YuMi Deadly Maths

Measurement

Prep to Year 9

Prepared by the YuMi Deadly Centre
Queensland University of Technology
Kelvin Grove, Queensland, 4059

Prep to Year 9: Resource 6 – **Measurement**

QUT

YuMiDeadly
Growing community through education

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VERSION 3, 30/03/16

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http://ydc.qut.edu.au

ACKNOWLEDGEMENT

The YuMi Deadly Centre acknowledges the traditional owners and custodians of the lands in which the mathematics ideas for this resource were developed, refined and presented in professional development sessions.

DEVELOPMENT OF THIS BOOK

This version of the YuMi Deadly Maths Measurement book is a modification and extension of a book developed as part of the Teaching Indigenous Mathematics Education (TIME) project funded by the Queensland Department of Education and Training from 2010–12. The YuMi Deadly Centre acknowledges the Department's role in the development of YuMi Deadly Maths and in funding the first version of this book.

YUMI DEADLY CENTRE

The YuMi Deadly Centre is a research centre within the Faculty of Education at QUT which is dedicated to enhancing the learning of Indigenous and non-Indigenous children, young people and adults to improve their opportunities for further education, training and employment, and to equip them for lifelong learning.

"YuMi" is a Torres Strait Islander Creole word meaning "you and me" but is used here with permission from the Torres Strait Islanders' Regional Educational Council to mean working together as a community for the betterment of education for all. "Deadly" is an Aboriginal word used widely across Australia to mean smart in terms of being the best one can be in learning and life.

YuMi Deadly Centre's motif was developed by Blacklines to depict learning, empowerment, and growth within country/community. The three key elements are the individual (represented by the inner seed), the community (represented by the leaf), and the journey/pathway of learning (represented by the curved line which winds around and up through the leaf). As such, the motif illustrates the YuMi Deadly Centre's vision: *Growing community through education*.

The YuMi Deadly Centre can be contacted at ydc@qut.edu.au. Its website is http://ydc.qut.edu.au.

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ABOUT YUMI DEADLY MATHS

From 2000–09, researchers who are now part of the YuMi Deadly Centre (**YDC**) collaborated with principals and teachers predominantly from Aboriginal and Torres Strait Islander schools and occasionally from low socio-economic status (SES) schools in a series of small projects to enhance student learning of mathematics. These projects tended to focus on a particular mathematics strand (e.g. whole-number numeration, operations, algebra, measurement) or on a particular part of schooling (e.g. middle school teachers, teacher aides, parents). They resulted in the development of specialist materials but not a complete mathematics program (these specialist materials can be accessed via the YDC website, http://ydc.qut.edu.au/).

In October 2009, YDC received funding from the Queensland Department of Education and Training through the Indigenous Schooling Support Unit, Central-Southern Queensland, to develop a train-the-trainer project, called the **Teaching Indigenous Mathematics Education** or **TIME** project. The aim of the project was to enhance the capacity of schools in Central and Southern Queensland Indigenous and low SES communities to teach mathematics effectively to their students. The project focused on Years P to 3 in 2010, Years 4 to 7 in 2011 and Years 7 to 9 in 2012, covering all mathematics strands in the Australian Curriculum: Number and Algebra, Measurement and Geometry, and Probability and Statistics. The work of the TIME project across these three years enabled YDC to develop a cohesive mathematics pedagogical framework, **YuMi Deadly Maths**, that covers all strands of the *Australian Curriculum: Mathematics* and now underpins all YDC projects.

YuMi Deadly Maths (**YDM**) is designed to enhance mathematics learning outcomes, improve participation in higher mathematics subjects and tertiary courses, and improve employment and life chances. YDM is unique in its focus on creativity, structure and culture with regard to mathematics and on whole-of-school change with regard to implementation. It aims for the highest level of mathematics understanding and deep learning, through activity that engages students and involves teachers, parents and community. With a focus on big ideas, an emphasis on connecting mathematics topics, and a pedagogy that starts and finishes with students' reality, it is effective for all students. It works successfully in different schools/communities as it is not a scripted program and encourages teachers to take account of the particular needs of their students. Being a train-the-trainer model, it can also offer long-term sustainability for schools.

YDC believes that changing mathematics pedagogy will not improve mathematics learning unless accompanied by a whole-of-school program to challenge attendance and behaviour, encourage pride and self-belief, instil high expectations, and build local leadership and community involvement. YDC has been strongly influenced by the philosophy of the Stronger Smarter Institute (C. Sarra, 2003) which states that any school has the potential to rise to the challenge of successfully teaching their students. YDM is applicable to all schools and has extensive application to classrooms with high numbers of at-risk students. This is because the mathematics teaching and learning, school change and leadership, and contextualisation and cultural empowerment ideas advocated by YDC represent the best practice for **all** students.

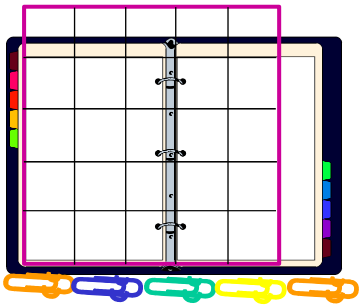
YDM is now available direct to schools face-to-face and online. Individual schools can fund YDM in their own classrooms (contact ydc@qut.edu.au or 07 3138 0035). This Measurement resource is part of the provision of YDM direct to schools and is the sixth in a series of resources that fully describe the YDM approach and pedagogical framework for Prep to Year 9. It focuses on the teaching of measurement, namely (a) the five stages used in the teaching of all measurement attributes; (b) basic measures (length, mass and capacity); (c) relationship measures (perimeter, area and volume); (d) non-metric measures (time and angle); and (e) extension measures (money and temperature). It overviews the mathematics and describes classroom activities for Prep to Year 9. Because YDM is largely implemented within an action-research model, the resources undergo amendment and refinement as a result of school-based training and trialling. The ideas in this resource will be refined into the future.

YDM underlies three projects available to schools: YDM Teacher Development Training (TDT) in the YDM pedagogy; YDM AIM training in remedial pedagogy to accelerate learning; and YDM MITI training in enrichment and extension pedagogy to build deep learning of powerful maths and increase participation in Years 11 and 12 advanced maths subjects and tertiary entrance.

Contents

1 Purpose and Overview

The measurement process begins with the partitioning of an attribute of a **continuous** whole object into a number of equal parts which can then be counted to give the size of that attribute. Examples of continuous whole objects that can be measured include a length of string, the amount of surface enclosed by a shape, or the amount of water in a glass. Measurement involves the **comparison** of an amount of an **attribute** with a **unit** in order to determine a **number** (as shown in the figure below).



The central big idea here is **continuous vs discrete**. All measurement topics are continuous and cannot naturally be counted or represented by numbers. However, the invention of unit has allowed the continuous to be **"discretified"** or partitioned into units and these units can be counted. This application of unit changes learners' perception of the attribute being measured and of number itself. In measurement, number can never be alone; measures are given by both number and unit.

Thus, the sequence for teaching measurement topics has three parts:

(a) understanding the measurement topic in its natural continuous state (Stage 1 – identifying the attribute, and Stage 2 – comparing and ordering without numbers);

(b) introducing unit and number to measurement topics (Stage 3 – non-standard units); and

(c) understanding the standard units adopted by Australia and the applications of, and relationships between, these units (Stage 4 – standard units, and Stage 5 – applications and formulae).

These three parts are expanded to the five stages defined above, which are the major emphasis and structure of this book as the book provides information on how to teach measurement.

This chapter provides the purpose for, and overviews, the material in the book. It covers connections and big ideas (section 1.1), sequencing (section 1.2), teaching and cultural implications (section 1.3), and overview of the book (section 1.4).

1.1 Connections and big ideas

YuMi Deadly Maths (YDM) centres itself on the development of big ideas in mathematics. These big ideas reach across the different strands and topics. They are based on connections between these different strands and topics. Measurement is a strand, similar to number, that is built around a common set of big ideas.

1.1.1 Connections across strands

Connections are important in relating other strands to measurement and relating the topics within measurement to each other. The following are important connections that are related around number, operations and geometry.

1. **Number, geometry and measures**. In general, measurement is connected to number and geometry. In fact, for many of the attributes or measures, measurement can be thought of as the application of number to geometric shapes.

GEOMETRY ↓

NUMBER → MEASUREMENT

2. **Unit, fractions and division**. Fractions divide a whole into equal pieces, division does the same, and so does a unit – it divides a length measure (e.g. length) into equal sub-measures (e.g. metres or hand span). Thus, division, fractions and units of measure all have inverse relations; that is, the less to share (divide) the more each sharer gets, the smaller the denominator, the larger the fraction, and the smaller the unit, the larger the number of units (e.g. a wall measures 7 m or 7000 mm).

3. **Place value (PV) and metric conversion.** If whole number and decimal place values are built around a pattern of threes macrostructure (... billions, millions, thousands, ones, thousandths ...) with a sub-pattern or microstructure of Hundreds, Tens and Ones, then thousandths can be millimetres, ones can be metres and thousands can be kilometres; or ones can be millimetres, thousands can be metres and millions can be kilometres. In this way, PV can be seen as the same structure as metrics. If multiplicative structure is built between the macrostructure, we can see that thousandths → ones → thousands → millions is ×1000 and millions → thousands → ones → thousandths is ÷1000. This leads to metric conversions, for example, mm → m → km is ×1000 (1000 mm = m) and km → m → mm is ÷1000 (m ÷ 1000 = mm).

4. **Line, plane shape, solid shape and dimensions**. Lines are 1D, plane shapes are 2D and solid shapes are 3D. Thus, measures of size relate to units that are linear, squared and cubed.

5. **PV, metric and algebra computation.** For 34 + 52, the PVs are separated so 4 ones is added to 2 ones to give 6 ones and 3 tens is added to 5 tens to give 8 tens. Thus, for 3 m 46 cm + 5 m 21 cm, the answer is 3 m + 5 m and 46 cm + 21 cm which is 8 m 67 cm, and for 3a + 4b add 5a + 2b the answer is 3a + 5a and 4b + 2b which is 8a + 6b.

1.1.2 Stages within measurement

Like the YDM Number book, each topic in this book follows the same teaching framework. Measurement is taught in stages for each measure. It is crucial to work through all five stages in order as follows.

Stage 1	→	**Stage 2**	→	**Stage 3**	→	**Stage 4**	→	**Stage 5**
Identifying the attribute		Comparing and ordering (no numbers)		Notion of unit/ non-standard units		Standard units		Applications and formulae

These stages cover measures from continuous to discrete forms as units become important. The crucial point in this development is Stage 3 where the notion or idea of measurement **unit** is introduced. This is the point where number appears. Number is not used in Stages 1 and 2. However, do not be misled into believing that the first three stages are unimportant because they do not deal with number and units – they lay the foundation for understanding measurement – particularly some of the big ideas of measurement.

These five stages apply to all measures and **this book is built around them for all measures**. They are so important that Chapter 2 (Overview of stages) looks at the five stages in detail before the rest of the book looks at particular measures (e.g. length, mass, capacity, and so on).

Stage 1: Identifying the attribute

This stage focuses on students understanding the attribute (or concept) of the measure. Activities to identify attributes should follow **rich experiences** with general sorting and classifying activities and much discussion of more general attributes, such as colour, sound, etc. They should also involve **developing meaning** for all the specialist attribute language that accompanies measurement topics.

The central idea in learning about an attribute is to experience it. However, if students have difficulty identifying the attribute from other characteristics of the experience, there are two general ways to introduce any attribute by providing examples where:

(a) the only thing that is the same is the attribute, and

(b) the only thing that varies is the attribute.

Stage 2: Comparing and ordering

This stage focuses on comparing (i.e. two examples) and ordering (i.e. three or more examples) the amounts of an attribute in the examples. The process of ordering is based on comparison; the ability to compare two examples is extended to ordering three examples by identifying the one that is **between** the other two. Stage 2 activities are learnt in two parts:

(a) direct comparison and order where examples are compared directly to each other, and

(b) indirect comparison and order through an intermediary.

These activities involve **no units** and **no numbers**; the total amounts of the attribute present are **compared or ordered holistically**.

Stage 3: Non-standard units

This stage has two foci:

(a) introducing the notion of unit, and

(b) the development of measurement processes and measurement principles (i.e. big measurement ideas that hold across all measurement topics).

The units are non-standard or class/learner chosen so that the learner is familiar with them. The measurement processes differ for different topics; they are related to the proper use of the measuring instruments. **Measurement principles** are the techniques for using units. These can be organised under three headings.

(a) **Common units** – common units must be used in measuring, comparing/ordering and calculating amount of attribute and, in this case, the example with the most attribute has the larger number of units (this leads to the need for a standard).

(b) **Inverse relation** – the bigger the unit, the smaller the number and vice versa.

(c) **Accuracy vs exactness** – smaller units are more accurate but more difficult to apply, so there is a need to choose appropriate units for the level of accuracy required and to develop skill in estimating as well as accurate measuring.

Stage 4: Standard units

This stage focuses on the introduction of the standard units accepted by Australia. It should be remembered that these units should only be introduced after the need for a standard has been determined by recognising the limitations of non-standard units. It is recommended that the introduction of standard units be preceded by the use of a class-chosen **common unit** (if appropriate).

To learn standard units, students need to:

(a) **identify** the unit through experiencing it or constructing it,

(b) **internalise** the unit through relating it to body or everyday activities, and

(c) **estimate** with the unit before measuring.

Activities at this stage need to:

(a) relate to the decimal number system to build understanding of conversion between units, and

(b) continue developing the measurement processes and principles.

Stage 5: Applications and formulae

This stage focuses on: (a) applications of measures to the real world (i.e. calculating the measure of things); and (b) formulae for determining measures (tends to be restricted to perimeter, area, volume and angle formulae).

The triadic big idea is important here. Measures applied to the world have three components: object, number and unit (e.g. the gate was 864 mm high). This means that there should be three types of application:

(a) number unknown (*What is the height of the gate?*);

(b) object unknown (*Find something that has a height of 864 mm*); and

(c) unit unknown (*The height of the gate is 0.864, what unit has been used?*).

The last application type is sometimes not taught.

1.1.3 Big ideas

YDM has divided the big ideas for mathematics into global, concept, principle, strategy, and teaching big ideas. The *YDM Supplementary Resource: Big Ideas* book details these big ideas and is available to download from the YDM Blackboard site. However, for measurement, the big ideas are in different groupings determined by the five stages of measurement teaching: global, Stages 1 and 2, Stage 3, Stage 4, and Stage 5. They are listed below, first the global big ideas and then the big ideas related to the different stages of measurement teaching.

