shapes. Ask: Is a square different than a rhombus? How? Draw out that a square is both a regular and an equilateral shape and that the rhombus is an equilateral shape, but may not be regular. (See Sample Lesson 2, opposite.) Encourage students to use mathematical dictionaries to find out how to use symbols to show that all the angles in a figure are the same size and symbols to show that the length of the edges in a figure are the same.

Geometric Symbols

Have students construct a chart showing some of the conventional geometric symbols. Encourage them to research mathematics dictionaries and other sources to find symbols for representing ideas about angles, parallelism and congruent sides in polygons.

SAMPLE LESSON 2

Sample Learning Activity: Later—'Straw Shapes', page 192

Key Understanding 3: There are special words, phrases and symbols that help us to think about and describe the shape and structure of things.

Key Understanding 4: People have developed useful ways to classify shapes. Knowing that a shape is one of the standard types can tell us a lot about it.

Teaching Purpose

My Year 7 class knew the names of polygons, such as pentagon, hexagon, heptagon and octagon, were based on the number of sides, but their image of these figures appeared mainly limited to regular shapes. I thought that they would be better placed to focus on the variation within each type of polygon if I could develop their spatial language and help them think about representing spatial ideas.

Action

I asked the students to watch as I strung three plastic straws of equal length onto a piece of thin hat elastic. As I tied the ends of the elastic together, pulling the straws into a triangle, I posed a problem:

'I am going to give you 25 plastic straws and ask you to make a set of polygons by tying them together with pieces of elastic. There are three conditions: your set of polygons has to have the largest possible number of polygons, the straws all have to be the same length and all the polygons have to be different figures and objects.

Drawing Out the Mathematics

The students worked in pairs and discussed their ideas. It was not long before Candice and Emma began to argue.

'Look,' said Candice, 'this is a square, and this one is a diamond shape.'

'But they both have the same sides,' said Emma. 'If it's got four sides the same it's supposed to be a square. I thought they all had to be different shapes.'

'Are the two polygons the same shape?' I intervened.

The girls agreed they were different shapes, so I continued, 'This one is a square and this one is called a rhombus. What is the same about them?'

'They both have four sides,' answered Candice, 'and the sides are all equal.'

I used the word 'equilateral' in an almost incidental manner here and hoped students would begin to use it for themselves. On a later day, I unpacked the word for them, talking about the 'equi' linking to equal and the 'lateral' meaning 'pertaining to the side'. I linked it to other related spatial words, 'equiangular' and 'equidistant'.

'So, they are both equilateral polygons,' I said. 'How are they different?'

'The corners are different,' observed Emma. 'The square has four right angles and the rhombus has two pointy corners and two flat corners.'

'So, for squares all four sides are all the same length *and* the four angles are the same. But for the other figure, all four sides are the same length but the angles are not all the same.

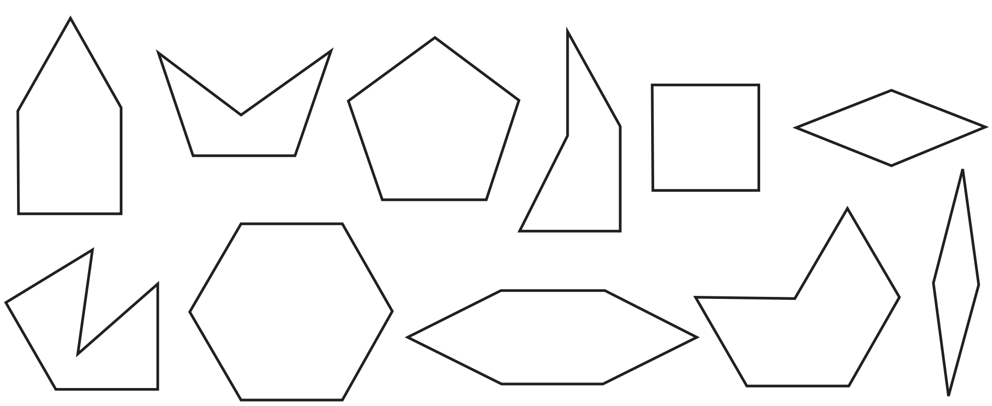
'We have a special word to describe figures that have all their sides the same length and all their angles are the same,' I explained. 'We say they are "regular".'

I asked Candice and Emma to help me explain the meaning of the word 'regular' to the rest of the class. I drew a square and a rhombus on the whiteboard.

Candice explained that both figures had all their sides the same length, but the square also had all of its angles the same size.

After they had finished, I reinforced what they had said by reiterating, 'So, the square is a regular shape because it has all its sides the same size (we say "equilateral") and all its angles the same size (we say "equiangular"). The particular rhombus we have here has all its sides the same size, but not all its angles, so we would not call it "regular".'

I then asked them to experiment with their straws and record the different shapes they could make and to highlight those that were regular.



I drew out that they were able to make many different equilateral pentagons with the straws, but only one was also equiangular and therefore regular. Similarly, they were able to make many different equilateral hexagons, but only one that was regular.

I then suggested, 'Have a look at your triangle and decide whether it is regular. Can you make a triangle that isn't?'

In this case, they found that they could not 'push around' the straws to make other triangles. The only triangle they could make with equal sides also has equal angles, so it is regular.

I was surprised when one student commented that because their straws were
all the same length, every figure they could make had equal length sides. She asked if you could have figures with all their angles the same but not all
their sides the same. I chose to have students return to our original problem, but promised to follow up her question on another day.

Several days later, I used her question to motivate some work on Key Understanding 4. I challenged students to find a quadrilateral that had four
equal angles but did not have four equal sides. They struggled with this for
quite a while until one student suddenly realised that all rectangles had
equal angles (were equiangular) but not all had equal sides. I then developed this further along the lines suggested in the Did You Know? on page 209.

The fact that once you have fixed the sides of a triangle the shape is 'rigid' is an important and useful property of triangles and is why they are used so much in construction (see Key Understanding 2).

The students worked out that they could make six different shapes with the 25 straws. They could have one triangle, three quadrilaterals and two pentagons; or, they could have one triangle, four quadrilaterals and one hexagon.

KEY UNDERSTANDING 4

People have developed useful ways to classify shapes. Knowing that a shape is one of the standard types can tell us a lot about it.

Shape is a property or attribute of things, and there are infinitely variable shapes possible just as there are infinitely variable colours. As for colour, we have developed standard classifications of shapes and given the classes names of their own. This naming helps us to distinguish shapes and remember them. The classes themselves become concepts ('triangle') with properties of their own. Thus a 'triangle' is the bearer of sets of properties. Students will go through a series of phases in developing this understanding.

This is a rectangle because it looks like a rectangle. Students initially identify figures and objects as being one of the conventional kinds by what they look like *as a whole*. They may come to know some properties of a particular figure (a rectangle has four sides) but that is not how they recognise them; a rectangle is a rectangle because it looks like one. Students should be assisted to recognise the wide variation included within such words. They should cut out rectangles, make rectangles with straws and rotate them, so that they are able to recognise and produce rectangles of any size, shape or orientation.

