YuMi Deadly Maths

Year 7/8 Teacher Resource:

NA – How many in the queue?

(developing linear relationships)

Prepared by the YuMi Deadly Centre
Faculty of Education, QUT

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YuMi Deadly Maths Year 7/8 Teacher Resource: **NA – How many in the queue?**

ACKNOWLEDGEMENT

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Prepared by the YuMi Deadly Centre
Queensland University of Technology
Kelvin Grove, Queensland, 4059

research.qut.edu.au/ydc

Year 7/8 Number and Algebra

How many in the queue? (developing linear relationships)

Learning goal

Students will

- develop a simple rule between two variables (positive values only)
- plot the points on the Cartesian plane to form a straight line
- extend the straight line in both directions.

Content description

Number and Algebra – Linear relationships

- Given coordinates, plot points on the Cartesian plane, and find coordinates for a given point (ACMNA178) (Year 7)
- Plot linear relationships on the Cartesian plane with and without the use of digital technologies (ACMNA193) (Year 8)

Big idea

Number – the Cartesian plane and algebra as a strategy to represent the relationship between two variables

Resources

Maths Mat. If you do not have a math mat, you can make one. It is merely a grid drawn on a large sheet of heavy plastic or mesh. We are using a 10 by 6 grid (the one in this link is 10 by 5: http://mathematicscentre.com/taskcentre/mathmat.htm)

Cards: two sets of labels 0 to 10, a card for the origin (0,0), an x and a y label which will later be replaced by new axis labels (f – *number of people in front* and T – *total number of people in the queue*), and labels for each point: (1,3) (2,5) (3,7) (4,9) (5,11) (6,13) etc.

A rope or thick cord, colourful if possible, to represent the line on the Cartesian plane.

Reality

Local knowledge

At an appropriate point (not necessarily the beginning), it is worth explaining why we want to investigate the relationships between variables (quantities). In fact a lot of maths that is used in the world is about relationships between variables. Maths can be about adding up a grocery bill, maths can be about percentage discount, but this is something quite different. Discuss with the students some of the real-world situations where we might need to know the relationship between variables.

For example:

- If we were to build a bridge we would be interested in the relationship between the weight of the cars, the strength of the steel, and the length of the bridge. How much weight can we put on the bridge before the steel starts to weaken? Does the bridge section need to be shorter so it will have more support?
- Other examples: A doctor would be interested in the relationship between the amount of medicine to give someone and the rate of healing; a grazier in the relationship between the amount of fertilizer to spray on a field and how much a crop grows; an aircraft engineer in the amount of fuel needed by a plane depending on the speed it flies at or to fly for a certain distance or time, and so on.

There are complex equations (or rules) for all of these relationships that people spend time developing. Explain to students that we are learning about this area of maths – relationships between variables – but we are going to start with something very simple.

The Cartesian plane is a key fundamental concept in modern mathematics and was put forward in 1637 by René Descartes, a French mathematician and scientist, hence it is named after him.

Prior experience **Revise plotting points on the Cartesian plane**

Lay out the Maths Mat (10 × 6) and have students carefully place cards for the numbering for each axis. Use the 6 side for the x-axis. Place the letters x and y on the appropriate axes. Replace the two zero cards with (0,0) the origin. Teacher models plotting a point. Use the notation (2,3) on a card, being clear about stepping out the coordinates (*two across, three up*). When at the point hold arms up at chest height at right angles to each other (one pointing to the horizontal axis and the other to the vertical axis) and say the values. Students repeat with various examples. Practise both ways (reversing): give a point, student plots it; student stands on the grid and we say the point.

Follow with pen-and-paper practice as needed.

Kinaesthetic **How many people in the queue?**

I'm standing in a queue waiting to buy my lunch and I notice that I am right in the middle of the queue. How many people might be in the queue?

We really don't know; we just know that if it was an even number there wouldn't be a middle. The answer has to be an odd number: 3, 5, 7, etc.

At this stage nominate someone to be the middle person (e.g. Roy) and form different queues around them. A queue of 3 or a queue of 5, a queue of 7, etc.

Abstraction

Body ***If I am standing in the middle how can I quickly work out how many in the queue?***

Start with three people in the queue. Complete the table and keep adding to the queue and the table. Emphasise the person has to be in the middle and how the result is found: e.g. *One in front, one behind, and Roy*; *Two in front, two behind, and Roy*.

The strategy we are aiming for is:
I can count how many people in front of me, double it and add 1.

Some students may see this immediately but develop the process step by step to allow everyone to see it.

You could discuss zero people in front, then how many in the queue? [1]. *If there is only one in the queue then is it really a middle?*

No. of people in front of <Roy>	Total no. of people in the queue
1	3
2	5
3	7
4	9
etc.	