Global big ideas

1. **Continuous vs discrete**. Attributes can be continuous (smoothly changing and going on forever – e.g. a number line) or they can be broken into parts and be discrete (can be counted – e.g. a set of objects). Units break continuous length into discrete parts (e.g. metres) to be counted.
2. **Notion of unit**. Anything can be a unit – a single object, a collection of objects, a section of a line, a collection of lines. Units can form groups and units can be partitioned into parts. (e.g. if there are six counters, each counter can be a unit, making six units, or the set of six can be a unit, making one unit.)
3. **Interpretation vs construction**. Things can either be interpreted (e.g. measure mass with a spring balance for this shape) or constructed (e.g. construct a rubber band mass measurer).
4. **Accuracy vs exactness**. Problems can be solved accurately (e.g. find 5 275 + 3 873 to the nearest 100) or exactly (5 275 + 3 873 = 9 148).
5. **Attribute vs instrumentation**. The meaning of an attribute leads to the form of measuring instrument (e.g. mass is heft or pushing down on hand, so measuring instrument is how long it stretches a spring).

Stages 1 and 2 big ideas

These are related to meaning and comparison/order and there are two clusters as follows.

1. **Meaning gives instrument**. The understanding behind measures leads to the forms of instruments that measures them (e.g. mass is heft and so instruments are beam balances and spring-based instruments).
2. **Order principles**. Order is nonreflexive (i.e. A is not > A), antisymmetric (i.e. A>B means B<A) and transitive (i.e. A>B and B>C means A>C and the order is A>B>C).

Stage 3 big ideas

These are related to the role of units and they can be considered in three clusters.

1. **Common units**. We must use same units when comparing and calculating (e.g. a 3 m by 20 cm rectangle does not have an area of 60) and, if we do so, the object with the biggest number has the most attribute.
2. **Inverse relation**. The bigger the unit, the smaller the number and vice versa.
3. **Accuracy vs exactness**. All units give rise to error, requiring a tolerance for this possibility, with smaller units being more accurate but more difficult to apply, so there is a need to choose appropriate units for the level of accuracy required for the job and a need to develop skill in estimating as well as accurate measuring.

Stage 4 big ideas

These relate to the introduction of metrics and can be considered in three clusters.

1. **Need for a standard**. In a world with modern Western interaction, each attribute has to develop a set of standard units that always apply for that attribute.
2. **Identification, internalisation and estimation**. This is a sequence for introducing standard units.
3. **Multiplicative structure**. Standard units are designed so that they reflect place value in that adjacent positions are related by moving left (× base); moving right (÷ base), where the base is 10 for metrics but 60 for time and angle and, in practical terms, 100 for dollars-cents and temperature.

Stage 5 big ideas

These lie behind applications and are classic big ideas in that meaning is determined by the relation between the parts not the parts themselves. There are two clusters.

1. **Triadic relationships**. When applications are considered in measurement, there are three components (object, number and unit) and so, as in all triads, there are three problem types – one each for each component to be unknown (i.e. problems can have object, number or unit as the unknown).
2. **Formulae**. Formulae are relationships between attributes which hold for all ways the attributes could be measured as numbers (e.g. $A = L \times W$ holds for all rectangles, all lengths and all widths).

Teaching big ideas

There are also teaching big ideas. There are two clusters under this heading.

1. **Reality–Abstraction–Mathematics–Reflection (RAMR) pedagogy.** The four steps in the RAMR cycle (see section 1.3.1).
2. **Five teaching stages**. The five stages and their sequence (see section 1.1.2).

1.1.4 Extra information

From the above lists of connections and big ideas, there are three big ideas/connections that are worth further attention.

Continuous vs discrete

Number applies to discrete objects. However, there are attributes in measurement that are not discrete, rather they are **continuous**, for example, length and area (in fact all of the measures). A child's height does not grow in jumps of 1 cm – it smoothly increases. Similarly, the ground and the ocean spread in all directions continuously, they are not naturally broken into square metres. Thus, number does not naturally apply to measures such as length, mass, capacity, area, volume, temperature, time, angle and money.

Western culture invented the unit to apply number to continuous measures; that is, a set amount of the measure that could be repeated across the object being measured to determine the number of the units that fitted into the object. This finding changed number from general to specific and means that, in measurement, it is necessary to state both the number and the unit, such as 56 cm, 4.5 m^2, 6 kg, and so on.

Thus, Western culture found a way to **discretify** the continuous – to break the unbroken into parts that can be counted. It is important that the units which enable this to happen fit together across what is being measured without gaps and overlaps. Therefore the type of unit, particularly its shape, is important, especially in area, volume and angle.

The notion of unit

Number enables discrete objects to be counted. Arithmetic provides rules that govern number in a world where the objects being counted do not spontaneously appear, disappear, split into two or more of the same object, or join to form one of the objects.

One counts a single object. For large numbers, objects are bundled into groups and then into groups of groups. For Western society, this is around a base of 10. This means 10 number names for 0 through to 9, and then names repeat as groups (10s) and groups of groups (10s, 100s, 1000s, and so on). For example (note "ty" means "ten"): one, two, three, ...; forty-one, forty-two, forty-three, ...; and six hundred and seventy-one, six hundred and seventy-two, six hundred and seventy-three, ...

The groupings are based on a pattern of three: hundred, ten and one ones; hundred, ten and one thousands; hundred, ten and one millions and so on (see table below).

Millions			Thousands			Ones		
H	**T**	**O**	**H**	**T**	**O**	**H**	**T**	**O**
2	**6**	**9**	**2**	**6**	**9**	**2**	**6**	**9**

269 million
two hundred and sixty-nine millions

269 thousand
two hundred and sixty-nine thousands

269 ones
two hundred and sixty-nine ones

The groupings enable large numbers and small amounts (through decimal fractions). For small amounts, one object is the start. The whole object is partitioned into equal parts, again in terms of tenths, hundredths, thousandths, and so on. These are related in a similar way to groupings, for example:

ten ones = one ten — hundred ones = ten tens = one hundred

one into ten parts = one tenth — one into hundred parts = tenth of a tenth = one hundredth

There are two standard units important for navigation (and, therefore, important for Torres Strait Islanders) that are not based on base 10 place value. These are time and angle; both are base 60, made up of 60 minutes in 1 hour or 1 degree, and 60 seconds in 1 minute. Flexibility with regard to grouping will help in this transfer.

Relationships between attributes

Finally, relationships between attributes are also important for students to understand. The first relationship type is that between length, area and volume. In the most common instances, area is related to a square of a length (l^2 or r^2) while volume is related to area of base and length (height) and to the cube of length (l^3 or r^3).

The second relationship type is between attributes that lead to rate. For example, length and time are related to give speed, and volume and time are related to give fuel consumption rate, both of which are vital in boating. The standard distance on water is a nautical mile (nm) which is the length at the equator that is given by a turn of the earth (or a movement of something on the sea) equal to 1 minute of a degree at the centre of the earth. A knot is 1 nm per hour. Hence, it is important to teach both length/distance and angle.

1.2 Sequencing

This section looks at the content to be covered in the book and discusses the reasons for the grouping and sequencing in this book.

1.2.1 Content

There are 10 attributes or measures that form part of YDM's view of measurement and these are divided as follows.

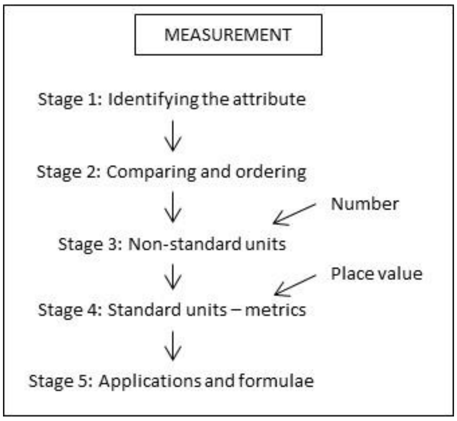
1. **Length, Mass and Capacity** (liquid volume). These are basic measures in that they are base 10, have metrics that follow the pattern of threes (i.e. relate to ones, thousands and millions, e.g. mL, L and kL) and do not have formulae.
2. **Perimeter, Area and Volume** (solid volume). These are relationship measures in that they are base 10 but have metrics whose relation is higher than 1000 (e.g. m^2 is 1 000 000 mm^2); they relate to each other in that perimeter is length, area is $length^2$ and volume is $length^3$, and they have formulae that represent this relationship.
3. **Time and Angle**. These are common measures that have a base of 60 and have formulae.
4. **Temperature and Money**. These are measures that are base 10 to some extent but not completely in terms of units (money is complete symbolically but does not have the one cent piece in reality).

It should be noted that time and money are also different to the other measures in that they involve more than understanding an attribute and using units through the five stages; they also have other outcomes to achieve.

1. **Time**. For time, there is **point** of time (which is reading a clock – telling time); there is **sequence** of time (which is hours and minutes in day, days in week and in months, months in a year, years in centuries and centuries across history); and there is **duration** of time (which is the meaning of time and its units and the five stages).
2. **Money**. For money, there is money handling (knowledge of notes and coins and how to use them); and there is understanding of money as **value** (using the five stages).

1.2.2 Sequencing

Each of the measures above has to cover the five stages. In all cases, this sequence becomes the way to teach each of the measures. This sequence is shown on the right. It applies to all measures.

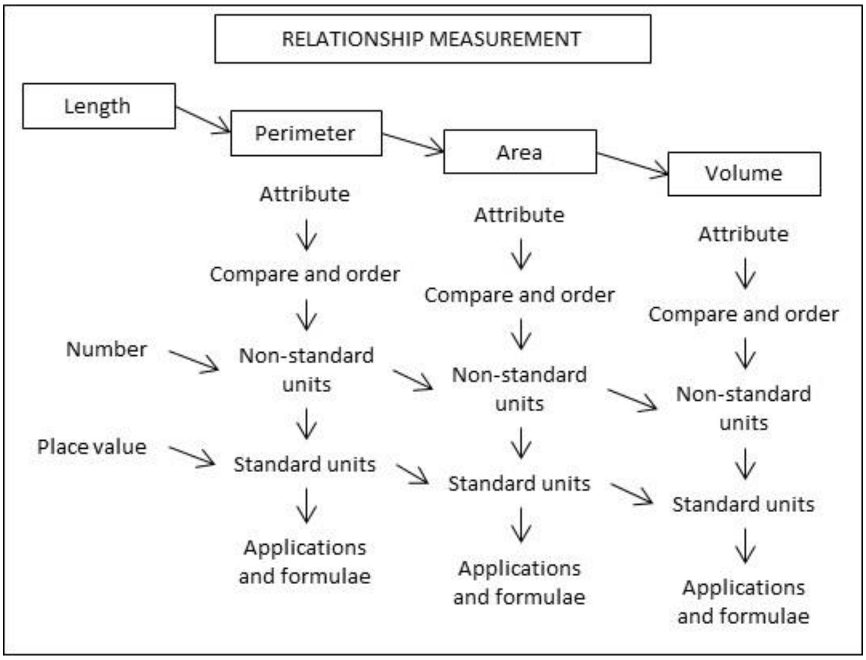


As can be seen in the diagram, the first two stages have **no number**, the third introduces unit and number, while the fourth introduces metrics, and therefore is based on place value. Number is introduced in Stage 3 and place value is introduced in Stage 4 to help understand conversions between units.

For time and angle and, to a lesser extent, for temperature and money, the place-value input is not the classical decimal place-value structure. This is because time and angle are not base 10 but base 60, while temperature has a relationship of 100 between freezing and boiling but only one unit. With the loss of the one- and two-cent coins, Money does not have a strong base of hundredths relating cents and dollars.

The five stages are described in detail in Chapter 2 of this book. The ways in which the stages are used in all measures mean that each measure is a separate sequence.

For teaching of the measurement topics perimeter, area and volume, there is a special sequence as below. In this, length begins additively with perimeter, becomes multiplicative with area and is based on the square of length, while volume adds another multiplier and is based on the cube of length. The important point is that if we have a rectangle and a rectangular prism, doubling the dimensions for the rectangle means that the perimeter is doubled but the area is quadrupled (×4), while the volume of the rectangular prism increases by eight times.



1.3 Teaching and cultural implications

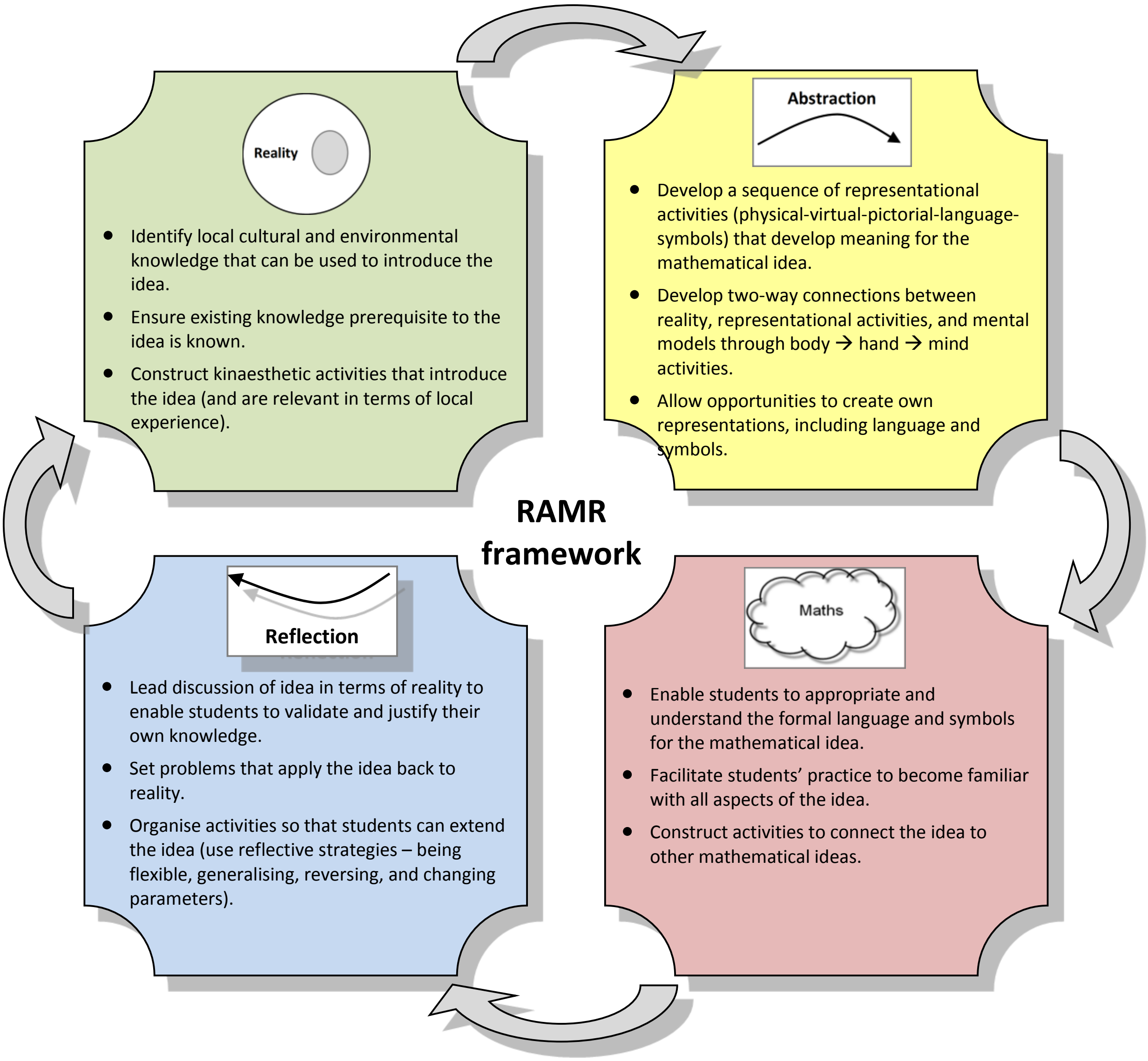
This section looks at implications of measurement for teaching and culture. Obviously, the five stages of teaching are crucial here, and so important that Chapter 2 covers them in more detail.