This is a rectangle and it has four sides. If students are to move beyond recognition to understanding that the figures they recognise as being rectangles all have certain properties in common, they need to manipulate (observing, cutting, folding, drawing, measuring and constructing) figures and objects so that the parts making the whole can be focused upon and properties drawn out. Students should develop the understanding needed to confidently assert that rectangles always have four sides and always have right-angled corners but they don't have to rest on a 'flat bottom'—they may even rest on a point.

This is a rectangle because it has four sides, opposite sides the same length and four right angles. It requires a major intellectual shift for students to realise that it is the properties that define the class. Rather than thinking ‘this is a rectangle *and* all rectangles have four sides, opposite sides the same length, all right angles’ they need to understand that ‘this is a rectangle *because* it has four sides, opposite sides the same length, all right angles’ and ‘this is not a rectangle because it does not have all these properties’. While the student will continue to think of a ‘rectangle’ as a whole and recognise it at a glance, properties become the means of convincing oneself and others that particular figures and objects do fit within a particular class.

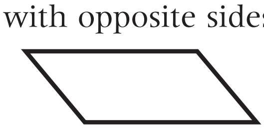
This ***square*** *is also a rectangle because it has four sides, opposite sides the same length and four right angles.* There is, however, more to learn. Students may know that a figure is a rectangle if it has certain properties but not see that properties of figures are related to each other (if a quadrilateral has four right angles it *must* have opposite sides the same length—a rectangle) or that classes of figures are related (all squares *must* also be rectangles). Primary students will benefit from activities focusing on relationships between properties. Only when students understand that properties are related to each other do they understand that knowing just a few properties of a figure or object enables us to work out other properties. And this is the most useful aspect of geometry.

Progressing Through Key Understanding 4

Initially, matching particular shapes exactly poses few difficulties for students, even when this requires them to change the orientation of the shape by turning it around or flipping it over. They recognise shapes that are close to the conventional triangles, squares and circles. As students continue to progress they are more precise in their identification of common figures and reject as a triangle something that has curved edges or edges that do not meet or cross each other. They know the names of some shapes, including ‘circle’, ‘triangle’, ‘rectangle’ and ‘square’, and ‘cube’, ‘cone’, ‘cylinder’, ‘sphere’ and ‘rectangular box’.

Next, students recognise conventional-named geometric concepts, such as triangle or prism, within natural and built environments and pictures, and standard drawings and models of them. As students progress further they can describe the geometric features of a collection of shapes saying ‘On each of these prisms, the total number of edges was three times the number of edges on one end.’

Later, students can make more abstract statements about ‘all rectangles’ or ‘all prisms’. Thus, they will say, ‘All squares are rectangles, but the reverse is not true’, and reject as inadequate a description of a rectangle as ‘a four-sided figure with opposite sides parallel’, offering the following as a counter example.



SAMPLE LEARNING ACTIVITIES

Beginning ✔✔

Scavenger Hunt

Give students as many opportunities as possible to find figures that match. For example, devise clues based on geometric shapes (e.g. *it has four sides all the same length, it has one line that curves and joins*) for games that resemble scavenger hunts. Have students interpret the clues and find the appropriate shapes. Encourage students to discuss their decisions and share with the class, telling what clues helped them decide a figure was or was not a rectangle (circle, triangle, cone, sphere, cylinder).

Shaped Formations

Have students take turns to arrange the class members into given shaped formations (e.g. square, circle, semicircle, triangle) for playing games or observing activities (e.g. visiting pets). Gradually support students to decide where they need to stand in the arrangement to make the shape. Ask: How many people do we need to make the corners of a square? Is there enough room for the rest of the class to stand along each side? What will the students at the corners have to do to make more room along the sides? How can we make all the sides the same length?

Triangle and Square

Place a square and a triangle in a bag and ask a student to select one and tell the class what it is just by the feel. Ask: How did you know it was a triangle (a square)?

Favourite Figures

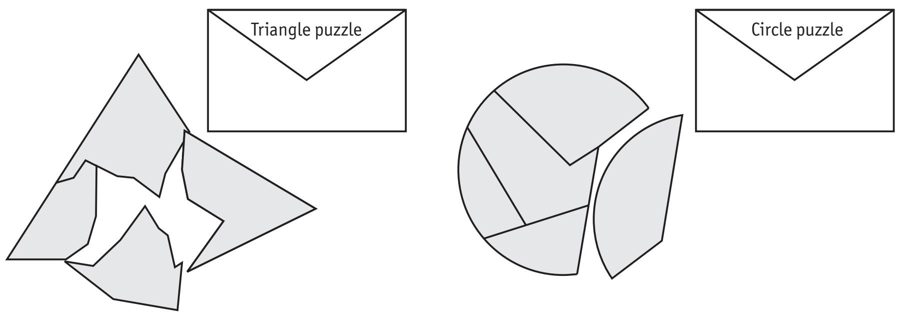
Invite students to make their favourite figures from straws or pipe cleaners. Ask: What is your shape? What do you like about it? How many sides (corners) does your shape have? What would you say a corner is? Say: Take away one side. What changed? What else changed? What happened to the corners? Does this figure have a different name?

Popsticks

Have students make a known polygon using popsticks. Ask: What is your shape called? Why do you think is it called that? Say: Add another side to your shape. Make up a name to describe this shape. Encourage students to continue to add to their shape and each time invent names for their new shape. Ask: How could we find out what these shapes are really called?

Puzzles

Cut geometric shapes (e.g. squares, triangles, circles) into a number of pieces and invite students to reassemble them from the pieces. When students complete a puzzle, have them tell their partners, group or class what clues they used to help decide how the shape should be reassembled. For example, *I knew the triangle had to have three corners, but if I had put it that way, it would have had four. Circles have no corners, so I knew the pointy bits had to fit on the inside.*



Shape Structures

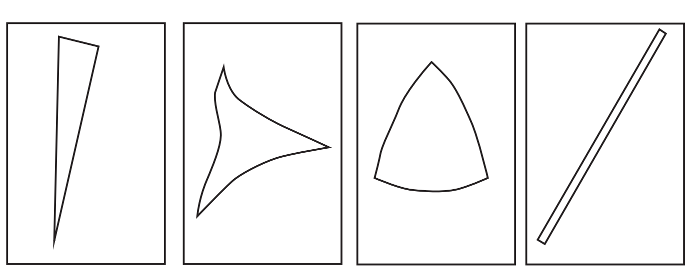
Provide groups of students with about ten to twenty of the same solid (e.g. spheres, cubes, pyramids, cylinders). Invite each group to build a structure made of their particular shape. Encourage them to discuss the suitability of using a different shape to make the same structure. Ask: Could you build such a high tower with spheres? Why? How? What problems did you have building with pyramids? Which shapes do you think were the best for building towers? Why?

Toothpick Shapes

Show students a range of figures including semicircles and hearts and tell them the names of each. Ask: Which of these shapes can be made from toothpicks? How do you know? Use toothpicks to make the shapes. Ask: Why can some shapes be made with straight material and others can't? (See Represent Shape, Key Understanding 1.)