Mind ***What is the pattern?***

By this stage we want students to be able to say verbally that the total number in the queue is the number of people in front multiplied by 2 plus 1. You can emphasise this as you go.

Now write this down using words.

Check the rule works by testing it on different values in the table.

No. of people in front of <middle person> f	Total no. of people in the queue T
1	3
2	5
3	7
4	9
etc.	

Total number in queue = (No. of people in front × 2) + 1

It is difficult to keep writing the words so let's use a shorthand method. Together choose suitable variables for each of these (avoid using x and y).

- Let f be the number of people in front of the middle person.
- Let T be the total number of people in the queue.

Using a capital *T* and lower case *f* is fine but be consistent. Use lower case *t* if you think this is too confusing

Add the variables to the table and we can rewrite our rule.

Total number in queue = (No. of people in front × 2) + 1

$$T = (f \times 2) + 1$$
$$T = 2 \times f + 1$$
$$T = 2f + 1$$

Notice we don't really need the brackets, because multiplication is done before addition. We generally put the number before the variable – this is a convention (a worldwide agreement). If students have learnt this already, leave out the multiplication sign, if not leave it in.

Hand/Mind **Extend the table**

Extend the table without building the queues, although be ready to build the queue with people or use blocks if students need confirmation.

Encourage them to see the queue with their imagination.

No. of people in front of <middle person> f	Total no. of people in the queue T
1	3
2	5
3	7
4	9
5	
6	

Mathematics

Language/ symbols table, variable, rule, relationship, equation, input, output, function machine

Practice Either move straight on to the next Abstraction activity, or practise the skills developed here with other tables and rules.

Abstraction

Body **Stepping out a linear relationship on the Cartesian Plane**

Remember our number pattern about how many people in the queue when Roy was in the middle of the queue? Put the table up again and the rule; clarify the meaning again.

Total number in queue = (No. of people in front × 2) + 1

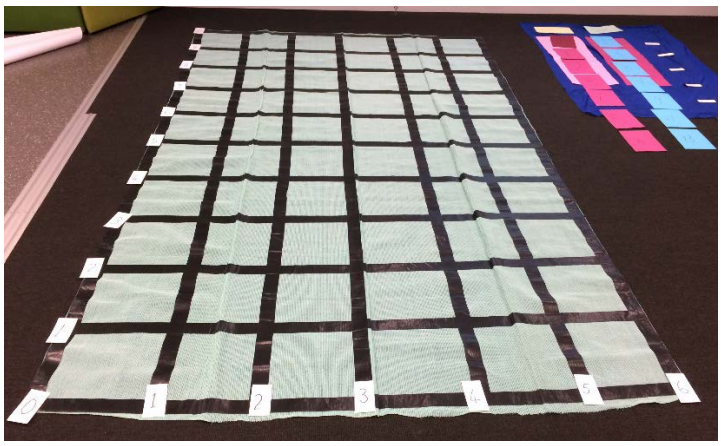
$$T = (f \times 2) + 1$$
$$T = 2 \times f + 1$$
$$T = 2f + 1$$

No. of people in front of <middle person> f	Total no. of people in the queue T
1	3
2	5
3	7
4	9
5	
6	

We are now going to make a picture of this number pattern on the Cartesian plane (grid).

Set up the Maths Mat. If using a 10 × 6 mat, set the horizontal axis on 6 units and the vertical axis on 10 units. Leave enough room to extend the vertical axis. Have students standing below the horizontal axis to view the graph as it is built.

- Emphasise vocabulary as you go: *origin*, *horizontal axis*, *vertical axis*, etc.
- Under the horizontal axis towards the right-hand corner (when standing below the axis facing inwards to the mat) put the label f – *number of people in front*.
- Along the vertical axis place the label T – *total number of people in the queue*.
- Now along each axis ask students to place down the cards indicating the values 0 to 6 (horizontal) and 0 to 10 (vertical) (see photo below). Discuss the values that would be there if the mat kept going. Replace the two zeros with (0,0), the origin.



Put the information from the table onto the grid.

On the table, circle 1, 3. *What do these values tell us?* [When there is one person in front of Roy in the queue ($f = 1$), the total number of people in the queue is three ($T = 3$).]

Introduce the notation (1,3).

Teacher models the first point:

- Take a card with (1,3) written on it. *We will start at the horizontal axis.*
- Stand at $f = 1$. *This means one person in front*. Walk forward to the point where $T = 1$ (*One in the queue? No?*), $T = 2$ (*Two in the queue? No?*), then $T = 3$ (*Three in the queue? Yes*). Stop.
- Hold out your arms at chest height (at right angles to each other – one pointing to 1 on the horizontal axis and the other pointing to 3 on the vertical axis): *This is the point (1,3). These are the coordinates of the point. It means when* $f = 1, T = 3$. *Notice this point is the intersection of the lines from the axes – from* $f = 1$ *and from* $T = 3$.
- *What does this point mean*? [When there is 1 person in front of Roy, there are 3 people in the queue.] Hold the card (1,3).
- Have a student take your place and crouch down on the point with the card visible.