1.3.1 Generic teaching strategies

YDM sees mathematics teaching as having three components – **technical** (handling materials), **domain** (the particular pedagogies needed for individual topics) and **generic** (pedagogies that work for all mathematics). Interestingly, and fortunately, the domain section is not as complicated as it could be because mathematical ideas that are structurally similar can be taught by similar methods. For example, fractions and division are similar and both are taught by partitioning sets into equal parts – except that the set is seen as one whole for fractions and a collection of objects for division.

Within measurement, the continuous to discrete movement provides five steps that all measures follow. Thus, this provides a generic framework. So also does the **RAMR framework** (see the figure on the next page) – it works for all mathematics topics.

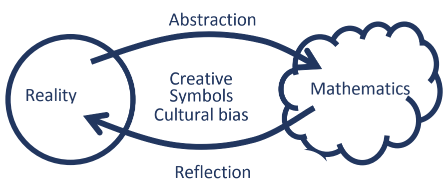
RAMR is very useful because of the generic teaching ideas contained in the framework. For a start, it grounds all mathematics in reality and provides many opportunities for connections, flexibility, reversing, generalisations and changing parameters, as well as body → hand → mind. The idea is to use the framework and all its components throughout the years of schooling and this will help prevent learning from collapsing back into symbol manipulation and the quest for answers by following procedures.



Note: Flexibility and reversing should be applied in all four components of the cycle and not restricted to Reflection.

1.3.2 Cultural implications for measurement

By its nature, mathematics is a creative cultural and contextual **abstraction** (or generalisation) of everyday life that empowers people to solve their problems. However, as shown in the diagram on right (see YDM Overview book), such abstraction has to take account of **local culture and context** in the abstraction process which gives rise to cultural bias. Unthinking mathematics teaching transfers that bias to students, and can negatively affect cultural beliefs and pride in heritage of students not of the dominant culture.

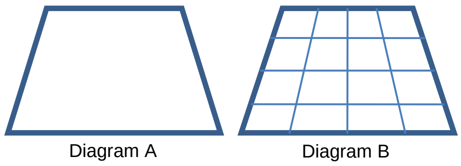


THE RELATIONSHIP BETWEEN LIFE AND MATHS

The nature of Western measurement

As already discussed in section 1.1.4, number applies to discrete objects, but measurement attributes are not discrete. Western culture found a way to apply number to these continuous measures by inventing the notion of unit. It is important to understand that using units can change the way a person perceives the attribute or measure. For example, land can look very different when seen as a continuous area than when seen as broken into units (see diagrams A and B on next page).

Diagram B tends to focus attention on parts and personal ownership and then onto ideas like “how much”, “how can I get more”, “what can I buy and sell”, “how can I exploit this bit”, while diagram A focuses attention on the whole and onto ideas like “how does it all interact” and “how can we keep it together”. Diagram B looks like house blocks while diagram A looks like a national park. This change in perception can affect cultural values. For example, some Indigenous cultures have no conception of “owning land”, and teaching area as square units can change this profoundly and challenge culture.



The impact of number and measure on Western culture

It could be argued that number is important in cultures where personal belongings are important (and, maybe, more important than the quality of the person) and status depends on the number of things that you have. You then need to count to determine who has the most things (e.g. cattle, goats) and to ensure that none of what you have is lost or stolen. Rulers gain their status from what they have and maintain this status by taking a share of what their people produce. They need numerically literate bureaucrats to gather their taxes. In this way, larger and larger numbers are needed.

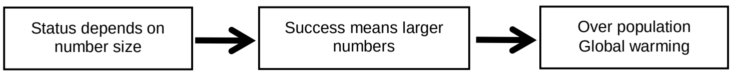
Over time, number becomes important for everything and begins to determine everything. We win the battle by having more guns. We determine whether we are better off by counting our Gross National Product. We work out whether we are happier by counting television sets. We work out whether we are doing better than our forebears by counting the number of hectares being farmed, the number of cars being sold, and the amount of money spent on shopping. Improvement appears to be determined by bigger numbers, so sales have to be higher and profits larger, more land has to be cleared and more coal and iron ore mined.

The **strength** of Western culture and Western measurement (if you can call it that) lies in the growth of science and technology that mathematics underpins. With better weapons and more products, Western culture comes to dominate the planet (with only the Chinese/Japanese and Hindu/Arabic cultures competing). Non-numerical cultures are driven aside and considered as primitive. The land and sea is developed to fit in with the modern lifestyle; the heat is cooled, the cool is warmed, and buildings, cities, and monoculture farms cover the earth.

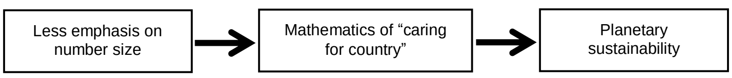
The **weakness** of Western culture and measurement lies in this same dominance. Success means increasing numbers and continued growth; no growth and negative growth are to be avoided (and are given names that signify failure, such as “recession”). The culture has little understanding of harmony and sustainability; it tries to dominate the land, the sea, the weather, and the animals, birds and fish. It has become arrogant believing all other cultures should follow in its footsteps. However, climate change is the latest in many indications showing that Western culture’s numerical addiction to growth could destroy the planet.

There needs to be a new way of looking at number and mathematics that leads to planetary harmony and sustainability. Such mathematics is most likely to emerge from a culture with less dominance by number, less need for growth, and a history of living in harmony with land and sea.

The secret is to sit outside of Western number and measurement and realise what it is – **to learn it but within an acknowledgement of its strengths and weaknesses** – and, if local culture is not one dominated by number, to not let it affect beliefs and pride about own culture.



The “progress” of Western number



A better mathematical progress

Questions

The mathematics taught in Queensland schools is based on Western culture, originating in Europe and influenced by America. As presented in schools, Western mathematics is based on number – whole numbers, fractions, money, measures, chance and data. As such, it reflects an imperative of Western culture regarding the primacy of number. In this, mathematics reflects an often unidentified component of its nature, that it is culturally based and biased and that its teaching has a large aspect of enculturation in relation to Western culture. (*Note:* Two other major cultures are also similar in their focus on mathematics and number, Hindu-Arabic and Chinese-Japanese, and Australia has many people from these cultures, but Australian mathematics education is still primarily European based.)

Therefore, questions that are basic for this subsection (and this book) are as follows.

1. Why would a society give high cultural importance to number? What affect does this have on that society? What strengths and weaknesses of such a society could be attributed to their number orientation? That is, what strengths and weaknesses does number give a society like Australia?
2. How would a number-oriented society interact with other societies, particularly societies that do not have its number orientation? What effect could Western teaching of number have on students from non-Western societies (e.g. Torres Strait Islander)?
3. What role does measurement have in relation to number? Could the teaching of Western measurement have a negative effect on non-Western societies (e.g. Aboriginal and Torres Strait Islander people)?
4. In particular, how can we teach Western measurement so that Aboriginal and Torres Strait Islander students understand it yet stay strong in and proud of their culture? (Both outcomes are necessary for a strong future for Aboriginal and Torres Strait Islander students.)

1.3.3 Teaching measurement to take account of culture

The recommended way to take account of cultural aspects when teaching measurement is to:

(a) make students aware of the implications of Western measurement – the way in which continuous entities are given numbers by the use of units, and how this changes perceptions of those entities; and

(b) first teach the measure as continuous before bringing in units, and use local culture-based understanding of attributes and units before the formal Western mathematics units.

Thus, we use a five-stage process for teaching measurement – Stages 1 and 2 are pre-number, Stage 3 introduces the notion of unit and applies number to continuous attributes for the first time, and Stages 4 and 5 are post introduction of number.

For Indigenous schools where the classroom culture differs from the mainstream Western classroom culture, it is important to use the stages of measurement teaching to maintain Indigenous culture as well as learning Western measurement. To achieve both these outcomes, the following are important.

Pre-number measurement

It is important to spend time on Stages 1 and 2 – identifying the attribute, and comparing and ordering (without units or number). This means spending time getting to know what the attribute means (e.g. what area, mass and angle actually are), and what it means for there to be more or less of the attribute (e.g. what does it mean for an area, mass or angle to be larger than another one).

The crucial part of this stage is to enable the students to understand length (perimeter), area, volume (liquid/solid), mass, time, value (money), angle and temperature as continuous entities and to relate this to the understanding of these attributes that are in the local culture. This requires an understanding of associated language, as attribute language is a crucial part of early measurement. For instance, lengths in different directions and of different types have different English words, such as long/short, tall/short, wide/narrow, thick/thin, deep/shallow, high/low, and so on. Some differences between words are subtle such as the difference between "tall" and "high"; other words also mean special types of length (e.g. "distance"). All these different words associated with length need to be understood. With students of a different culture and home language, this requires initially associating Western meaning with local cultural understanding and Western mathematics language with home language.

To begin the process of teaching Stages 1 and 2 of measurement to Indigenous students, it is recommended that teachers approach local teacher aides, parents, elders and members of councils and community groups for their support and ideas. Find what the local culture means by length, area, and so on, and more/less, and so on. Find the local words and phrases that mean the same as the Western words that you would like to teach. Act out what the Western words mean – sometimes it is hard to find a local way of saying what the Western word actually means until people see the actions. Acting out will also allow the students to tell you what they would call it in their local language. It is also recommended that teachers give special attention to attributes that relate to strong local cultural and contextual needs. For example, in the Torres Strait Islands it is important to give ***angle*** *greater emphasis than may be given in another culture because of its role in navigation.*

Notion of unit

Stage 3 becomes the pivotal stage in the development. Here the invention of unit is introduced and students are shown how this has enabled a discrete entity, number, to be applied to continuous entities such as length, area, and so on. Students should be made aware of how this can change the way things are perceived – it gives us something but it takes away something too. Because introducing two things at once – the notion of unit and the actual Western units – can cause difficulties in learning, Stage 3 is designed so that the notion of unit is introduced with everyday or child-chosen units based on body parts or highly familiar things.

The application of unit and number to attributes also brings with it a baggage of principles that can be best introduced in Stage 3 before the standard units of Stage 4; for example, knowing that: (a) we must use the same units when comparing; (b) if using the same units, the largest number means more; (c) using larger units means a smaller number and vice versa; and (d) small units give greater accuracy. There are also skills to be acquired when using units (e.g. no gaps and overlaps, keeping sticks in line, starting at 0 for sticks marked into sections).

When introducing Stage 3 to Indigenous students, it is recommended that teachers approach their students, aides, parents and elders to find local non-standard units (e.g. measuring distance between islands in terms of drums of fuel) and familiar everyday items that can be used as non-standard units. The local name for these items can also help, as can any local techniques for using them or recording numbers. History can be useful here. Even Western history about hours, minutes, pints, gallons, acres, and so on can build understanding. However, the imperative behind units, to enable commerce and determine wealth and belongings, should be understood.

Formal units, applications and relationships

Once into Western measurement and the standard units, it is important to ensure that the new material is connected to number, but differentiated from number. The most important of these is to build the relationship between metrics and place value. Finally, relationships between attributes are also important for Indigenous students as formulae are important in many jobs. So ensure that metrics and formulae are taught for this reason, not as part of the greatness of Western mathematics.

Teaching

Mathematics teaching needs to be from the local world of students to the formal abstract mathematics and back again so that it imitates the nature of mathematics. This means that mathematics teaching should move between three worlds:

(a) the culture and context of Indigenous culture;

(b) informal understandings (materials, pictures, patterns) and language that can bridge from everyday life to abstract mathematics; and

(c) formal abstract mathematics (language and symbols).

1.4 Overview of book

This book outlines a teaching sequence for measurement and then provides details for this teaching sequence for each of 10 measurement attributes. Length, mass and capacity are direct or uni-dimensional measures. Each of these attributes can be measured by comparison to a single linear scale. Perimeter, area and volume are measurements that are calculated after the measurement of dimensions have been taken. Time and angle are two attributes of measurement that are not decimal but work from a base of 60. Temperature and money are the last two attributes which are often included in the curriculum as relating to measurement. Both of these attributes are related to their use of 100 as a base for the relationship between units.

The book has the following major sections:

Chapter 1: Purpose and overview – describing connections and big ideas, sequencing, and teaching and cultural implications.

Chapter 2: Overview of measurement stages – outlining the teaching-learning stages for measurement that will be the basis for each of the following chapters.

Chapter 3: Length, mass and capacity – detailing how to teach these measures which reflect decimal-number place value in their metrics.

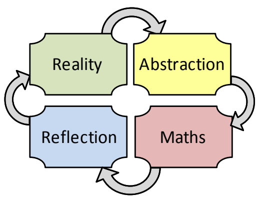
Chapter 4: Perimeter, area and volume – detailing how to teach these measures which are related through formulae.

Chapter 5: Time and angle – detailing how to teach these measures which are not metric (based on 60).

Chapter 6: Temperature and money – detailing how to teach these measures.

Appendix A: Teaching tools.

Within Chapters 3 to 6 of this book, activities for teaching are often but not always provided using the RAMR framework in 1.3.1 and on right. The framework provides the best way to teach anything and also enables the big ideas to be provided in relation to the stages. Activities that are given in RAMR framework form are identified with the symbol on right and are written in italics to help distinguish them from the main text.



2 Overview of Measurement Stages

All measurement topics are continuous and cannot naturally be counted or represented by numbers. However, the invention of unit has allowed the continuous to be discretified or partitioned into units and these units can be counted. This application of unit changes learners' perception of the units. Thus, the sequence for teaching measurement topics has three parts:

(a) understanding the measurement topic in its natural continuous state;

(b) introducing unit and number to measurement topics; and

(c) understanding the standard units adopted by Australia and applications of, and relationships between, these units.

These three parts are expanded to five stages as below. (*Note*: In measurement number can never be alone: measures are given by **number and unit**).

Stage 1: Identifying the attribute → Stage 2: Comparing and ordering → Stage 3: Non-standard units → Stage 4: Standard units → Stage 5: Applications and formulae

This learning sequence is not linear but provides a general sequence with overlaps. All attributes and concepts can be explored through this process at all year levels. The following sections of this chapter describe each of these stages in general.