Rectangles and Triangles

Invite students to construct different-shaped rectangles and triangles and pin them onto a chart labelled 'Rectangles', 'Triangles' and 'Not Rectangles or Triangles'. Contribute some uncommon shapes to the chart, including very long thin rectangles, low flat scalene triangles, a triangle orientated on a point and a figure resembling a triangle with curved edges. Invite students to say which category the figures go into and why. (See Sample Lesson 3, page 206.)



Beginning ✔✔

Part of a Figure

Show students a part of a triangle, and say: This figure has a piece cut off it. I wonder what figure it was to start with. Draw what you think the missing part looks like. Ask: Why do you think that? What else could it be? Encourage students to use matchsticks or straws to explain their thinking. Repeat this with other figures.

Paper Squares

Provide students with paper squares and have them cut the figure so it is no longer a square. Ask: What name would describe the figure you have made? How do you know it is not a square any more? Add students' suggestions to a wall glossary of terms.

Rectangles

Invite students to draw pictures of different types of rectangles, including squares, and describe each example in their own words (e.g. *a long thin rectangle, a small square rectangle, a fat rectangle*). Ask: Why do these figures belong to the rectangle family? Show other figures (e.g. a parallelogram, a rhombus) and ask: How could this figure be related to the rectangle family? What is the same? What is different?

SAMPLE LEARNING ACTIVITIES

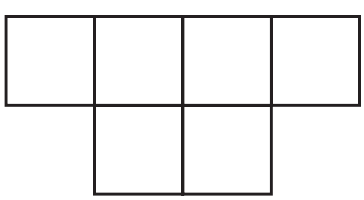
Middle ✔✔✔

Trains

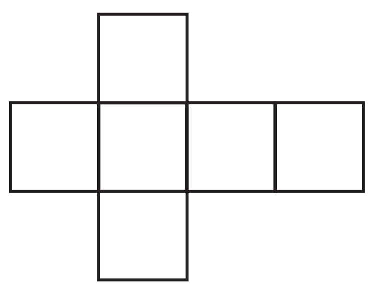
Invite a student to describe the properties of a box and suggest a name for the box based on those properties (e.g. two square faces). Have another student find a box with one matching property and place it end-to-end with the first box. Encourage students to take turns to continue the process until all of the boxes of that type are in line. Ask students to recall why each of the two boxes were placed together. Ask: Which parts of these two boxes match? How is that box different from this one? (See Key Understandings, Beginning Activities.)

Hexominoes

Ask students to decide by looking and visualising whether a hexomino figure (an arrangement of six squares with at least one side of each meeting) could be folded into a cube. Ask: If it won't fold into a cube, how will it need to be changed? Why? (Link to Represent Shape, Key Understanding 2.)



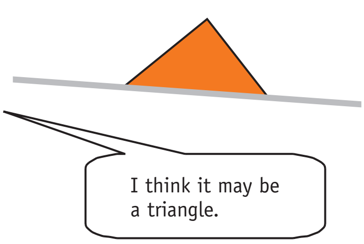
This hexomino cannot fold into a cube.

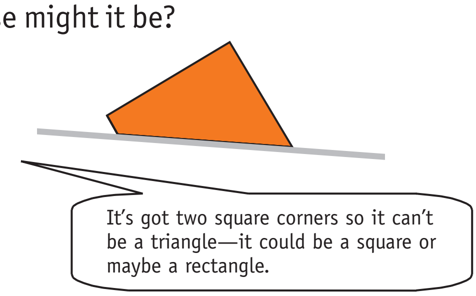


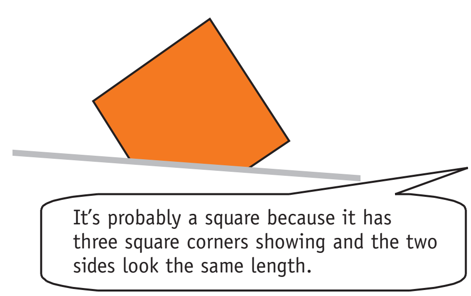
This hexomino can fold into a cube.

Guess My Shape

Have one student hold a figure behind a screen and then slowly move it up for all to see. Encourage the student to stop periodically and invite the other students to say what shape it could be, based on what they can see so far. When they change their minds about which shape it is, ask: Why? What made you decide it could not be a triangle any more? What made you think it might be a square? What else might it be?







Middle ✔✔✔

Choose a Shape

Invite students to choose a 2D figure or 3D object, find out all they can about it and create a chart to record the information about their shape in an interesting way. Display the charts and ask students to say which figures and objects they think go together and why.

Concentration

Invite students to choose a 3D object and make pairs of cards about that object: one card showing a drawing of the object and one showing each of the faces by tracing around them. Organise students into groups and have them combine their cards to play concentration. Each time they match cards, ask them to explain how the faces match the picture of the 3D object. (Link to Represent Shape, Key Understanding 2.)

Fields and Tables

Have students use what they know about properties of shapes to solve problems. For example, say: A farmer has a square field. He measures one side. Ask: How can he use this to know how much fencing to buy? Will he need to measure the other side? What does he need to know about squares? Have students measure one side of a table. Ask: Do you know how long any of the other sides of the table are? What if the table is a square?

Covering Containers

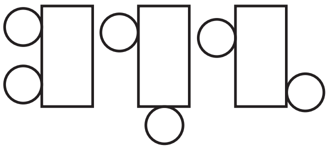
Have students use paper to cover one face of a cube, cardboard box or jar. Ask them to decide on the type and number of shapes they will need to cover the whole object. Have them draw a diagram of the faces of the object and have it verified by a partner before making the shapes and covering the object. (Link to Represent Shape, Key Understanding 2.)

What Am I?

Have students choose a 2D figure or 3D object and write a 'What Am I?' for others to guess the shape. For example, *I have six edges. I have four corners (vertices). I have four flat faces. I am a 3D object. What am I?* Ask: How did you know which shape it was? Which bit of information convinced you? What other things could you say about the shape?

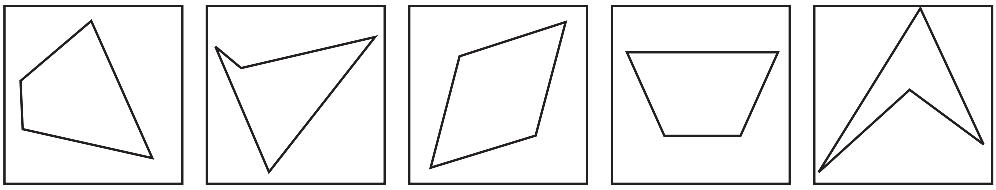
Cylinder Nets

Show students the following nets and ask them to say which of them can be folded to make a cylinder. Encourage them to justify their choices by referring to the properties of a cylinder. Ask: What do you know about a cylinder that helped you to decide it would (wouldn't) work? (Link to Represent Shape, Key Understanding 2.)



Quadrilaterals

Have students draw three different quadrilaterals on separate cards approximately the same size and describe how each is different from the others. Ask: How are they different? What is different about the length of the edges? What is different about the angles? Organise students into groups and invite them to work together to sort their figures. Encourage them to share the different ways they sorted. Ask: How is this group different from that group? Have students label each group with an appropriate name and justify their choices.