Students repeat for each point 2, 3, 4, stepping from the horizontal axis to the point, holding their arms up and voicing the meaning of their position or questioning the rest of the group.

When points 1 through 4 have been plotted, take the rope and stretch it between the students. *Is the line straight?* Hopefully yes (ensure students are in line). Emphasise that the points are all in a straight line.

Extend the line beyond $f = 4$. Ask: *When* $f = 5$, $T =$? Ask a student to plot this position, noticing that the T value has gone off the mat. Repeat for $f = 6$. Students step out the position by starting at the horizontal axis and stepping up to the point, crouch in the approximate position and hold the rope as well.

Extend the line before $f = 1$. Ask: *When $f = 0$, what does this mean?* You may have discussed this earlier: *When no-one is in front and you are in the middle, how many in the queue?* [1] Hand a student the card (0,1) and have them step out their position from the horizontal axis to the point.

Consider a point not on the line. For example, have someone stand at (3,2). *Why doesn't this point fit on the line? Does this fit our queue story? Does this fit our rule? $f = 3, T = 2$?* [No – that is why it is not on the line.]

Teacher note: The aim is to continually link the story, the numbers fitting the rule and the location of the points on the grid at each step.

Mathematics

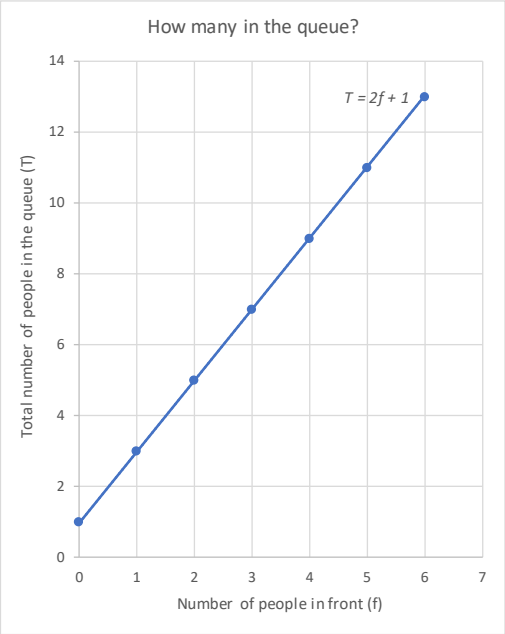
Language/ symbols

variable, rule, equation, Cartesian plane, axes, horizontal axis, vertical axis, the origin, coordinates, (2,3), point, straight line, linear, function

Note: *function* has a specific meaning, generally introduced in Year 11; for now it can be considered interchangeable with the word *rule*.

Practice

Now repeat the process on paper (see graph below). Ensure students name the axes and give the graph a title.



Introduce the word *linear* as a maths word meaning *in a straight line*. Emphasise the meaning of the points being on the line – they obey or satisfy the rule.

Connections

Connect to rate, e.g. distance and time – plot a line for travelling at a constant speed of 20 km/hr (with time in hours on horizontal axis and speed on vertical axis), then look at what happens to the line if the speed increases or decreases.

Reflection

Validation

The purpose of validation is to encourage students to relate this new maths knowledge back to their own context in some way. In this case they could look at line graphs from the media. Find current examples relevant to your students; some examples are in the **Appendix**.

Application/ problems

Provide applications and problems for students to apply to different real-world contexts independently; e.g. *I planted a small tree that had 1 leaf. Each day it grew 2 more leaves; how many leaves after 5 days? How many after 8 days? How many after n days?* (Note: You could vary this by having it grow 3 leaves per day or start with 4 leaves, etc.)

Extension

Flexibility. The Cartesian plane on the Maths Mat can also be used to demonstrate different equations and their gradients as students step from one point to another. For example, the slope of a hill or road might be expressed as "1 in 5", meaning that you climb 1 vertical unit for every 5 horizontal units – see Year 9 lesson on gradient: *How does my garden grow?*

Reversing. Start with a picture of a straight-line graph and have students work out the rule (equation) from the graph.

Generalising. The starting point can change and the gradient can change. These change the slope and parameters of the graph.