2.1 Identifying the attribute

This stage focuses on students understanding the attribute (or concept) of the measure. Identifying the attribute enables students to understand length, mass, capacity, time, money and area as continuous entities and to relate this to the understanding of these attributes that are in their local culture. This requires an understanding of attribute and comparative language, as attribute language is a crucial part of early measurement. As discussed in section 1.3.3, lengths in different directions and of different types have different English words, such as long/short, tall/short, wide/narrow, thick/thin, deep/shallow, high/low, and so on. Some differences between words are subtle such as the difference between "tall" and "high"; other words also mean special types of length (e.g. "distance"). Using items from the real world will assist students with learning about the attribute of length and its associated attribute and comparative language.

2.1.1 Importance

Identification of the attribute needs to be strongly established before undertaking formal measuring activities, for three reasons. Firstly, length, mass, capacity, angle, time and temperature are all uni-dimensional measures. They can be measured on a single calibrated scale. The other attributes of perimeter, area and volume are multi-dimensional and require calculation after measurement of dimensions. This difference is identified in attribute identification.

Secondly, understanding an attribute provides insight into possible measuring devices. It is also the cause of many problems with measurement. If the attribute is not properly defined then the measuring instrument does not provide a good indication of amount. When a student knows what attribute is to be measured, then the student is more likely to select an appropriate measuring device (instrument). For example, before they can compare or measure an attribute such as mass, students must be aware of what mass is and this has to come from experiencing instances of the attribute. Thus, students need to experience the "heft" of objects (the distance the hand is "pushed down"). When students have experiences such as these, the use of pan, spring and scale balances makes more sense. As well, students need to be able to see differences between measures – many students mix up solid volume and mass.

Thirdly, identification is a good time to ensure language is known. Objects that students encounter in their world have many attributes which can be measured for different purposes. Someone could measure the height of a table to see if it matches other existing furniture, they could measure its length to check how many people could be comfortably seated on each side, they could measure its width to see if it can fit through a doorway, they could measure the area of the table top to see if it is suitable for completing a particular jigsaw puzzle on, or they could measure its mass to consider costs of transporting it to a new house. Attribute identification and associated language can be developed through activities in which the **only** thing that varies is the attribute and activities in which everything varies **except** the attribute to be established (see 2.1.2 for more on activities).

It is important for attributes such as length, area, volume, and so on, to be understood in their continuous state. In this state, they appear to be understandable in all cultures. It is an opportunity for two-way sharing of understandings and language.

Attribute	Language (opposites)
Size	big/small, big/little
Length	long/short
Height	tall/short
Mass	heavy/light
Capacity	full/empty
Volume	full/empty
Time	early/late, day/night
Width	thick/thin, wide/narrow
Area	covers more/less
Distance	far/near
Temperature	hot/cold, high/low

2.1.2 Activities

Students need experiences with activities that require them to explore attributes of objects so they can identify which attributes change and which don't in different circumstances. For example, moving a table from one place to another in a room does not change its length but filling a bottle with water will change its mass. Once students are familiar with the different attributes they can begin to make comparisons about different amounts of each attribute.

Activities to identify attributes should follow **rich experiences** with general sorting and classifying activities and much discussion of more general attributes, such as colour, sound, etc. They should also involve **developing meaning** for all the specialist attribute language that accompanies measurement topics.

The central idea in learning about an attribute is to experience it. However, if students have difficulty identifying the attribute from other characteristics of the experience, there are two general ways to introduce any attribute by providing examples where:

- the only thing that is the **same** is the attribute (e.g. ribbons, sticks, pens that are all the same length), and
- the only thing that **varies** is the attribute (e.g. pink ribbons that are made of the same material that are all different lengths).

2.2 Comparing and ordering

Comparing is the process of determining whether two objects or events are the same or different in relation to specified attributes. Comparing is the forerunner of **ordering** and **measuring** and to be able to compare, the student must be familiar with the attribute, have acquired the ability to perceive similarities and differences, and know the specific language that is used for describing particular comparisons. Therefore, comparison activities cannot be undertaken until the attribute and its associated language have been established.

Students should undertake comparing activities involving two objects, before ordering three or more objects. The change from comparing two to ordering three or more is difficult as it requires a focus on **between-ness** (i.e. identifying the example which is between the other two).

Once students have identified the attribute in focus for a measurement task it is possible to compare objects according to the attribute and to order objects according to increasing or decreasing amounts of the attribute, e.g. length. Identify objects that are long or short and compare two objects to identify which object is longer or shorter. The focus of language here is on “er”.

The important aspect of this stage of the learning sequence is that it does not involve any numbers because the focus is not a number of units but direct or indirect comparison and ordering of various measurement attributes. The total amounts of the attribute present are compared or ordered holistically.

In the picture above the two objects can be compared in a number of ways. The straw is longer than the stick. However the stick is wider than the straw. The stick is heavier than the straw. The attribute or aspect of the attribute that is in focus will determine the comparison that is made.

There are two ways to compare measurement attributes:

- **direct comparison** and order where examples are compared directly to each other; and
- **indirect comparison** and order through an intermediary.

Sequentially, comparing activities should proceed from **direct to indirect** comparison activities to ordering of three objects and then ordering of more than three objects.

2.2.1 Direct and indirect comparison

Direct comparison involves, as the name suggests, a direct comparison of two attributes by focusing on a particular attribute. To directly compare the height of two people they can stand back-to-back and the taller will be identifiable by sight. To directly check if a table is wide enough or long enough to fit through a doorway the table is taken to the door and the comparison is made by trying to get it through. To compare the mass of two bags a person can pick them both up and feel which is heavier. This process is called hefting.

Direct comparison activities can often be made visually or sensorially (e.g. by feel when hefting) and can be validated by directly aligning the attributes to be compared. These activities should involve the following types of situations:

- comparing similar objects where only one attribute varies (e.g. two pencils of different lengths); and
- comparing different objects (e.g. comparing the length of a pencil and a pair of scissors).

Indirect comparison involves the use of an **intermediary** item that can represent the measure of the attribute being compared for one of the items and then this intermediary item is compared to the second item. For example, comparing the length of the tray of two utility vehicles by using outstretched arms to represent the length of one tray, then walking to the other ute and comparing the outstretched arms to the tray of the second vehicle. The measurement of one object is represented by the intermediary and then this is compared to other objects rather than the objects being positioned side by side for a visual or sensory comparison.

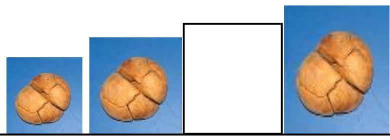
Indirect comparison is also the time that the equals and order principles should be known, particularly that equals is symmetric (A=B means B=A) and comparison is antisymmetric (i.e. A>B means B<A) and transitive (i.e. A>B and B>C means A>C and the order is A>B>C). Put together, these mean that if B<C and A=B, then A<C (the intermediary is the B).

2.2.2 Ordering

Ordering is the arrangement of more than two objects on the basis of a particular attribute (e.g. height). Ordering builds on comparing and develops flexible thinking, visual thinking and logical thinking. Sequentially it moves through four steps as in the table below. Once two objects are compared more objects can be systematically compared to create an order. Therefore ordering is a process for more than two objects.

The process of ordering focuses on between-ness and starts with identifying the extremes (e.g. longest and shortest) then comparing each new object to either extreme to choose which is more like. The process of comparing involves two objects, but multiple comparisons are needed to place a group of objects in order. The focus of language use here is on "est"; e.g. to find the longest of three ribbons.

To develop students' abilities to order a number of objects a sequence of activities can be used (see table below).

1. **Copying a given order**. Begin this step with same materials (e.g. all shells as in example on right) and then use different materials (different shapes or different objects). Assist students by identifying the attribute through discussion. Also allow the students to determine the attribute.	
2. **Extending a given order**. Once again, using same materials should precede using different, and again there should be discussion with students on the attribute.	 . . .
3. **Supplying the missing parts** (terms) in a given order. Assist the students at the start by giving them a set of objects to choose from. After they have mastered this, allow them to create the missing part on their own.	
4. **Inventing an order** for a given set of objects. Provide the objects and allow students to order in their own way and describe the measurement attribute chosen for the ordering.	

2.3 Non-standard units

The notion of a unit is very important in measurement. The whole idea of measurement being the **discretification of continuous quantities** underlies the need for units. This stage of the learning sequence is the first time that numbers are introduced to the measurement activities. Any objects can be used as units at this stage of the learning sequence, e.g. hand spans, leaves, pencils, counters, paces. By using units that are not standard, students can be helped to realise the need and benefits in having standard units. A progression from student-chosen objects, often non-uniform non-standard units (e.g. leaves to measure the length of the path between two buildings [length] or hand prints to measure the size of a table top [area]) to uniform non-standard units (e.g. the same size exercise books to measure length or sheets of A4 paper to measure the desktop) can assist students to learn about the use of units and then to learn about the need for standard units.

When measurements do not need to be highly accurate the use of non-standard units can also assist students to understand about estimating measurements.

Many important concepts about measurement can be taught through the use of non-standard units, as described below.

Nature of measurement

- Measurement involves **both a number and a unit** of measurement. To find out how much of an attribute an object has it can be broken into units and counted. Then the count can be compared for the same attribute in different objects.

Common units

- Measurements of particular attributes can easily be compared if they are taken in common or same units. Measurements can only be compared if the measurements are taken in common units. If the length of one table is taken in paperclips and the other in pencils then even though the measure of each can be counted, the numbers obtained cannot be compared because the units were different.
- When measuring an object the unit being used should be common throughout, that is, it should not change. This means that you might need a large quantity of a particular unit.
- If common units are used then the number of units determines which object is larger with respect to that attribute. In economies like Australia, this means that everyone has to use the same or common unit which leads to the need for a standard.
- Standard units are particularly useful when measurements need to be communicated to others. Using feet or hand spans depends on the person but metres are understood across contexts and countries.

Inverse relation

- The size of the unit is inversely proportional to the number that will be needed to measure a particular object. The smaller the unit used the more units needed and vice versa.

Accuracy vs exactness

- Measurements are best taken using a unit that possesses the same attribute that is being measured. Measuring mass is best done with non-standard units that have a noticeable mass. Measuring the mass of an item using feathers can be done but it would be better to use marbles. Measuring area with square tiles or other shapes that tessellate works better than using counters that leave gaps.
- The unit chosen for a measuring task can change according to the context. Measuring the amount of water in a drink bottle can be done using cups and it would be possible but tedious to measure the capacity of a fish pond using the same cups. Therefore students learn the units are not absolute but can be chosen for convenience.

- The smaller the unit used the more accurate the measurement will be. To compare the mass of two items using large marbles might show no difference, but if smaller, lighter blocks are used the difference can be identified.
- All the above points require students to see measurement as needing a tolerance for error and to be able to choose units appropriate to the error tolerated.

2.4 Standard units

Once students have had experience working with non-standard units they should realise that the use of standard units can be preferable in many situations, especially when measurements need to be communicated to others. By using units that have a standard, fixed amount of an attribute, measurements can be communicated clearly between people and across countries. Studying early measurement units, e.g. cubits, feet and so on, can assist students to see why standard units were developed.

2.4.1 Formality of standard units

Measuring with standard units brings in other important skills and considerations involving formal notation for recording measurements, conversions between units and using specific measuring tools accurately, e.g. rulers, protractors, and reading scales on various measuring devices.

To learn and understand standard units it is important to ensure that students:

- **identify** the unit through experiencing it or constructing it;
- **internalise** the unit through relating it to body or everyday activities; and
- **estimate** with the unit before measuring.

Students should become familiar with the **base unit of measure** in each family of measures – for example, metre, kilogram, and litre respectively in the linear, mass, and liquid volume families – through undertaking measuring activities in which whole and part-whole units of measures are required. Abbreviations for units should be introduced when students feel a need for speedy recording. Note that no plurals are used and the only capital letter is "L" to avoid confusion with the number "1". (In high school, students will learn that units of measure that take the name of the person who identified them are capitalised, e.g. kiloWatt/kW; Joule/J; Celsius/C.)

The recording of parts of units of measure should follow the development of common and decimal fraction understanding. Therefore, in Years 3 and 4, students should use a smaller unit of measure to denote the parts (e.g. 6 m 25 cm). From Year 5 on, they could also use fractions to denote the parts (e.g. $6\frac{1}{4}$ m or 6.25 m).

Other units within a family of measure should be introduced in relation to the base unit. For example, the centimetre and millimetre are smaller than the unit, metre, while the kilometre is larger. At this stage, students need to become aware (or consolidate their understanding) that smaller units of measure give more precise measurements because the smaller the unit of measure, the larger the number of units required to measure an attribute (and the converse).

2.4.2 Relation to place value

Students should understand the language in units of measure, namely, that units of measure within each family of measures all contain the base unit (e.g. kilo<u>metre</u>, <u>metre</u>, centi<u>metre</u>, milli<u>metre</u>). As with most of mathematics, there is an exception, namely, the units of mass measure (tonne/megagram, kilogram, gram) where the kilogram is the base unit but "gram" is the base word. Students should also understand that the prefixes give the clue as to whether the measure is larger or smaller than the base unit. For example, "kilo" always means 1000 times larger, "centi-" and "milli-" end with an "i" and indicate parts of the unit (one hundredth and one thousandth respectively). Students could make up names for measures that are one hundredth of a gram (centigram) or litre (centilitre) both of which are used in European countries.

Once into Western measurement and the standard units, it is important to ensure that the new material is connected to number, but differentiated from number. The first way in which to do this is to build the relationship between metrics and place value as in the table below.

Thousands	Hundreds	Tens	Ones	tenths	hundredths	thousandths
kilo kilometre	hecta hectametre	deca decametre	unit metre	deci decimetre	centi centimetre	milli millimetre

This strong relationship between metric prefixes and place value is important for metric conversions (knowing the relationships between metric units).

2.4.3 Teaching sequence

An effective **sequence** for introducing a standard unit, such as metre, is the following four steps:

Need for a standard/common unit – examples are looked at which support the sense of a standard (e.g. measuring a wall by pacing gives different answers depending on the measurer's stride), including some use of a common unit (e.g. a length of wood) by the class.

↓

Identification – constructing actual units out of materials, or experiencing examples of them (e.g. making a metre out of 10 cm straws).

↓

Internalisation – finding examples/referents in students' own bodies or in common everyday items, that equal units in size (e.g. a metre is the length from tip of left hand to top of right shoulder).

↓

Estimation – being able to "think" in the units and visualise them sufficiently to be able to make good "educated" guesses at the measure of examples (pick something to measure in metres, such as a wall, estimate it, measure it, reflect on any difference, and then measure something new).

The four steps above will be very important if the local culture of students does not have many experiences with Western measurement units, or uses different units. This is an opportunity to explore the dimensions of playing spaces for sports as a method of internalising standard units. It will also require care to be given to the two forms of distance measurement that are needed for sea-based economy, namely, millimetres, metres and kilometres, and nautical miles (particularly as these have a different base).