Geoboard

Have students work on a geoboard to change an irregular quadrilateral to a trapezoid, a trapezoid to a parallelogram, a parallelogram to a rectangle, a rectangle to a square. Encourage them to say which part they needed to change to make each new shape and explain how this relates to the properties of the shapes.

SAMPLE LEARNING ACTIVITIES

Later ✔✔✔

Sketching Solids

Have students visualise and sketch a solid such as a cylinder. Display all the sketches and list things that are the same and things that are different about the cylinders. Ask: How do we know that an object is a cylinder? Repeat the activity for a pyramid. Ask: What makes a pyramid a pyramid?

Sketching Objects

Have students use standard 3D shapes as a basis for sketching a range of objects. Ask: Why can we represent a candle, mug or a bottle as a cylinder? What objects around you could be represented by a rectangular prism (a cube, a cone)?

What Am I?

Invite students to write 'What Am I?' riddles for 3D objects for others to solve. For example, *I am a polyhedron. I have five faces, and one face is a square. I have five vertices. I have no parallel faces. I have four triangular faces. What am I?* Ask: At what stage were you sure of the shape? What information was not really necessary? (See Key Understanding 3.)

Shape Clues

Uncover a list of increasingly specific clues about a shape one at a time until students work out the shape. For example, say: It has straight sides. All the corners are the same size. All the sides are the same length. It has four sides. At least one angle is 90°. Ask: Would it have been possible to identify the shape from an earlier clue? Why? Why not?

True or False?

Say: When Jim was making up rules to describe a square, he said *Four sides equal so four angles equal. All sides equal means opposite sides equal. Right angles are equal, so that means opposite sides are equal.* Invite students to explain why they think each statement is true or false. Ask: Is the first statement true? Why? Why not?

Practical Problems

Have students use their knowledge of the properties of shapes to solve practical problems. For example, ask: How can the mats for t-ball be correctly placed on the field? How can a circle be drawn on the asphalt for games like dodge ball? How can an accurate rectangle be drawn for games like volleyball? How does knowing the diameter of a tennis ball make it possible to design a cylinder that will hold three tennis balls?

Identifying Prisms

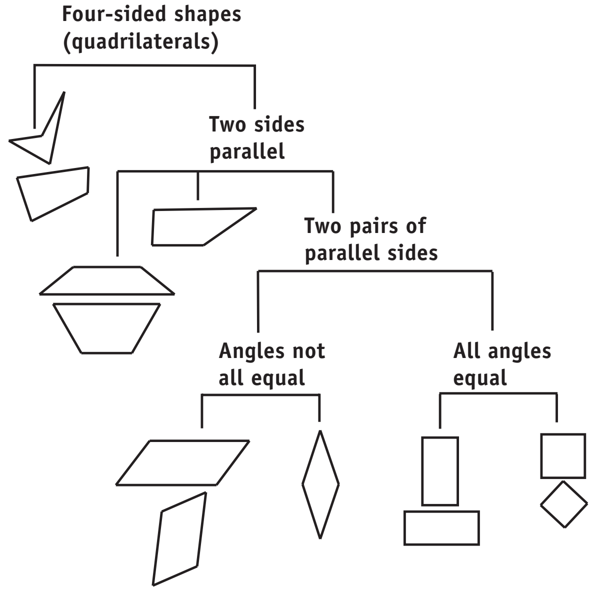
Give students a sheet showing diagrams of a range of geometric solids (e.g. cylinders, prisms and pyramids). Ask: What would you tell someone to look for if they had to identify all the prisms on this sheet? Can you shorten your list and still give your partner sufficient information to identify only prisms?

Every Square a Rectangle

Say: My sister said that every square is a rectangle but every rectangle is not a square. Ask: How can that be so? Invite students to list all the properties of each and tick those that are the same for both shapes. Ask: What is the only property that a square has that a rectangle doesn't? Are there any properties of a rectangle that a square doesn't have?

Quadrilaterals

Give students a mixed set of quadrilaterals to sort. Ask: How many groups can you make? How did you decide on the groups? Encourage students to choose from a collection of labels for each group (trapezoid, parallelogram, rhombus, kite, rectangle, square). Ask: Can you sort the groups again? How did you sort? Choose labels for each group. Can you re-sort the groups? Why? Why not? (Link to Key Understanding 1.)



Relationships

Give students names of quadrilaterals on cards and cardboard arrows and ask them to show relationships between the shapes as a tree diagram. Start with the quadrilateral card and decide on possible positions for the trapezoid, parallelogram, rhombus, kite, rectangle and square. Invite students to compare their trees by starting at the square and reading each arrow as … *is a special* … For example, *A square is a special rectangle, that is a special parallelogram, that is a special quadrilateral.* Encourage students to discuss the properties that informed their judgments. Ask: What are the properties that make all rectangles parallelograms, but not all parallelograms rectangles?

Sorting Triangles

Have students sort a range of different triangles, decide on a label for each group and say how each group is different. Invite them to compare their categories with other students and refine their categories if they need to. Encourage students to refer to a mathematics dictionary to see how their categories compare.

SAMPLE LESSON 3

Sample Learning Activity: Beginning—'Rectangles and Triangles', page 199

Key Understanding 4: People have developed useful ways to classify shapes. Knowing that a shape is one of the standard types can tell us a lot about it.

Teaching Purpose

My Year 3 students could recognise triangles, rectangles and squares in the environment, although they didn't yet know that squares are special sorts of rectangles. Most used the words correctly when talking about the shapes, but they saw rectangles as close to 'book' or 'door' proportions and sitting on their ends or sides, and triangles as close to equilateral triangles and sitting on a flat side, not a point.

I planned a tactile construction activity that might force the students to focus on some of the features of each type of shape.

Action and Reflection

Three sections of the notice board were labelled 'These are all rectangles', 'These are all triangles', and 'These are not rectangles or triangles'.

The students were told that we were going to make some different-shaped rectangles and triangles and pin them onto the board. If they accidentally made any that didn't turn out, they could put them in the third space.

Groups of students used one of the following sets of materials. Each type of material forced students to focus their attention on different features of polygons:

- **drinking straws, lengths of wool, scissors**—cut straws to size and string together and tie to make the shapes (focus on the size relationships of the sides)
- **coloured paper squares**—fold to make figures, overlaps can be cut off, but the sides must be formed by folds (focus on the shape of the corners)
- **paint, brushes and white paper**—paint shapes on the paper (focus on the need to create straight sides and sharp angles)

To create the need for students to compare and discuss properties of the figures, groups were asked to ensure that each figure they made was different in size or shape to any of the others in their group.

When examples had been placed on the notice board, I gathered the class for a discussion, asking, 'Which shape was easiest to make?'

'Triangles are easy, you just cut three straws and tie them together, but I had to cut two long ones and two short ones for a rectangle, and then I had to do it little one, big one, little one, big one. And then it's still no good, it was all floppy. We had to pin it up to get it even.'

'Any old folds make a triangle, you just have to fold it to make a point, then fold the other side over, but for rectangles, it's got to be two and two and it's got to be just right.'