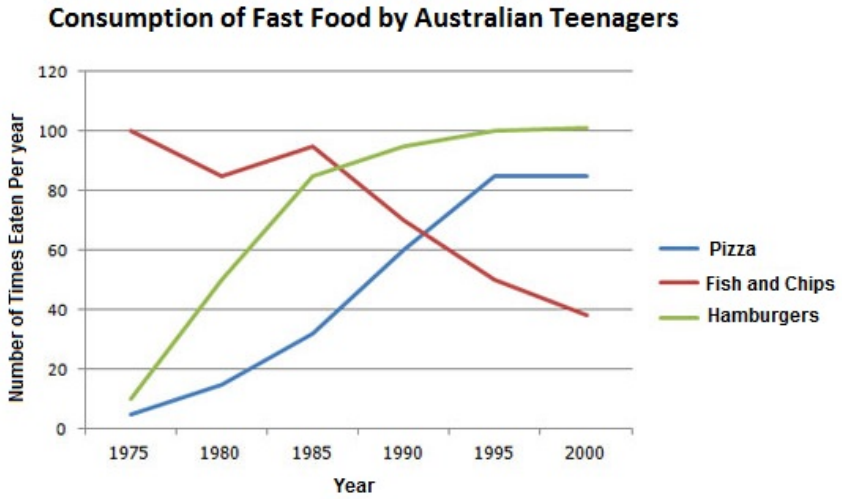
Changing parameters. Starting at (0,2), step out 1 across and 3 up; starting at (1,0.5), step out one across, one up; starting at (3,0), step out 1 up. *What points do you stand on? Is it a straight-line graph?* [No]

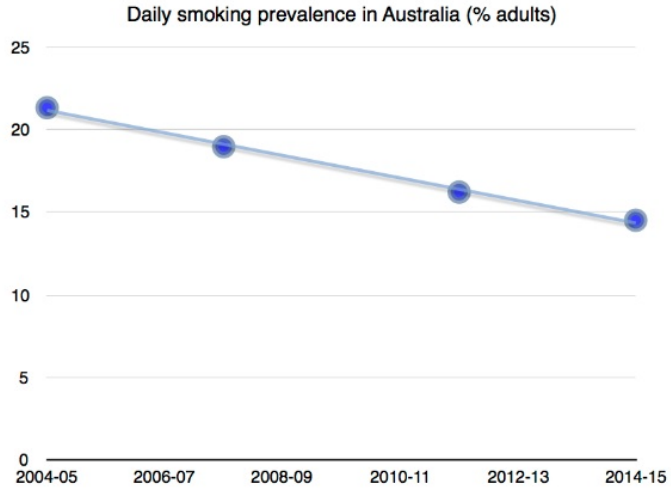
Teacher's notes

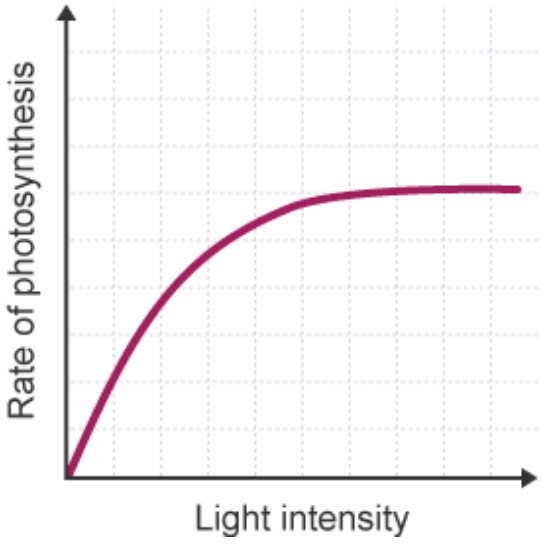
- This lesson includes **three body activities**. It is likely that you would need to complete this material over at least two lessons:
  - the Prior Experience activity where students practise or revise plotting points on the Maths Mat (prior to this lesson)
  - the activity to determine the length of a queue and develop a rule between variables
  - the activity to represent the linear relationship on the Cartesian plane.
- Suggestions in Local Knowledge are only a guide. It is very important that examples in Reality are taken from the local environment that have significance to the local culture and come from the students' experience of their local environment.
- Students need to be taught the skill of visualising: closing their eyes and seeing pictures in their minds, making mental images; e.g. show a picture of a graph, students look at it, remove the picture, students then close their eyes and see the picture in their mind; then make a mental picture of a different graph.
- Useful websites for Aboriginal and Torres Strait Islander perspectives and resources: www.rrr.edu.au; https://www.qcaa.qld.edu.au/3035.html
- Explicit teaching that aligns with students' understanding is part of every section of the RAMR cycle and has particular emphasis in the Mathematics section. The RAMR cycle is not always linear but may necessitate revisiting the previous stage/s at any given point.
- Reflection on the concept may happen at any stage of the RAMR cycle to reinforce the concept being taught. Validation, Application, and the last two parts of Extension should not be undertaken until students have mastered the mathematical concept as students need the foundation in order to be able to validate, apply, generalise and change parameters

Appendix: Examples of graphs of the relationship between two variables

Sometimes they are straight lines but very often not. Using time as one of the variables is a very common graph.







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https://www.bbc.co.uk/education/guides/zsf82hv/revision