2.4.4 Imperial and metric units

Early units of measure were derived from the measurement of different parts of the body; for example, an inch was the length of a thumb from the tip to the first knuckle and a cubit was elbow to fingertip. Difficulties arose when this measurement differed on different people. The need for a standard for these units was identified. Imperial measurements were standard in that an inch was a recognised length, as was the foot and mile, and so on. However, imperial units of measure lacked easy conversion factors. There were 12 inches in a foot and 3 feet in a yard and so on. In some countries imperial units of measurement are still used. The difficulty of these units is the complexity of the relationships between units in particular attributes, and the lack of a relationship between the units used for different attributes.

It was the French who determined the need for another system of measurement units and developed the metric system, which related to the decimal number system that had been widely adopted and was based on multiples of 10. A measure of length, the metre, was determined, based on the length of one ten-millionth of the distance from the North Pole to the equator. From this length the other metric units were derived. This international system of units is called the SI system, derived from the French term *Système International d'Unités*. The SI system of metric units allows conversion not only between units of the same attribute, e.g. 1000 mL = 1 L, 10 mm = 1 cm and so on, but also between attributes, e.g. 1 mL = 1 g = 1 cm^3 (of water at 4° C at sea level).

2.5 Applications and formulae

The final stage of the learning sequence should be introduced once students are comfortable with measurement process, units and measuring devices. The application of measurement in standard units is to use these measurements to solve problems, including calculating other measurements. There are two possibilities that are worth exploring.

- **Triadic relationships:** When applications are considered in measurement, there are three components (object, number and unit) and so, as in all triads, there are three problem types – one each for each component to be unknown (e.g. problems can have object, number or unit as the unknown). The application which focuses on the unit being unknown is sometimes not given its importance. For example, when measuring area, it is possible to: (a) find the area of objects for different area units; (b) find objects that have a particular area (number and unit); and (c) work out the area unit being used when given an object and a unit.

- **Formulae:** Formulae are relationships between attributes which hold for all ways the attributes could be measured as numbers. In other words, they are general rules that apply regardless of dimensions (e.g. A = L×W holds for all rectangles, all lengths and all widths). Not all measures have an important focus on formulae. The ones that do are perimeter, area, volume, angle and money.

The attributes of perimeter, area and volume can be measured as a count of units but it is possible to calculate these and the realisation that these calculations are possible can help students to understand and develop powerful tools for generalisation. The formulae we use for calculating perimeter, area and volume have evolved into their current efficient form over a long period of time and involved observations, theory development, generalisation and refinement. Students need time to emulate this process of evolution, starting with them constructing their own ideas for formulae based on a deep understanding of the attribute. Self-constructed formulae enable students to make sense of their calculations. They also provide a solid base on which to develop the accepted efficient versions of these formulae.

Unfortunately, formulae are often introduced to students without them having experienced the previous learning sequence stages or gaining familiarity with the attributes and the measurement of them. For example, many students have their first real experiences of the concept of area with the formula for the area of a rectangle. Students who have only had experiences of measuring these attributes through the use of formulae have difficulty in identifying appropriate formulae for particular circumstances and in applying the appropriate formulae to find the area of irregular shapes.

2.6 Sequencing learning activities for measurement attributes

Using the five stages of measurement described in this chapter, the sequence below is recommended for the development of measurement concepts relating to any attribute.

1. **Identifying the attribute**
    - Direct comparison of two objects where only the attribute being compared varies, e.g. comparing two pencils of different lengths.
    - Direct comparison of two objects where more than the attribute being compared varies, e.g. comparing the length of a pencil and a stick.

2. **Comparing and ordering**
    - Indirect comparison of two objects using an intermediary, e.g. comparing the distance around a can to the length of a pencil using a piece of string.
    - Comparing different representations of the attribute, e.g. comparing the diameter of a bicycle wheel and the width of a door.
    - Ordering three objects focusing on between-ness, e.g. finding the longest stick, the shortest stick and noting the third is between the other two in terms of its length.
    - Ordering more than three objects by focusing on between-ness, e.g. ordering four blocks by their mass by finding the heaviest block, finding the lightest block, comparing one of the other blocks to the heaviest and lightest and noting which it is more like, comparing the last block to the heaviest and lightest and noting which this is most like, and lastly comparing the two in-between blocks to confirm the order is correct.
3. **Non-standard units**
    - Measuring with informal, non-uniform non-standard units to focus on the count of units representing the measurement of the attribute in focus.
    - Measuring with uniform, non-standard units.
4. **Standard units**
    - Measuring with standard units including use of the correct names, labels and conversions between units as needed.
5. **Applications and formulae**
    - Solving problems involving applications of measurement and the use of formulae.

3 Length, Mass and Capacity

These three measurement attributes are uni-dimensional measures. Each of these attributes can be measured as a count of units and in particular they can be measured using a linear scale.

3.1 Length

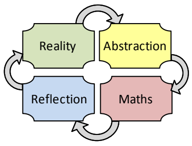
Length is the attribute on which most other attributes are built. Perimeter is an adaption of length where a number of lengths are added together. Area is a two-dimensional attribute which requires the measurement of the length of both dimensions. Volume is a three-dimensional attribute which requires the measurement of the length of all three dimensions. The metric system of measures is based around the attribute of length. Length can be identified as a straight or curved distance between two points.

3.1.1 Identifying the attribute of length

Background

Length is a one-dimensional concept related to the geometric notions of direction and line. It is a measure of the separation of two points along a straight line (a curve is measured by dividing it into small straight segments). Perimeter is a particular application of the measurement of length and is discussed in section 4.1. Length has many manifestations in real life. When we investigate how long, how tall, how thin, how deep, how wide, how far around we are focusing on the attribute of length. There are a number of words that students are likely to encounter and use which are about the attribute of length, for example:

Length: long, longer, longest; short, shorter, shortest
Width: wide, wider, widest; narrow, narrower, narrowest; thick, thicker, thickest; thin, thinner, thinnest
Depth: deep, deeper, deepest; shallow, shallower, shallowest
Distance: near, far, close, next to
Height: tall, taller, tallest; high, low, up, down, above, below



RAMR lesson for identifying the attribute of length

Reality

Use relevant real-life contexts to embed the activities – use local objects or situations. For example, distance, heights, legal limits for fish, size of trucks or road trains.

Abstraction

Body

Use actions, mimes and dances that represent growing higher or growing longer and shorter (e.g. a tree growing taller, a fish getting longer). Act out with students' bodies all the length words.

Cut strips of ribbon to students' heights and stick on walls or mark heights on walls; draw around a student lying down; draw around a student's foot.

Experience objects in class and home that are long/short, and so on (e.g. the board is wide, the door is high, a pencil is narrow, a book is thick or thin, and so on). It is always comparative.

Check the different sizes of tree trunks by hugging trees. Or trace a finger around the outside of an object.

Hand

Experience a variety of materials. Put out five pencils of different lengths (what is different?) or put out a feather, a pencil, a strip of paper, a can, and a duster that are all the same length (what is the same?).

Hold up an object; look for things that are the same length as it, look for things that are different lengths to it.

Experience different ways to get length – distance around a can, diagonally across a blackboard.

Get students to follow directions to find things – it's far from the desk, beside the bin, and so on.

Mind

Students shut eyes and you tell them a particular object and ask them to think of things that are thinner, wider, taller etc. than that object. Think of things that are the same length.

Mathematics

Practice

Worksheets where students sort objects into long and short, thick and thin, and so on.

Connections

Show non-connections – find long and thin, short and wide, and so on.

Reflection

Application

Take new understanding of length out into the school grounds and identify long/short, wide/narrow, and so on.

Reversing

Get students to draw their own long and short, or own thin and thick, and so on. (Note: Go from word to drawing or object as well as object or drawing to word.)

Generalising

Look at high and tall. Is a tall student or object on the ground taller than a short student or object on a chair (or just higher)? What does this mean when thinking about length?

Hold up a piece of wood, say is this long or short? Discuss what people might say in different situations (e.g. it's long for a piece of firewood but short for the trunk of a tree). Discuss that long/short depends on perspective or situation or experience of people. Discuss what the attribute of length is – its "longness" or "shortness".

Discuss ways in which we could determine which of two examples is longer.

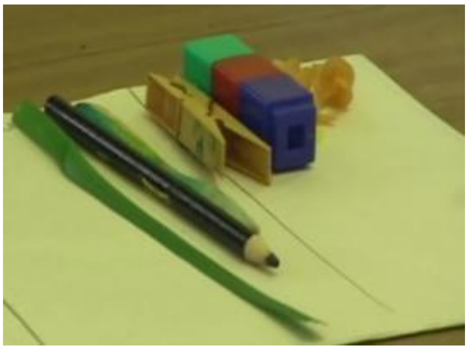
Changing parameters

Consider length not being a straight line. Is a curly line long or short? What about a spiral line? What happens when a tall student lies on the ground?

3.1.2 Comparing and ordering length

After experiencing objects that possess the attribute of length, students can begin comparing and then ordering objects according to their length.

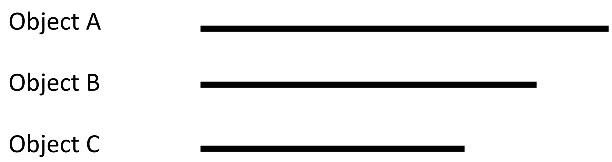
Direct comparison of length



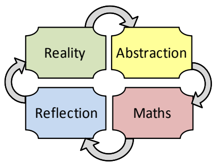
The length of two objects can be directly compared by placing the objects side by side on a common baseline. The object that is longer (or which has a greater length) is able to be identified. When comparing length and all the variations of the attribute of length (width, depth, height, etc.) students need to be made aware of the importance of using a common baseline to ensure the comparison is direct. Small differences in length are difficult to identify using direct comparison.

Indirect comparison of length

The indirect comparison of length without using units requires an understanding of the transitivity principle or big idea. This idea is that if object A is longer than object B, and object B is longer than object C then object A is also longer than object C, and it does not need to be measured to prove this. The figure below shows this relationship visually. The transitivity principle also states that object B is both longer and shorter because it is longer than object C but shorter than object A at the same time. So objects can be compared indirectly without placing them side by side through repeated direct comparisons with an intermediary object. Here, object A and object C can be indirectly compared by comparing both to object B. (*Note:* The way to move from comparing to ordering is to focus on finding the object that is "in between").



A real-life application of this could be the comparison of two pieces of furniture in a showroom by using your outstretched arms as the intermediary. If the first piece of furniture is longer than your arms and the second is shorter then you know the first is also longer than the second.



RAMR lesson for comparing and ordering length

Reality

Where possible, find real-life contexts to embed the activities in; for example, using relevant objects or situations.

Abstraction

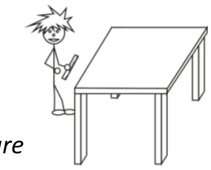
Body

Comparing. *Directly compare and order using students' bodies. Begin with comparing length attributes of two bodies (e.g. height). Continue to develop vocabulary for length by creating situations that will allow students to compare which body is taller/shorter, higher/lower, longer/shorter distance around and so on.*

Ordering. *Compare length of parts of body and extend to ordering experiences by using more than two examples of each.*

Include situations where students experience comparison and ordering between thickest/thinnest, nearest/farthest, deepest/shallowest, and so on.

Hand



Comparing. *Directly compare lengths where* ***both*** *objects can be physically moved together for the comparison. Also should compare thicknesses and widths of objects.*

Directly compare lengths where ***only one*** *of the objects can be physically moved. Compare different types of length, including thicknesses, perimeters and depth.*

Indirectly compare lengths where ***neither object*** *can be moved by using an* ***intermediary*** *(e.g. string, paper strips, part of own body, and so on). Compare different types of length, including thicknesses, perimeters and depth.*

Ordering. *Repeat all the cases above but this time* ***ordering*** *more than two examples. Focus on finding the "between" examples. Introduce language such as tallest/shortest, longest/shortest, widest/narrowest, and so on. Introduce language and categorise objects by near and far.*

Virtual manipulatives can be a very powerful way to learn about ordering and comparing.

Models can be longer/shorter, taller/shorter, wider/narrower, etc. Different versions of things can be moved around and objects, animals, and so on built from parts that are different lengths (e.g. put the shortest tail on the longest dog, put the tallest boy into the shallowest end of the pool, and so on). As well, things can be moved around contexts (e.g. put the ball next to the chair, put the bat away from the chair, and so on). Repeat the above for pictures (e.g. circle the wider, or widest, door in the picture, and so on).

Mind

Shut eyes and imagine someone getting taller/shorter, wider/narrower, and so on. Us this as an opportunity to discuss all the different words for length comparison – shut your eyes, imagine you have jumped into the deepest end of the pool, what will happen; now you jump in the shallowest part, what will happen?

Imagine, then draw and describe a variety of objects, demonstrating comparisons and orderings of length. For example: a taller and shorter tree; a higher or lower book in a bookcase; a wider or narrower fish; a set of fish swimming at different depths.

Special activity. *What is longer – the height of a glass or the distance around its rim?*

Mathematics

Practice

Give students many opportunities to compare and order the length of objects both directly and through the use of an intermediary.

Introduce notation for comparison (e.g. A>B and B<A). Make sure the rules of comparison are known – nonreflexive: A is not < A; antisymmetric: A<B → B>A; and transitive: A>B and B>C → A>C.

Connections

Connect to line – draw a straight line, take two objects of different length and, in turn, put one end on the start of the line and mark the end points on the line. Discuss what the positions of the end points mean [longer is further along the line].

Reflection

Application

Discuss real-world situations of longer/shorter, and so on (do all the words). Find things that are between (e.g. wider than the door but narrower than the desk, and so on).

Flexibility

Think of situations where you would use the length words (i.e. longer/shorter, longest/shortest, wider/narrower, widest/narrowest, and so on). Mix things up – find the object which is the shortest and widest, and so on; find the object which is far from the chair, near the post, taller than the car, wider than the post box, and so on.

Reversing

Make sure you do all the directions – words ←→ objects (e.g. you give the comparison such as wider or widest, and the students find objects where one is wider or widest; you provide the objects, students determine which is wider/widest, taller/tallest, thinner/thinnest, and so on).

Remember that comparison can be considered as a ***triad****, with three parts: first object, comparison word, and second object – thus there are three "directions":*

(a) *give first object and word (e.g. thinner) and require a thicker object;*

(b) *give second object and a word (e.g. thinner) and ask for an object that the second object is thinner than; and*

(c) *give two objects and ask for word(s) to relate them. (This is developing the* ***triadic relationship*** *big idea.)*

Generalising

Try to find things that hold true for all comparisons of length. Put two objects together in a way that shows the shorter is longer (leads to the generalisation that you must have a common starting point).

Generalise the connections activity above (the object whose end point is further along the line is the longer). Discuss how this could lead to a way of measuring length. (This begins understanding of the Attribute leads to instrumentation big idea.) Show that an object can be both longer and shorter (comparison depends on the size of the object to which it is compared).

Generalise the reversing activity above (when three things exist, there are three directions and three problem types). Comparing involves first object, second object and comparison. Problem types are: (a) give the two objects, ask for the comparison; (b) give first object and comparison, find second object; and (c) give second object and comparison and find first object.