'I couldn't get the corners of the triangle pointy enough because the paintbrush was too big. I could do the rectangle though.'

Challenge Existing Knowledge

To extend and challenge the limited range of figures students made, I placed some non-stereotypical rectangles and triangles, which I had previously made, into the categories. This stimulated argument.

'That's too long and skinny to be a rectangle,' they exclaimed.

I asked Kieran what he had to think about when he was making his straw rectangle.

'It had to have two long ones and two shorter ones, and you had to go short, long, short, long to make it work,' said Kieran.

'Well, mine has got two long and two short sides, and look, mine goes short, long, short, long as well,' I said, as I pointed to my rectangle.

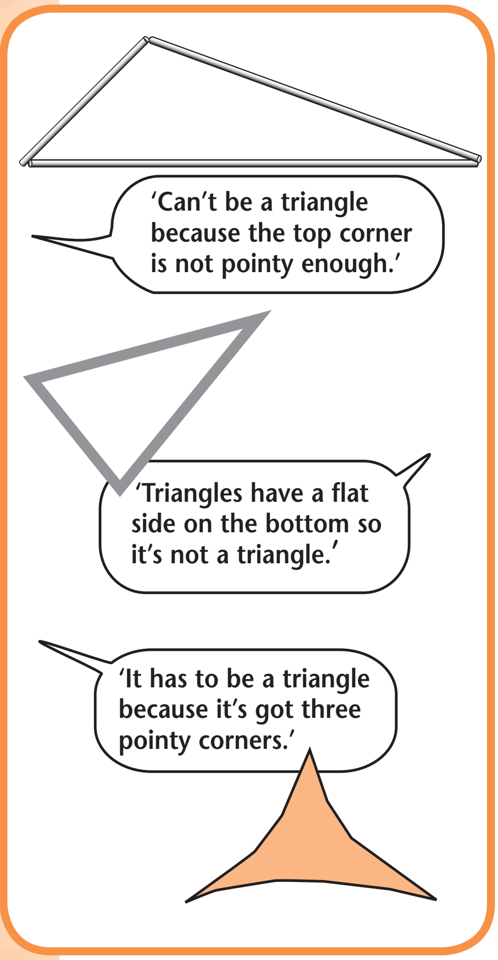
We examined all the rectangles we'd made and reached a consensus that they all fitted this description, so the long skinny one did belong.

They were adamant my squares were not rectangles. I told them they were, in fact, a special kind of rectangle, but as I could see most were confused by this idea, I decided to leave it to a later lesson.

They also questioned my placement of other figures.

I focused some discussion on the triangles.

'So, what do we know about triangles? What do all triangles have to have?' I asked.



They suggested they all needed three sides and three 'points'. I asked them to have another look at each of the triangles alongside.

'So, hasn't the first one got three sides and three points?' I enquired.

'No, it's got three sides but only two points,' said Leila.

'But what about this bit, isn't that a point too?' I asked, indicating the other corner.

After looking at the range of corners in other figures that the students had classified as triangles, they accepted that the three corners of a triangle didn't have to be as 'pointy' as they first thought. I had to tell them, however, that the sides of triangles (and rectangles) have to be straight.

I dealt with their rejection of the triangle 'standing' on its point by reorienting their straw and folded paper triangles in different ways, asking if they were still triangles. They quickly reasoned that the sides and corners of their triangles didn't change no matter how the figures were placed.

Did You Know?

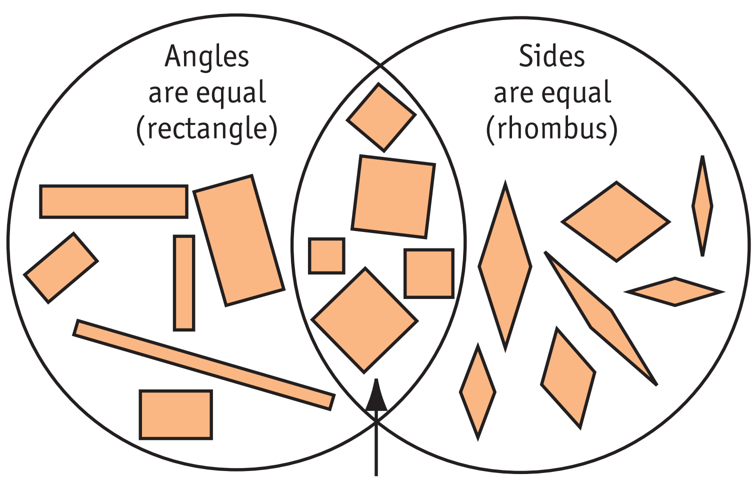
When we see a four-sided figure, with four right angles, we think 'rectangle', we call it a rectangle and we expect students to learn to do so also. Although we know that rectangles are four sided and therefore are a subset of a bigger class called quadrilaterals, we often leave that unsaid. Similarly, when we see a four-sided figure with four right angles and four equal sides, we think 'square' and students learn to do so also.

The challenge comes, however, when they have to learn that just as rectangles are special kinds of quadrilaterals (the *equiangular* quadrilaterals), so, too, squares are the special rectangles that have all their sides the same length (the *equilateral* rectangles). So, squares belong in the bigger set called rectangles.

In a similar way, rhombuses are special kinds of quadrilaterals (the *equilateral* quadrilaterals) and squares are the special rhombuses that have all their sides the same length (the *equiangular* rhombuses).

At the end of Sample Lesson 2 on page 195, one student asked, *Can you have figures with all their angles the same but not all their sides the same?* The teacher returned to that question a few days later. What she also plans to do, however, is to develop some classification activities that help students build up the relationships in the diagram shown here.

Some Special Quadrilaterals



CHAPTER 7

Markers of Progress

This chapter elaborates on how students progress through:

- Represent Location
- Represent Shape
- Represent Transformation
- Reason Geometrically

Represent Location

Initially students follow and give simple directions to describe position and to locate things in their classroom, school or home. They can draw pictures, make things or use computer drawing packages that illustrate the meaning of words relating to position and orientation; for example, a picture with the crocodile behind the boy or a triangle next to a square. They demonstrate an understanding of everyday words associated with position and orientation, by using words such as 'under', 'behind' and 'in front of', and words associated with movement such as 'back', 'forward' and 'turn'.

Students respond appropriately to the language of position, orientation and movement such as 'under', 'behind', 'in front of', 'below', 'on', 'alongside', 'near', 'right' and 'left'; for example, they can 'Put the bear behind the chair', and tell that 'The book is under the table' and that 'I walked past the taps'.

As students continue to progress they put key features in their environment in the right order on maps; for example, placing the taps between the classroom and the cafeteria correctly on their map of the school. Their maps may even look like a 'bird's-eye view' but they may not match the actual locations of the places. However, students' sense of direction, of how close things are together and of scale may not be particularly well developed. Furthermore, their maps and plans are likely to be somewhat egocentric; for example, they draw their own desk larger than those of other children in the class, and in drawing a map they are likely to start with their own class or house, which may be placed in a key location on their map.