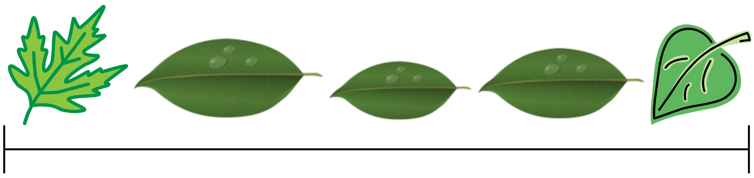
Changing parameters

Instead of looking for, for example, longest or shortest, look for a length ***in between*** *(e.g. find something whose length is in between the board's height and the door's width). Look at length which is not straight. Use intermediaries to find which of a wriggly line and a spiral is longest.*

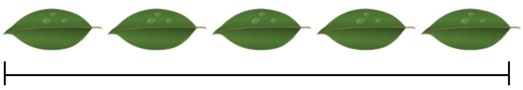
3.1.3 Non-standard units for length

Background

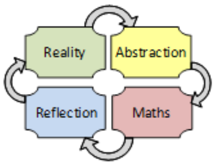
Non-standard units for length need to be objects that possess the attribute of length. Using objects from the environment is a good way to start the focus on units and learning that the count of units of length can be used to describe the length of objects. Environmental objects like leaves, sticks, stones etc. are all suitable initially. These items are unlikely to be uniform lengths. Students can be helped to see that the count of these units can describe the length but that comparing or describing lengths using different length units is not ideal.



Moving to uniform non-standard units allows students to focus on the unit and the count of units as representing the length of objects.



Notes: 1. Focus on introducing the idea of a constant unit to measure length and the development of the measurement process and principles of length. 2. The Reflection stage of RAMR is very important to non-standard units because this is where we develop the principles.



RAMR lesson for non-standard length units

Reality

Where possible, find real-life contexts to embed the activities in; for example, using relevant objects or situations.

Abstraction

Body

*Introduce the idea that a given length, like a pace, can be used to give a number to a larger length. Pick a wall, determine how many teacher paces it takes to "walk" the wall (e.g. 9 paces). Then say that this means that the wall is 9 teacher-paces long – a **number** and a **unit**. Get the students to measure things with their own paces and write down the number and the unit.*

Always get students to estimate before measuring and to give answers as numbers and units. Make students aware of the correct use of paces in measuring. Measure a wall by correct pacing. Then, act out various "incorrect" ways to do the pacing, as follows. (Note: When we get different numbers for the same wall, ask the students what is unfair about these examples. This method is called "torpedoing".)

1. ***Not starting at the beginning of the wall.*** *Start 2 or 3 paces along the walls and ask why the number is less. Start well before the wall and ask why the number is larger.*

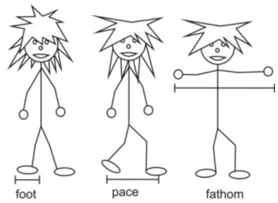
2. ***Not ending at the end of the wall.*** *Try this and ask why the number is larger or smaller depending on when the walking finishes.*

3. ***Not walking in a straight line.*** *Try a really wobbly walk and ask why the number of paces is larger.*

4. ***Not keeping paces the same.*** *Walk the wall with a variety of different length paces. Ask why the number is larger or smaller depending on whether paces are mostly short or long. (Note: This leads to the first aspect of the* ***common units*** *big idea –* ***units in a measure must be the same****.)*

Repeat the above two steps for different lengths, short and long, fitting the body parts on the right into the lengths (e.g. how many digits along the side of an A4 sheet?)

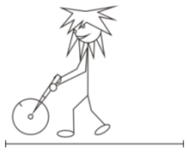
Again use "torpedoing" to get across correct measurement processes: (a) start and end units in line with the start and end of the object being measured; (b) place units beside each other in a straight line; (c) use the same units throughout the measuring; and (d) do not have gaps between the units.



Hand

Use a variety of non-standard units to measure the lengths (e.g. how many dusters long is the board, how many pencils wide is the desk, how many sticks wide is the playground?). Objects that could be used as units to measure length could include pencils, pencil cases, sheets of paper, blackboard dusters, Smarties, Cuisenaire rods, blocks, straws, cardboard strips, pegs, paperclips, lengths of dowel, lengths of string, and so on.

Good units are ***multiples of lengths of everyday things****. For example, cut out a copy of your foot and use this as a non-standard unit. Use anything that* ***rolls*** *and count the revolutions turned when the object is "wheeled" along the length (e.g. bicycle wheel, cotton reel, can, dish, and so on).*



Make up a ***non-standard ruler*** *– string beads or pieces of straw onto string, stick short sections of paper end on end, and so on. Count the number of units which are alongside the object being measured. Use* ***virtual materials and pictures*** *to experience non-standard units.*

Things that also should be done*: (a) repeat the activities from Body to reinforce the measurement processes that give accurate measures, (b) always estimate before measuring, and (c) always give answers as number and unit.*

Mind

Have students draw units beside the object being measured. Have students imagine units being used to measure the length of an object. Also imagine accurate measurement processes used in the Hand experience and represent how the length was measured.

Mathematics

Practice

Continue to provide situations for students to measure length – use materials, pictures and worksheets. Estimate first. Reinforce accurate measurement processes while the practice is being undertaken. Allow opportunities for students to identify inaccurate measurement processes.

Connections

Connect non-standard measurement to number-line division. For example, the diagram on the right can be considered as the length of the rod divided by the paperclip length. For division, the more people there are to divide the cake among means that each person gets less cake. It is the same here: increasing the length of the paperclip means fewer paperclips. (This leads into the ***inverse relation*** *big idea.)*

Reflection

Application

Measure with non-standard units in real-world situations. Set up measurement problems based on non-standard units.

Flexibility

Find non-standard units in local community (e.g. number of cans of fuel to measure distance travelled by boats). Try to get students to think of ways non-standard units are used in the world, local and otherwise. Use history and look at other units used in the past (e.g. the mile which was 2000 paces of the Roman army).

Reversing

*The components of a non-standard measure are object, non-standard unit and number. These form a classical triad and result in three problem types, as in Stage 5. Thus, make sure all three directions are taught. (This leads to the **triadic relationship** big idea.)*

Generalising

*Here the objective is to extend the understanding from Abstraction and Mathematics to teach **continuous vs discrete** and the **three measurement principles** as follows. This is a major part of this unit.*

1. ***Continuous vs discrete.*** *Discuss how the act of measurement is the same as counting on a number track, ladder or line. Look at the line **without** regular marks or divided into units. Look at the line **with** these things. Elicit that the line is continuous and cannot be naturally counted. Discuss how the unit breaks it into parts that can be counted. Relate to ruler and measuring with rulers. (This leads to the **continuous vs discrete** big idea – that there are two ways that number is applied: (a) to discrete objects, and (b) to continuous things such as lines by the use of units to discretify the continuous line.)*

2. ***Measurement principle 1: Common units.*** *Use torpedoing to show that we cannot know how long something is if we do not know the unit or have the **same unit**. For example, come into the class and say you caught a 24-unit long fish and draw on board. Then say also that a friend caught a 36-unit long fish and ask students to mark beside the drawing of the 24-unit fish where the 36-unit fish would end. Then put up a smaller fish for the 36 unit fish and ask how can this be? (Most students will say that using smaller units.)*

   *Another example is to set a tall student and a short student to pace a wall and write their answers up in number of paces. The numbers should be different and you can ask why this is so. (Once again most students will say because pace length is different – the bigger number is caused by the smaller pace.) Repeat activities like this as much as needed. (This leads to the second aspect of the **common units** big idea – that **units must be the same length when comparing objects**.)*

   *Then set a **common class unit** – let the students choose it – with which all can measure the lengths of objects. Discuss what a bigger number will mean in this case when all are using the same length unit. (This leads to the third aspect of the **common units** big idea – that **when units are the same, the larger number specifies the longer object.**)*

   *Move discussion onto situations where something, say an item for building a house, is being made in one town or country and wanted in another town or country. What is needed to ensure that the thing being made is the right length? Discuss buying long and selling short – using a tall person to buy fabric by the fathom and a small person to sell fabric by the fathom – discuss how this would lead to anger and difficulty in buying the right amount of fabric. (This leads to the fourth aspect of the **common units** big idea – that **there is a need for a standard**.)*

3. ***Measurement principle 2: Inverse relation.*** *Measure things with large and small units, with units that are $\frac{1}{2}$ and $\frac{1}{3}$ the length of other units. Record the results as follows.*

UNIT	OBJECT	NUMBER
Small stick	Desk	13
Medium stick	Desk	9
Large stick	Desk	6
½ large stick	Desk	12
⅓ large stick	Desk	18
Double large stick	Desk	3

*The pattern is easy to follow – the larger the unit the smaller the number and this is in inverse relation to the unit length. It is like division, decrease or halve the divisor is to increase or double the quotient and vice versa. (Activities like this lead to the **inverse relation** big idea – **the larger the unit, the smaller the number and vice versa.)***

4. ***Measurement principle 3: Accuracy vs exactness.** Get students to cut ribbons to 12 small units (e.g. 12 fingers). Get them to compare ribbons and they will find slight differences in length. Ask them why? (Most students realise that in measurement you cannot be exact, you can only be as accurate as you want or are able.) Get students to measure a length in **complete** larger units and smaller units and then cut ribbon to the length shown with both units. Compare the ribbons with the object. Students should see that the smaller units are more accurate. (This leads to the first consequence of the **accuracy vs exactness** big idea – that **smaller units give greater accuracy**.)*

   *Discuss what smaller units being more accurate means. Discuss whether this means that we should always measure in small units. Argue about whether there are times when such accuracy is not needed. Provide students with measurement tasks for contexts (e.g. measure the window for a replacement window sill, measure the side of the classroom so we can work out if it is longer than other classrooms, and so on). Discuss what would be the best units for each task. Continue this line of thought by giving students a variety of objects of differing length and a variety of measurement instruments. Have them select and record the appropriate instrument to measure the length of each object. Practise these concepts with more tasks where students devise their own measurement plans, involving them selecting the unit of measurement and estimating and checking: Students must reflect on the quality of their measurement instrument and accuracy of estimation. (This leads to the second consequence of the **accuracy vs exactness** big idea – students require **skill in being able to choose appropriate units.)** Discuss whether estimation is ever good enough. Find situations in which it is. (This leads to the third consequence of the **accuracy vs exactness** big idea – students require **skill in estimating**.)*

*As for length, perimeter and circumference can continue to extend the understanding from Abstraction and Mathematics to teach **continuous vs discrete** and the three measurement principles: **common units, inverse relation,** and **accuracy vs exactness.***

Changing parameters

What if the units were the standard ones, would they act the same with regard to measurement processes and principles as non-standard units? What if the units were not length – say they were mass or area? Would they need similar study of measurement processes and principles? Would this study have common points?

3.1.4 Standard units for length

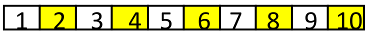
The standard units used for length are important as they form the basis for all metric units of measure. The metre is the base unit and other units are derived from smaller and larger multiples of this unit. The smallest standard unit of length in the metric system is the millimetre (mm) and the largest is the kilometre (km). The decimetre exists as a standard unit but is not commonly used; 1 decimetre (dm) = 10 cm. The decimetre is important for conversions between the attributes of volume, capacity and mass (see section 4.4).

The standard metric units of length and their relationships are:

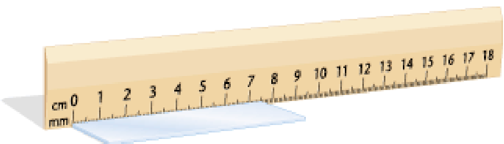
1 centimetre (cm) = 10 millimetres (mm)
1 decimetre (dm) = 10 cm
1 metre (m) = 10 dm = 100 cm = 1 000 mm
1 kilometre (km) = 1 000 m = 100 000 cm = 1 000 000 mm

Measuring devices for length using standard units

To measure length in standard units requires students to be able to use a range of measuring devices. A ruler is a length of wood or plastic usually marked in cm or mm or both. Simple early rulers can be made where 1cm sections are coloured alternatively to help students focus on the count of units representing the measure of length. Marking each centimetre using numbers in each section can assist young students to move to identifying the numbers with the length. Then they can be shown that on standard rulers the numbers are at the end of each length.



Some rulers do not have the zero aligned with the end of the ruler requiring students to align the zero with the edge of the object they are measuring (see figure below). This can cause measurement errors for inexperienced students. The figure on right below shows a student correctly using a rule to measure a length of straw by aligning the zero with the end of the straw.



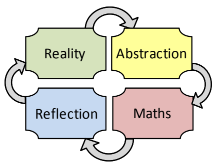
Zero not aligned with the end of the ruler

A student correctly aligning the zero on a ruler to measure length

Measuring tapes that are flexible can be used for measuring objects that are not straight, e.g. curved edges of tables. Using an intermediary like string can also help the measurement of lengths that are not straight, e.g. roads on maps. The length of string can then be measured using a ruler.

Trundle wheels are measuring devices for measuring distances. The circumference of the wheel is one metre and at the completion of each rotation the device makes a clicking noise. Students can count the number of “clicks” to measure distance in metres.

RAMR lesson for standard length units



Reality

Where possible, find real-life contexts or use relevant objects or situations to embed the activities in.

Abstraction

Common unit

After the need for a standard has been developed through the use of non-standard units in Stage 3, time can be spent measuring length with a class-chosen unit – e.g. a piece of dowel, a pencil. This can be used by students to show that with a common unit, a higher number really means a longer length.

Identification

Cut 1 cm pieces from different coloured drinking straws. Thread these pieces along a string in groups of 10 of one colour followed by 10 of another colour. This can be used to identify centimetres and groups of 10 centimetres to make up a metre.

Using 1 cm grid paper, cut ten strips that are 10 cm in length. Tape these together to form a folding 1 m measuring strip. This should be placed on cardboard to make it more durable. This again identifies a cm and a m.

Attempt to cut a 1 cm piece of paper into 10 equal slices. Discuss how small they are. Find a long distance (along a road or around an oval), get students to estimate how far to make a kilometre, then use a trundle wheel to have everyone walk an actual kilometre.

Internalisation

Use a measuring tape to measure and record your personal body measures:

Height	Arm span	Head circumference	Leg length
Around the wrist	Foot	Length of hand	Ankle to knee
Wrist to elbow	Left hand	Thumb	Index finger

Find a reference length in your body which is approximately:

1 cm ……………………………………………………………………………………………......

10 cm …………………………………………………………………………………….........

1 m ………………...

1 mm ....

Use this measure to estimate different items around the room – measure to check how close.

Mark out a 10 m distance using a measuring tape. Determine how many of your paces equal this 10 m. Pace the following distances, make some up, and use this value to convert paces to metres:

DISTANCE	**PACES**	**Estimated conversion to METRES**
Length of room		
Width of board		
Length of veranda		
Distance around the classroom		
Distance around the building		

Mark out 100 m. Use a stop watch to time how long it takes you to walk this distance. Use this time to determine how long it would take you to walk a kilometre.

Find a local well-known distance that is around 1 km – check if your time matches.

Estimation

Estimate, using a variety of techniques – internal measures, time, paces, etc. a variety of items or lengths – don't forget perimeters! Then measure the same lengths and see how close you get.

Estimate larger distances that students may walk every day – classroom to canteen, school to shop or significant landmarks in the community and then measure these distances. Try to develop the distance of a kilometre.

Place-value connections

Set up the place-value cards:

One Millions	Hundred Thousands	Ten Thousands	One Thousands	Hundred Ones	Ten Ones	One Ones	Tenths Parts of One	Hundredths Parts of One	Thousandths Parts of One
				←	3	7	→		

Move numbers left and right and use a calculator to determine relationships in moves (×10 to left and ÷10 to right, and so on).

Look at names of units – research what centi, milli and kilo mean.

Place metric units under place-value positions as follows:

One Millions	Hundred Thousands	Ten Thousands	One Thousands	Hundred Ones	Ten Ones	One Ones	Tenths Parts of One	Hundredths Parts of One	Thousandths Parts of One
			Kilo metre	Hecto metre	Deka metre	metre	deci metre	centi metre	milli metre

Use relationships from above to show relationships between metric units.

Use a marker on the unit you are looking at to show the conversions.

***Example**. Place 37 m on the place-value chart. This is m, so put an orange (or other marker) on the metre place. This shows 37 m. If we wanted to convert to millimetres, leave the 3 and 7 where they are and move the marker to the millimetre position. Fill the three "empty" places with zeroes. This shows 37 000 mm which is the same as 37 m.*

Mathematics

Metric Expanders

Construct a larger copy of Expander A (kilometres, metres and millimetres) located in Appendix A and cut it out. Fold the expander like number expanders. Use them to relate km, m and mm as for place-value cards.

Metric Slide Rule

*Copy the metric slide rule in Appendix A. Using scissors, cut out the slides and the scale, and slit the scale along the dotted lines. Then, using the rounded end of the slide as a tongue, thread each slide **from the back** up through the slit on the left of the scale and across the front and out the slit on the right of the scale. Use the slide rule to relate metrics and decimal numeration.*

Practice

Give students experience with specialist measuring instruments such as callipers, verniers, etc. Undertake outdoor activities such as height measurement, orienteering and scale drawings/surveying.

Consolidate the metric conversions through drill – some examples include:

1. *Dominoes*
2. *Bingo*
3. *Mix and Match cards*
4. *Card decks(for Concentration, Gin Rummy, Snap etc.)*

1000 mm	10 m	1000 cm	1 mm

2 m	200 cm

1000 mm	10 m	1000 cm	1 mm

Note: Metrics should be introduced along with decimals. They apply decimal understanding and reinforce decimal concepts. For instance: two decimal places are related to money (dollars and cents) and length (m and cm); and three decimal places are related to length (m and mm), mass (kg and g, and t and kg) and volume (L and mL).

3.1.5 Applications and formulae for length

Applications for length and distance around (perimeter) should be built around the idea of a triad – there is an object, a unit of measure and the number of units. Thus, applications for length can be built around three types of problems:

- **Number unknown** – *measure this wall in metres*.
- **Object unknown** – *find an object that is 16 cm long*.
- **Unit unknown** – *this object is 35 units, are these units cm or mm?*

An application of length is finding perimeter. The vocabulary and concept of perimeter, as the distance around the outside of an object, should have been built during the first four stages of Length. In this book, perimeter is in Chapter 4. However, it is useful to integrate length and perimeter in teaching.

Once length measurements have been taken it is possible to develop and use a range of formulae to find perimeter, area and volume of a variety of shapes. These measurements are calculated from length measurements. Each of these new attributes is related to length. Each of these attributes is described in a separate section of this book in Chapter 4 (Perimeter – section 4.1, Area – section 4.2, Volume – section 4.3).

3.2 Mass

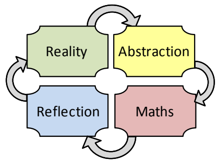
Mass is the measurement of the amount of substance in an object. Mass is different from weight which is influenced by gravity. The only way we can measure mass is to use gravity. On the moon, where there is less gravity, an object would have the same mass but would weigh less.

Mass is the measure of the inertia of an object – how much force it takes to move the object and how much force it takes to stop the object from moving. Mass may not be proportional to the volume of the object. It can be understood as the amount of pressure the object pushes down on something holding it up.

3.2.1 Identifying the attribute of mass

This involves building the meaning of mass as **heft** and resistance to push and using appropriate vocabulary. Mass is a complex concept for students to understand fully. Although solid shapes have mass as a property, mass is not directly related to solid shape. Rather, it is the measure of the inertia of an object, that is, how much force it takes to move the object, and how much force it takes to stop the object from moving. This may not be proportional to the volume of solid shapes. For school students, it is related to the force exerted by an object as a result of gravity (the weight of the object) — and can be intuitively understood as the amount of pressure the object exerts down on the hands holding it up (the "heft" of the object). It should be remembered that weight varies depending on the planet the person is standing on. (On the moon, our weight is approximately $\frac{1}{6}$ of what it is on earth.) Mass does **not** vary. Invariant mass has replaced varying weight as the "inertial" attribute of an object which is studied by school students. Weighing is a process used to measure mass. So we weigh objects to find out their mass.

RAMR lesson for identifying the attribute of mass

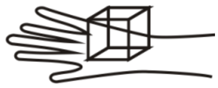


Reality

Find relevant real-life contexts to embed the activities in – use local objects or situations.

Abstraction

Body



Give students many experiences using their bodies with ***hefting*** *objects, trying to lift different objects (e.g. "how much can I lift?" Be careful here). Look at and feel how hard things press down on the hand.*

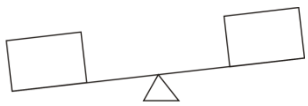
Make the students' bodies into a beam balance by hanging plastic bands off each hand. Put weights in each side and let students feel the heft of each side and which is heavier. Display two bags, for example, one filled with dried leaves and the other with sand, for the students to identify by hefting the bags and determining which bag is heavier or lighter. Use a seesaw as a balance beam with students on each side.

Make sure all mass language is introduced and experienced – mass, weight, heavy/light, heavier/lighter, heaviest/lightest, heft, and so on.

Hand

Provide a variety of experiences showing the effect of mass on pushing down on objects. For example: (a) placing things on the end (or middle) of a stick and seeing how much the stick bends (or bows); (b) using a spring or a rubber band to see how long different objects stretch out the spring or rubber bands when lifted with it, or how much the objects compress a spring when the object is placed on top of it.

Use the idea of pushing and pulling as well as hefting. Show students two large cartons and ask which one is heavier. Tell students that the cartons are too large to be lifted and ask if there is another way to compare them. Let students take turns to push or pull the cartons. Discuss objects that have to be pushed or pulled rather than lifted (e.g. beds, couches, and so on). Discuss how the effort in pushing and pulling is also an experience of mass.



Experience the balance beam. Try different objects on each side. Develop the notion of balance and how we can balance and unbalance. Look at questions such as "how can we get this side to go up?"

Experience hefting masses that are large yet light, small yet heavy, and so on. Make decisions regarding heft and pushing effort for virtual examples and pictures.

Mind

Students close eyes and imagine lifting a light and then a heavy object. Find a way to visually represent the difference in effort displaced to show the difference in mass. Repeat this activity for pushing and pulling two objects of significant mass difference.

Mathematics

Practice*. Continue experiencing hefting and pushing masses through activities, virtual materials, pictures and worksheets. Use rubber band measurers and beam balances.*

Connections. *Similar to the "measuring container" idea in capacity (see section 3.3), the rubber band or bending stick mass measurers can be shown as a connection between mass and length. The further they stretch down in length, the heavier the mass.*

Reflection

Application. *Explore examples of mass in everyday world of students. Set activities to investigate what kinds of masses exist in students' life and whether heavy or light.*

Flexibility. *Think of many things that are heavy or light. Relate this to size. Does this relationship always hold? What could make a larger thing lighter – even if it is a larger version of the other one?*

Reversing. *Get students to experience mass both when teacher gives object and student experiences its mass, and when teacher describes a mass and students find or construct an object with that mass.*

Generalising. *Two things as follows:*

- ***Relativity***. *Look at the relativity of heavy and light as words. Discuss/show how, in different situations, the same object can be heavy or light. For example, a motor bike is light compared to cars but heavy compared to back packs.*
- ***Big idea – attribute leads to instrumentation.*** *Discuss the attribute of mass. Think of it as heft. Discuss how we might measure this heft [stretching a spring]. Try to elicit that the length the spring is stretched is a way of measuring mass.*

3.2.2 Comparing and ordering mass

After experiencing objects that possess the attribute of mass, students can begin comparing and then ordering objects according to their mass but without reference to number.

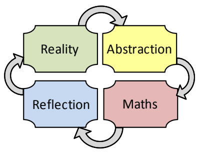
Direct and indirect comparison

Mass can be directly or indirectly compared without reference to number. Mass can be directly compared in two ways. One involves a measuring device (but no units) and the other relies on a person feeling the differences.

1. The first method involves someone holding one item in one hand (or in a plastic shopping bag) and the other item in the other hand. By using their senses, a person can feel the difference in mass. This process is called hefting. It is important when hefting to swap hands as our dominant hand is likely to be able to hold a greater mass and the difference might be masked by this.
2. The second method uses a balance scale. Place one item in one side of a balance scale and the other item in the other side (arms of the balance must be the same length). Observe the way the pans tilt to determine which is heavier.

These methods have difficulty in identifying small difference in mass. Indirect comparisons can be done by using a third intermediary – water, gravel, the length a string is stretched or the amount the "boat" sinks.

RAMR lesson for comparing and ordering mass



Reality

Use relevant real-life contexts to embed the activities – use local objects or situations.

Abstraction

Body

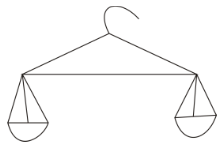
Have students make their body into a beam balance (e.g. a plastic bag in each hand). Experience weights in each side and determining which is heavier or lighter. As people have different strengths in each side, always transfer the weights to the opposite side before making decision – heavier weights should be heavier regardless of which *arm they are on. Move on from comparing to ordering by finding the "between" weight/mass. Introduce comparing words – heavier/lighter, heaviest/lightest, and so on.*

Have students directly compare their masses with each other using a seesaw type beam balance – a plank across a low brick will do but be careful.

Hand

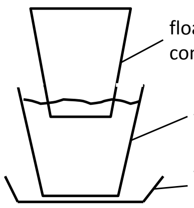
*Compare two objects/masses by a variety of ways – beam balance, rubber band mass measurer, flotation measurer, and stocking measurer. Compare **directly** with the beam balance. Compare **indirectly** with the other balances as the objects have to be placed in turn and the length of the stretch gives the comparison.*

1. *A homemade beam balance can be constructed from a wire coat hanger, string and two containers (see right).*



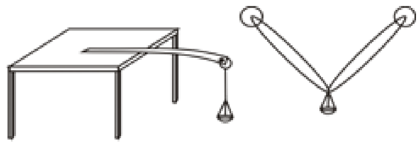
2. *A homemade spring balance or rubber band balance can be constructed from a wire hook, rubber bands, string, and a container (see right). This balance can be used to compare mass by seeing which object stretches out the rubber band the furthest.*

3. *A flotation measurer can also be constructed to compare masses (see right). This is a tall container floating in water with sufficient plasticine in the bottom to prevent it tipping. Objects placed in the container cause it to sink deeper into the water. The object causing the furthest sinking is the one with the most mass.*



4. *A stocking measurer is easily made – a mass in the leg of the stocking will stretch like a rubber band with the heavier mass stretching further.*

Other homemade mass measurers can be made from pieces of wood or plastics or metal as on right (rubber bands can also be used). How much the material bends or stretches determines the mass of the object.



Extend to ordering three or more objects using the mass measurers.

Extend to ordering three or more objects using the mass measurers.

Spend time ensuring accuracy in measuring with spring/rubber band measurer: (a) focus on the increase in stretch due to the mass (not the lowest point) unless you make sure that the un-stretched length remains in one place; (b) put a small mass in the tray or bottom of stocking so there is tautness in the un-stretched length; and (c) check that rubber bands, springs, and stockings have not been overstretched. Also spend time on beam balance measuring – the balance arms must be the same length.

Use virtual and pictorial means to experience mass comparisons – drawings of beam balances with pictures of the masses on each side can easily show weight comparisons.

Mind

Imagine, then draw and describe a variety of objects, demonstrating comparisons and orderings of mass using all the measurers.

Mathematics

***Practice**. Continue to give many opportunities for students to compare and order mass, estimating before calculating. For example, use a seesaw balance and have students make a seesaw using soft-drink cans/bottles* *and shoe box/plastic container lids. Ask students to balance the empty lid on the can first. Then put one object such as a toy car on one side and ask students to find things that make the lid balance. Make shapes out of play dough or clay that have equal mass but a different shape.*

Make sure that students understand comparison notation (i.e. > and <, = and ≠).

***Connections**. Continue link between mass and length for spring balances, rubber band measurers and other mass measurers which go up and down depending on mass.*

Reflection

***Application**. Relate understanding back to everyday life – what things in the world have large and small mass (e.g. a truck has greater mass than a car, a pencil has less mass than a stapler, and so on). Set problems to do with ordering mass which relate to the students' world.*

***Flexibility**. Think of as many pairs of things as possible that have more/less mass than each other. Think of situations where mass has to be taken into account (e.g. small planes or boats) and different lifts – elevators, goods on a truck, a forklift.*

***Reversing**. Make sure teaching goes from: (a) teacher provides masses→ students use measurers to give comparison word, and (b) teachers give comparison word →students provide containers that meet that word.*

*Also remember that comparison can be considered as a **triad** – first container, comparison word, and second container– thus there are three "directions", or three problem types: (a) give first mass and word (e.g. heavier) and students find a lighter mass; (b) give second mass and a word (e.g. greater) and students find a heavier mass; and (c) give two masses and ask for word(s) to relate them (e.g. "the second mass is lighter"). (This is the **triadic relationship** big idea.)*

***Generalising**. Generalise the accuracy rules that have already been stated: (a) more/less mass means a greater/less increase in stretch or the side of the balance going down/up (note that more mass means side goes down); (b) don't overstretch the rubber bands, springs, etc.; (c) make sure the beam balance's arms are the same length; and (d) for comparison, like all triadic relationships, there are three problem types as in reversing. (Again this is developing the **triadic relationship** big idea.)*

***Changing parameters**. Discuss what happens in a beam balance if the arms are different lengths or in a rubber band measurer if the bands are overstretched.*

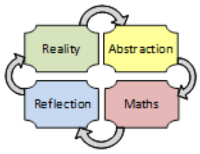
3.2.3 Non-standard units for mass

Any objects that have a noticeable mass can be used as non-standard units for mass. All objects have some mass but it would be difficult to use feathers as units of mass as their mass is barely detectable either on simple scales or by feel. Objects like stones, shoes, blocks or marbles can be used successfully for measuring mass.