Students understand ideas about order and 'betweenness', visualising the arrangement or order of objects within the environment; for example, reading a simple map sufficiently well to say, 'The bus stop is after the shops'. They use these same ideas when describing a path or the location of an object that meets a simple criterion; for example, they can describe the 'safest' path from school to home or say, 'Mum's work is between our house and school because we drop her on the way' and follow directions, such as finding the lost keys when told they were dropped between the two trees, but past the tap.

Students locate objects and describe paths on simple grids or computer screens that require movements to be done in order, such as 'It's three spaces up and two to the right' or 'Forward seven, turn right, forward four'. They find paths on informal maps and mazes; for example, they can find a path between two parts of the playground or through computer-generated mazes.

Next students understand that direction and distance are important and will attempt to represent orientation and proximity in maps of familiar locations. Their maps and plans may include personalised knowledge and non-essential information, but they can now think of a plan or map as a 'bird's-eye view'. They recognise and use a top view to represent familiar locations and plans. While they notice order and proximity among some objects, they may not attend to scale or the overall frame of reference used in a plan of familiar locations and paths, such as mapping their classroom. They can place and locate key features on a map; for example, they place the bus stop correctly between the park and shop but closer to the shop.

Students understand directional language associated with quarter and half turns, compass bearings and directions, such as north, south, east, west, right angle, quarter turn, right, left, to describe a route; for example, 'Go east for 20 metres and then due north for …' and give directions such as 'As soon as you get to the tree look to the right …'.

Students also recognise and use repetitions systematically to produce paths and arrangements. Thus they can predict the effect of following a rule involving a simple sequence of movements by imagining looking down on the path mapped out; for example, 'Two steps forward, turn right, two steps, turn right, two steps, turn right …'.

As students progress further they understand and use grids, including everyday map coordinates and whole-number scales, such as one centimetre for each metre, to interpret and make maps. They draw maps and plans that show a sense of scale and interpret a particular map drawn 'to scale' by comparison, saying, 'The map shows that the river is about ten times further away than the road'. They can also draw simple topological maps to show place and position but not distance, such as stations on a railway line.

Students place and describe key features on a map or path relevant to their school, home or community with sufficient care that others can use them; for example, they can provide a tour map of their school for visitors or a map of the location of some important community resources. They interpret maps to find their way around their environment, such as using a street directory to plan a journey.

Students understand that plans represent a given space containing an arrangement of particular objects. They plan and describe routes on road maps to fit specifications such as the shortest, the safest or one that does not involve retracing their steps. They use both distance and direction and conventional map grids to give unambiguous and clear instructions for moving and locating things in their environment or on models, plans, maps or computer screens. While they attend to direction, they may not connect compass directions with the general idea of angles and degrees of turn. They attend informally to general direction and relative size, but are not able to make use of formal scale relationships in making or interpreting plans.

Later students understand correct mathematical terms and measurements associated with representing paths, maps, plans and locations. They interpret and sketch paths, maps, plans and locations described in degrees of turn, compass directions and key bearings, distance, scale and coordinates; for example, they use a map of Western Australia to estimate distances between towns and the key bearing of one town from another. They can interpret and represent size and position accurately when describing, making and using maps and plans for a wide range of purposes; for example, they can sketch the plan of their school building using a scale of 1:100.

Students follow a path through a location, given directions and vice versa. They may construct or follow a path through local parkland or through the school grounds or give directions of a path through a map, plan or the Cartesian plane using coordinates and compass directions.

Students understand and produce diagrams, such as networks, showing the essential features of a location in a familiar environment in which what is 'key' depends upon the purpose; for example, they use different features of their town to plan a tourist route, a shopping trip or a quick route through the location. They can draw a network representing their school, but realise that they cannot determine which drinking fountain is closer based on the drawing alone. They use diagrams to represent arrangements and movements when solving problems; for example, they solve problems such as the river crossing problem by representing diagrammatically the systematic movements to describe the solution of the problem.

Represent Shape

Represent Shape

Initially students attempt to show important spatial features when they draw and make things; for example, their drawings of a triangle will show three roughly straight lines that more or less join at their ends and a seesaw will show the essential idea of a plank balancing on a point. They attend to the special features and purpose of the shape when making an object; for example, they select circular pieces for wheels, rectangular pieces for tabletops and choose suitable building blocks to make a house. They are beginning to remember the shapes of objects and their component parts and use descriptions that involve spatial language or imply shape; for example, 'Draw a tall witch's hat with a pointy top', 'Use modelling clay to make a snake' or 'Select a shape most like the ice cream on top of the cone.'

Students can convey the essential spatial features of common mathematical figures in their drawings with pen and paper or by using a computer drawing package to make a thing from an oral description, such as a circle, triangle or complex shapes within a shape; for example, drawing a rectangular door and windows on a rectangular wall.

As students continue to progress they understand simple requirements relating to shape and structure; for example, when constructing a model, they can select one type of building material and use it repeatedly to build a wall that will remain standing when they remove and replace a 'door' piece. They can fit component parts of a larger shape together in a range of practical tasks. They also understand criteria or needs relating to function, such as, 'will fit together', 'will roll', 'will stack', 'will stand up by itself'.

With prompting, students can sort according to a familiar attribute, such as points. They identify common mathematical shapes such as cones or rectangles and can draw figures that show the essential spatial features of named geometric shapes; for example, they can draw five straight sides that join end-to-end to form a pentagon.

Students are beginning to develop the necessary skills to make recognisable (although not necessarily accurate) copies of pictures composed of geometric shapes, such as a table setting, so that the main components are recognisable in shape, position and orientation. They can draw freehand, or use a template or a computer to construct shapes or arrangements that meet simple criteria.

Students match 2D figures to the faces of a provided 3D object and can list and draw the component faces and say how many there are of each.

Next students understand the features of 3D objects in solid forms, such as those made from clay; hollow forms, such as plane shapes cut from cardboard; and skeletal forms, such as those made from straws. They can compare these forms to examine which features are emphasised and best represented in each form. They identify the key features that must be represented on a polygon, prism or pyramid, such as the shape and number of sides, faces, edges and vertices (corners).

Students attend to the shape and placement of parts when matching, making and drawing things, including matching 3D models that can be seen and handled with conventional drawings of them and with their nets. They use their knowledge of a suitable 'net' to make common prisms and pyramids and predict which of different flattened nets fold to make particular, clearly different 3D objects by considering the number, shape and placement of the faces. They may be confused by subtly different nets, however, or by an arrangement of the faces that will not actually fold to make the object; for example, they can predict which pentominoes can be used as nets for an open box.

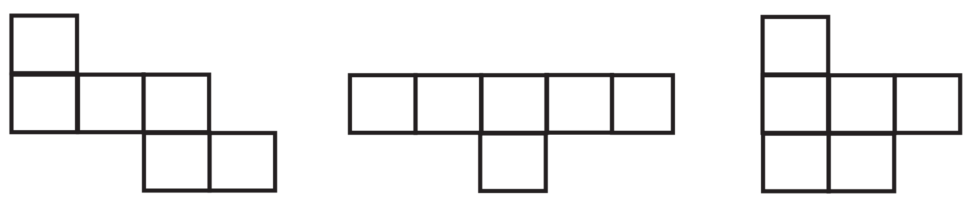
Students can turn, position and rearrange everyday and geometric objects to match drawings of them. They interpret conventional diagrams of objects, such as prisms and pyramids, although they may not be able to produce them, recognising, for example, a hexagon with some lines drawn inside as a diagram of a cube.

In their own drawings of objects, students show what can actually be seen; for example, they will show whole and part wheels on a trolley using ovals if that is what can be seen, rather than drawing the four circles that they know to be there. They also give some attention to depth in their pictures, showing things that are further away as being smaller.

Students imagine and draw different cross sections of simple 3D objects; for example, they sketch the cross sections that they can 'see' and make when slicing through a cube of cheese (rectangle, square and perhaps a hexagon).

Represent Shape

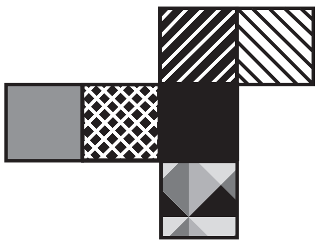
As students progress further they understand shape, structure and scale when making recognisable models of simple 3D objects; for example, they can make a box to match a provided wooden block by drawing around each of its faces to make a net. They trace each face of the block onto paper to make a net, ensure that adjoining faces are placed correctly with respect to each other, and then cut, fold and tape the paper to make the model. Their ability to understand the process of folding a net into a simple prism or cube has improved; for example, they predict whether the following nets will or will not fold to make a cube and they can predict which edges will match.



Students understand and apply the basic skills of attending to shape, structure and informal scale to making models of familiar and imaginary objects and scenes, such as television sets, a football field or puppet shows. They can use cubes to copy models made from cubes, attending to cubes that cannot be seen but must be there.

Students understand some of the more common conventions for drawing 3D shapes; for example, that solid lines on a drawing indicate edges that can be seen, while dotted lines indicate edges that cannot be seen, objects that are further away are drawn smaller and ellipses represent circles in perspective. They select actual objects to match exploded drawings or front, back and side views of various art forms; for example, given a tray of objects, they can select one to match a particular drawing and explain why. They draw prisms, pyramids, cylinders and cones recognisably, if not with precision, and use these basic shapes to help draw other things such as a building.

Later students show considerably more precision and forethought in producing models of 3D shapes; for example, they say 'six small straws and three larger straws can be used to make a model of that triangular prism'. They can construct independently a net for a prism or pyramid and will consider the final size of the model, the effect of fold lines, how their models will be held together and where to place tabs in the planning before beginning the construction of the model. They measure and fold with care. Their capacity to visualise what a net will look like when folded (or a 3D model when unfolded) has improved to the extent that they can predict which particular figures on a net will become faces on the resulting object; for example, given the net shown here, they know which faces will be parallel, which will be opposite and which will be adjacent. They interpret and compare a range of representations of 3D models.



Students understand the conventions used to produce orthogonal, oblique, perspective and isometric drawings of common 3D shapes accurately. They construct an arrangement of cubes from an isometric drawing. They visualise an object in different orientations and draw possible 'other views' of an object from the information contained in 2D drawings; for example, given an oblique drawing of a complex arrangement of cubes, they can construct or draw the arrangement from a different position. They consider what cannot be seen, but must be there, when drawing or constructing an arrangement of cubes and then draw the arrangement from a different view.

Represent Transformation

Initially students may rely largely on their memory of previous attempts to predict whether shapes will fit into specified spaces, such as postal boxes, or fit together according to shape and orientation, such as tangrams or jigsaws. They reproduce a simple arrangement using tiles or pattern blocks. They use computer software to move, position and match shapes; for example, manipulate tangram shapes to solve a tangram puzzle on a website.

Students can match figures one to one and can make or copy a picture, pattern or object when provided with a choice of component parts. They can fill in pictures and patterns where lines are provided to enable direct matching of component shapes, such as picture sticker books, and also where no lines are provided.

Students recognise pictures that do and do not have mirror symmetry and can use a fold line to produce simple symmetrical pictures by drawing freehand, folding, cutting or tracing. They may use the phrase 'the same as' rather than 'symmetry'.

As students continue to progress they rearrange and fit together shapes to form larger shapes, perhaps by completing puzzles such as tangrams.

They explain their choice of simple figures which are 'the same' with language such as 'turn', 'turn over', 'flip', 'slide along'; for example, they say, 'We turned the tall rectangle around and then we could slide it on top of the wide one'. In this way they can also explore symmetrical pictures using a variety of means including cut-out figures and flipping and drawing around templates.

Students understand simple movement rules for generating such patterns and use multiple copies of a figure or object to make patterns and arrangements. They describe how they make the patterns using words, such as 'flip it over and slide along one space'; for example, they can make friezes or borders by sliding, turning and flipping a shape, or several shapes, in a repetitive way and make pictures with mirror symmetry by flipping shapes about a central line.

Next students are able to imagine how things will look from different positions; for example, they predict the shape of a cross section or order photographs taken as a photographer moved around an arrangement.

Given a complex pattern, such as that in a fabric design or a beehive, students identify the basic shape or shapes from which it is formed and show how repetitions are used systematically to make the pattern. They also create patterns based on systematic movements of the shape; for example, they make a border from two different shapes and describe informally the movements used. They experiment with multiple copies of shapes, such as a triangle, rectangle or pentagon, deciding which can and which cannot tessellate and demonstrate the result.

Students identify repetitions of component parts in symmetrical objects and arrangements by moving one component over another; for example, they can trace around one arm of a windmill and turn the tracing around the centre to show how each arm fits over the others. They also identify and describe figures or arrangements that are symmetrical and can justify this by showing the movement required to place one part onto another, saying that a shape has line (or mirror) symmetry because it can be folded along a line so that one half fits over the other half.

As students progress further they no longer need multiple copies of a shape in order to decide whether or not it will tile; for example, they can use a single piece by moving it systematically and tracing around it to cover the surface. They can use a computer drawing package that will produce a tiling pattern by translating, rotating or reflecting a shape systematically. They use appropriate language of transformations to describe how one shape can be superimposed on another or moved to form a tessellated pattern, such as 'rotate it at right angles around the centre and translate it to the left'.

They recognise the translations, rotations and reflections embedded in the arrangements and patterns of such things as curtains, supermarket displays and flights of birds. They know that each of these transformations produces a different type of symmetry; for example, they recognise the translation symmetry in a series of electricity pylons and the rotational symmetry in a windmill.

Students identify the transformation(s) used to produce a spatial sequence and continue the sequence. They visualise and reproduce the folds and cuts used to make a complex symmetrical pattern such as on a frieze or a snowflake. They can also decide which transformation is involved in producing a particular symmetrical pattern or arrangement; for example, they explain that a logotype will be repeated each third turn and so it has rotational symmetry.

Later students follow instructions for moving or sketching things according to one or more transformations: translations, rotations, reflections or dilations. They can describe and predict the effect of a translation, rotation or reflection on the position and orientation of a figure; for example, they can imagine what a given shape will look like if rotated through a 90° turn clockwise around a specified point and may use tracing paper to show the effect of this rotation. They describe the effect of the transformation on the position and orientation of the figure or object. They can also explore, with a mirror or computer package, and report, 'each bit of the reflection was as far behind the mirror as that bit was in front of the mirror'.

Students use transformations to modify tessellating shapes to produce other tessellating shapes and explain informally why this works; for example, they use a quadrilateral to produce an Escher-type design. They can use a grid to enlarge or reduce a figure by a specified whole number or unit fraction scale factor; for example, they dilate a rectangular prism made with six cubes by a scale factor of three. They make distortions of 2D figures and 3D objects by, for instance, doubling the widths but not the heights.

Students move from selecting shapes with specific symmetries to visualising and making figures and objects that fit such specifications using paper folding, mirrors or computer graphics; for example, they use circular grid paper to make a figure that has rotational symmetry and that can be moved six times to get back to where it started.

Reason Geometrically

Initially students recognise likenesses and differences between things that can be seen or handled and begin to connect shape, movement and function. They understand and begin to use everyday language such as 'flat', 'straight', 'curved', 'side', 'round' and 'corner'; for example, they select tree trunks, posts and pencils to show 'long but round' and spatial language such as 'rolls', 'slide' and 'stacks' to describe an object's function. They also describe how two objects are 'alike' and how they are 'different'; for example, they say that two leaves are alike because they are both flat but that they are different because one is almost round and the other is long and thin. Matching of basic shapes poses few difficulties, even when this requires students to change the orientation of the shape by turning it around or flipping it over. Thus, they can select and stick on the component shapes needed to cover a picture or make their own copy of a picture composed of simple shapes.

Students recognise shapes that are close to conventional triangles, rectangles, squares and circles in everyday things (such as windows and traffic signs) and on mathematical shapes (such as cubes). They have begun to see a relationship between the shapes of things and the purposes to which they are put; for example, they will choose 'round' (circular) things for wheels because the wheels they have seen are that shape. They will also reject shapes that have corners as not being useful for wheels and attempt to explain why, perhaps by talking about 'being bumpy'. They classify objects by a familiar attribute relating to shape, movement or function; for example, as being 'good' or 'not good' for holding a drink.

As students continue to progress they sort according to everyday spatial criteria and, when prompted, use their own spatial language to describe the shape of things and represent classifications including in two-way tables; for example, they classify objects by how many faces they have and the shape of the faces. Students recognise the names of some shapes, including circle, triangle, rectangle and square, and cube, cone, cylinder, sphere and rectangular box, and say, 'If I remove the handle from this mug it looks like a cylinder'. They are more precise in their identification of common figures and reject as a triangle something that has curved edges or edges that cross each other or do not meet. If asked to describe the shape of something, students will talk about shape rather than, say, colour or size. They use language such as 'flat', 'round', 'straight', 'curved', 'edge', 'face' and 'side'.

Students discriminate between shapes that are alike; for example, separating circles from ovals, rectangles from squares and so on. They can describe some slices of salami as being circular but others as not, and explain that some slices are not circles because they are 'longer' one way. They can also distinguish between a 3D object and its 2D representations; thus they can tell circles apart from spheres or cylinders and squares from cubes using the correct mathematical terms in their descriptions.

Next students understand common spatial language and use it to describe and compare features of things they can see and handle, saying how they are alike or different. They use conventional names of shapes and component parts of shapes in their descriptions of things, such as 'side', 'face', 'edge', 'vertex', 'base', 'surface', 'curved', 'triangular' and 'circular'. They can select an object from a collection given a description of its spatial features, such as 'I have two flat faces and one curved face. I also have two edges. What am I?' They accept that the same collection can be sorted in different ways.

Students understand the difference between a prism and a pyramid (including a cylinder and cone) and recognise these shapes in the world around them; for example, they say that this tent is very like a prism with triangular ends but the other tent is a pyramid on a square base.

Students predict, sketch and describe different cross sections of familiar objects and then test and improve their prediction or drawing; for example, by slicing cheese sticks, they can justify that elliptical, circular or rectangular cross sections can be made. They link the spatial features and structures of familiar objects to their production and uses; for example, they compare milk bottles and milk cartons and conjecture that differences in their shapes relate to the materials used, stability and strength. When prompted, they suggest the advantages and disadvantages of each shape in terms of production, storage and display. In doing this, they use spatial terms to improve the clarity of their explanations.

As students progress further they recognise figures and objects on the basis of spatial features, using conventional geometric criteria. They select objects or drawings on the basis of a geometric description, such as 'has five faces and eight edges' and can compare these objects and drawings based on their spatial features.

Although they may not yet be able to produce a model to meet given specifications, students select figures and objects that meet geometric criteria related to sides, faces, angles and edges; for example, they choose an object given the description 'I am made from two triangles that are the same shape and size and three rectangles that are not the same shape and size'. They also use common geometric language; for example, they can describe clearly the school logotype to someone over the telephone or explain the movement needed to shift one part of a pattern onto the top of another part of it.

Students conjecture about whether a certain shape will or will not tessellate and can explain their thinking, saying such things as 'I think parallelograms always tile because you can slide them together to make strips and then slide the strips together'.

Students link features of structures, such as flexibility or rigidity, fragility or strength, to shapes and objects; for example, they say, 'If I put a popstick across from corner to corner it will make the shape rigid, because triangles are rigid'. They can explain why they think certain shapes will predominate in certain situations, locations or places.

Later students understand the geometric features that distinguish one class of shapes from another; for example, they know that 'prisms have two parallel faces that are exactly the same but pyramids don't'. They describe the common features of collections of shapes; for example, they reason that, for all the prisms, the total number of vertices is double the number on each of the opposite parallel faces and that the altitude of isosceles triangles is perpendicular to the base.

Students generate and classify shapes that satisfy given conditions; for example, they use straws to produce quadrilaterals that have two equal diagonals, and determine what these quadrilaterals have in common if the constraint that the diagonals cross at their mid-points is added. They refine their descriptions using properties of the shapes and identify inadequacies in their description; for example, they deduce that stating that a shape has triangular faces is not sufficient to describe a triangular pyramid.

Students have, however, made a shift in their thinking and can make more abstract statements about 'all rectangles' or 'all prisms'. They apply the distinguishing features of common classes of shapes to determine 'inclusive' relationships between them; for example, they show a variety of different quadrilaterals in a Venn diagram and conclude that 'All squares are rhombuses but the reverse isn't true, because ...', and reject as inadequate a description of a rectangle as 'a four-sided figure with opposite sides the same length', offering a parallelogram as a counter-example.

They can use the properties of shapes to produce informal arguments about classes of shapes and the relationships between tessellations, symmetry and transformations. They may deduce that all triangles tessellate, so as a triangle and its reflection form a parallelogram, all parallelograms tessellate.

Students explore and use angle relationships such as the sum of the angles in a triangle and in a quadrilateral, vertically-opposite angles, supplementary and complementary angles.

Classroom Plan for Week ___, Term ___ Year Level: ___

Outcome/Key Understanding	Mathematical Focus	Activities	Focus Questions	Observations/Anecdotes

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