Measuring devices for mass using non-standard units

The use of a balance scale for mass can introduce students to the use of units to represent the measurement of mass. Students can place an object on one side of the balance scale and place numerous items on the other side and observe when the two objects balance each other. When they are balanced they are equal so the count of units on one side will equal the mass of the object on the other side. Initially, non-uniform objects can be used as units to balance an item being measured. Students will realise that by using uniform non-standard units the count better represents the mass and this number can be used to compare the mass of this object to the mass of another object by using the same units. Other devices can be used such as springs, rubber band measurers, stocking measurers, a ruler on the end of a table or something floating in water. For these devices, it is the length of stretch or sinking that should be measured.

RAMR lesson for non-standard units of mass



Reality

Use relevant real-life contexts to embed the activities – use local objects or situations.

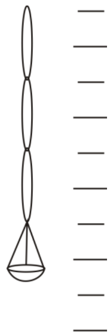
Abstraction

Body

*Use seesaw large beam balance to work out how many bricks balance a student – be careful. Use plastic bag beam balance to determine the number of small masses that balance one larger mass. Small objects that can be used include dusters, ball bearings, marbles, books, MAB ones, stones, pencils, small cans of baked beans and spaghetti, and so on. Always **estimate** first.*

Hand

*Use a variety of measurers to find the mass of an object in terms of smaller masses (e.g. the litre bottle of water is five spaghetti cans). Again, always **estimate** or **predict** first.*



1. *The beam balance is easy – how many of the masses will balance the object?*
2. *The homemade spring balance can be placed against a blackboard or a sheet of paper. The container can be weighted with plasticine to make the rubber bands taut. This position can be marked and called 0. Then regular intervals can be marked on the blackboard/sheet going down. When objects are placed in the container, their mass can be read off the position that the rubber band stretching allows the container to move down to.*
3. *The flotation measurer can also be used for non-standard units by marking regular intervals on the side of the flotation measurer; similarly for other stretching measurers.*

Both the spring balance and the flotation measurer can also be used similarly to the beam balance. An object is placed in the container and the position where the container ends up is marked. The object is removed and smaller objects added until the same position is reached.

Undertake a variety of activities including:

(a) *find an item that is bigger than a particular object but lighter than it;*

(b) *find the mass of different objects and order them;*

(c) *find objects that are of different volumes but same mass and different mass but same volume;*

(d) *compare very different objects (e.g. does object X have the same mass as object Y?; how many object X might have the same mass as object Y?; how can you find out?);*

(e) *find an object that has the same mass as a given object but is a different size; and*

(f) *use a beam balance to order five identical containers filled with different types of objects (place in order from lightest to heaviest) – because the containers are not perceptually different, this task requires students to make multiple comparisons on the beam balance.*

Look at what is needed for accurate measurement, for example:

(a) *arms of balance the same length; and*

(b) *lines behind rubber band balance starting at taut position, and being equally spaced apart.*

Use virtual and pictorial activities (e.g. beam balance pictures). Ensure students know how to be accurate with the scales.

Mind

Imagine larger and smaller mass being compared on a measurer in the mind with eyes shut. Draw examples of what is imagined.

Mathematics

Practice

Continue to provide situations for students to experience measuring mass with non-standard units using all the measurers, and also use virtual materials and worksheets with pictures. Estimate first.

Reinforce accurate measurement processes while the practice is being undertaken. Have worksheets where students have to identify inaccurate measurement processes.

Make sure that students understand comparison notation (i.e. > and <, = and ≠) and the rules of comparison (i.e. nonreflexive, antisymmetrical, and transitive).

Connections

Connect non-standard measurement of mass to division. For example, working out how many small masses balance an object is the same as dividing the object's mass by the small masses. This means that the rules of division apply to measurement. The most powerful of these is inverse relation – that bigger units or masses mean fewer of them to balance the object and vice versa. (This leads into the **inverse relation** *big idea.)*

Reflection

Application

Measure mass with non-standard units in real-world situations. Set up mass problems based on non-standard units in everyday life situations.

Flexibility

Find use of non-standard mass units in local community activity. Try to get students to think of all the ways non-standard mass units are used in the world, local and otherwise. Use history (e.g. the talent *was the weight of gold to buy an ox). Look at old stories – like the one where the prince discovers how to weigh the elephant.*

Reversing

The components of a non-standard measure are object, non-standard unit and number. These form a classical triad and result in three problem types:

- ***Number unknown*** *– for example, how many dusters to balance a book?*
- ***Object unknown*** *– for example, find an object which is 7 dusters in mass.*
- ***Unit unknown*** *– for example, the mass of the book is 8, what is the unit?*

Make sure all three directions are taught. (This leads to understanding the **triadic relationship** *big idea.)*

Generalising

Here the objective is to extend the understanding from Abstraction and Mathematics to teach ***continuous vs discrete*** *and the* ***three measurement principles*** *as follows.*

1. ***Continuous vs discrete.*** *Discuss how mass non-standard units have broken up continuous mass into small parts that allow mass to be counted. (This leads to the* ***continuous vs discrete*** *big idea – that there are two ways that number is applied: (a) to discrete objects, and (b) to continuous things such as the use of units like spaghetti cans to discretify the continuous mass.)*

2. ***Measurement principle 1: Common units.*** *Use torpedoing to show that: (a) we cannot measure accurately if we vary the masses being balanced, and (b) we cannot know if an object is heavier/lighter than another unless we use the* ***same unit****. Do activities like balancing an object with a variety of units of different mass and ask what is the problem here? Why can't we count these units? Do activities where you say that a heavy object is 4 units and a light object is 16 units and ask why this could be. (This leads to the first and second aspects of the* ***common units*** *big idea – that* ***units must be the same size when measuring and comparing objects****.)*

   Then set a ***common class unit*** *– a common mass that all will be measured against and measure using this. Elicit what bigger numbers mean now. (This leads to the third aspect of the* ***common units*** *big idea – that* ***when units are the same, the larger number specifies the heavier object****.)*

   Discuss if different mass units were used in different towns and countries, what would this mean? We could be paying a lot of money and end up with, say, a small load of concrete because the company you buy from has small units. Set up spring balances with different-spaced lines and different rubber bands and show that the same object is a different number in each measurer. Ask why. (This leads to the fourth aspect of the ***common units*** *big idea – that* ***there is a need for a standard****.)*

3. ***Measurement principle 2: Inverse relation.*** *Measure things with large and small masses and record results on a table as in section 3.1.3. Study the results for patterns. This is best done with the students working in pairs. (Activities like this lead to the* ***inverse relation*** *big idea –* ***the larger the unit, the smaller the number and vice versa*** *– and to the understanding that* ***measuring in units is like dividing****.)*

4. ***Measurement principle 3: Accuracy vs exactness****. Three activities:*

   (a) *Get students to measure an object twice with a rubber band measurer. Was it the same answer both times? There should be a difference in the second measure because of error. Discuss this error and whether we always want to be exact. Discuss how to make the measuring more accurate.*

   (b) *Measure the mass of an object with large and small units. Discuss which is more accurate if giving answers in whole numbers of units.*

   (c) *Give students a variety of masses as non-standard units (e.g. books, dusters, cans, and so on) to use in a variety of measuring activities. Have the students select the appropriate mass to measure with and record the mass as number and object.*

   Discuss situations where estimation would work. (This leads to the three consequences of the ***accuracy vs exactness*** *big idea –* ***smaller units give greater accuracy****, students require* ***skill in being able to choose appropriate units****, and students require* ***skill in estimating****.)*

Changing parameters

What if the spring loses its elasticity so it cannot stop from becoming stretched and therefore changing how far it stretches? How do we handle this?

What if the units were the standard ones, would they act the same with regard to measurement processes and principles as non-standard units? Would they need similar study of measurement processes and principles? Would this study have common points?

3.2.4 Standard units for mass

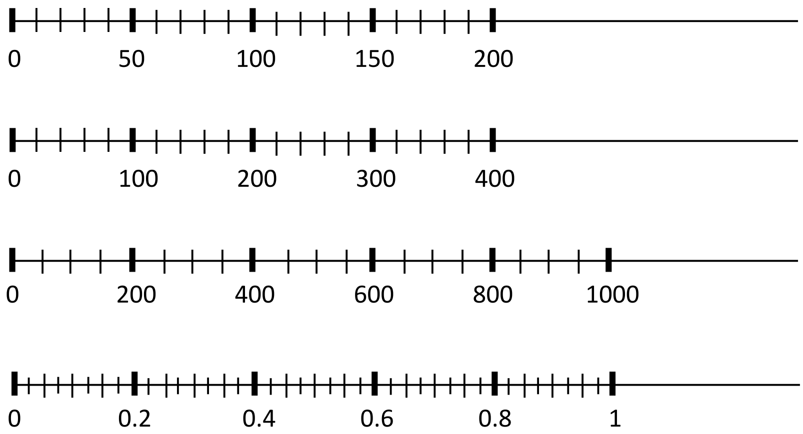
The kilogram is the SI base unit for mass. However, to maintain consistency with the other attributes, the gram is considered the base so the prefixes describe the relationships as they do with other attributes. Mass is an attribute where the units commonly used range from very small (milligrams and micrograms) to very large (tonnes).

The standard units of mass and their relationships are:

1 gram (g) = 1 000 milligrams (mg) = 1 000 000 micrograms (µg)
1 kilogram (kg) = 1 000 g = 1 000 000 mg
1 tonne (t) = 1 000 kg = 1 000 000 g

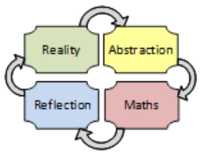
Measuring devices for mass using standard units

Measuring mass in standard units using measuring devices relies on students' ability to read and interpret number lines. Measuring scales display the mass using a needle on a linear (or curved) scale. The difficulty of the scale depends on the number and levels of graduations and which of the graduations are marked. The figure below shows some potential variations in the level of difficulty of reading scales (*Note*: the scale is often circular on devices like kitchen scales which can also add to the complication of using these tools to measure mass).



Different levels of difficulty for common scales on mass measurement devices

RAMR lesson for standard mass units



Reality

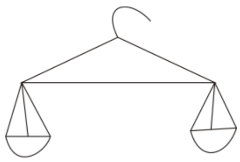
Use relevant real-life contexts to embed the activities – use local objects or situations.

Abstraction

Common unit

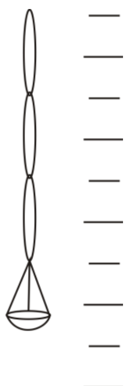
After the need for a standard has been developed through the use of non-standard units in Stage 3, time can be spent measuring mass with a class-chosen unit – e.g. a stapler, a particular rock. This can be used by students to show that with a common unit, a higher number really means a greater mass.

Identification



Construct mass measurers as follows:

> *Method 1 – a wire coat hanger, string and two containers (plus masses).*
>
> *Method 2 – a long piece of paper, 3 rubber bands, string and a container (calibrate the "spring balance" with masses – mark lengths on the paper).*



Use these measurers to make up plastic bags, or other containers, containing 100 g, 250 g, 500 g and 1 kg of various materials (pasta, sand, marbles, rice, etc.). Feel the weight of a centicube – this weighs 1 gram (if no access to these, feel the weight of a paperclip – close to 1 gram).

Find examples of utilities and trucks that hold 1, 2, 5 and 10 tonnes. A normal home trailer will hold ½ tonne. A metre cube of water is 1 tonne.

Internalisation

Use a bathroom scale to measure your own mass in kilograms.

Measure 1 litre of water.

Find objects in the environment that measure approximately 1 kg, 500 g, 250 g, 100 g, 50 g and 1 g. Make up lumps of plasticine to these measures.

Find out the mass of a car and a 4-wheel drive, an elephant, and so on.

Estimation

Estimate first and then measure the masses of the following objects/people (find some more to measure). Complete estimates and measures of object before moving onto the next.

OBJECT	ESTIMATE	MEASURE	DIFFERENCE
Duster			
Suitcase or port			
Shoe			
Another student			
Teacher			

Look up pictures of utilities and trucks, estimate how many tonnes they can carry, and then check by research.

Place-value connections

Set up the place-value cards with place-metric cards as follows:

One Millions	Hundred Thousands	Ten Thousands	One Thousands	Hundred Ones	Ten Ones	One Ones	Tenths Parts of One	Hundredths Parts of One	Thousandths Parts of One
			Kilo gram	Hecto gram	Deka gram	gram	deci gram	centi gram	milli gram

Again analyse the meaning of kilo and use relationships between place-value positions to reinforce relationships between metric units.

Mathematics

Metric expanders

Construct a larger copy of Expander C (tonnes, kilograms and grams) in Appendix A and cut it out. Fold the expander like number expanders. Use them to relate t, kg and g as for place-value cards.

Metric slide rule

*Copy the metric slide rule in Appendix A. Using scissors, cut out the slides and the scale, and slit the scale along the dotted lines. Then, using the rounded end of the slide as a tongue, thread each slide **from the back** up through the slit on the left of the scale and across the front and out the slit on the right of the scale.*

Use the slide rule to relate metrics and decimal numeration.

Practice

Consolidate the metric conversions through drill – some examples:

1. *Dominoes*
2. *Bingo*
3. *Mix and Match cards*
4. *Card decks (for Concentration, Gin Rummy, Snap, etc.)*

1000 g	1000 kg	1 t	1 g

2 kg	2000 g

1000 g	10 kg	1000 kg	1 t

Note: Metrics should be introduced along with decimals. They apply decimal understanding and reinforce decimal concepts. For instance: two decimal places are related to money (dollars and cents) and length (m and cm); and three decimal places are related to length (km, m, cm and mm), mass (kg, g and t) and volume (kL, L and mL).

3.2.5 Applications and formulae for mass

There are no formulaic applications of mass except through the conversion of units. The relationship between the attributes of mass, volume and capacity (1 g = 1 cm^3 = 1 mL) is described in section 4.4. This relationship can provide opportunities for students to solve problems involving conversion between these attributes.

Providing students with opportunities to solve real-life problems that require the measurement of mass helps them to apply their understandings of this attribute in ways that relate to life beyond school. Applications can be built around the triad of the three types of problems:

- **Number unknown** – *measure this book in grams*.
- **Object unknown** – *find an object that is 2 kg in mass*.
- **Unit unknown** – *this object is 350 units, are these units g or kg?*

3.3 Capacity

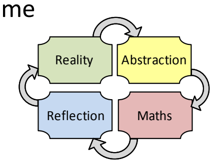
Capacity is the measurement of the amount of liquid in a container and can be measured when the container is full or part full. It refers to liquid volume – solid volume is covered in Chapter 4.

3.3.1 Identifying the attribute of capacity

Background

Capacity and volume are three-dimensional concepts related to the geometric notion of a solid. Both are measures of the amount of space enclosed by a solid shape. Both volume and capacity can be measured by filling the interior of solid shapes with liquid (e.g. water), liquid substitutes (e.g. rice, sand) or solid materials (e.g. blocks) and measuring the amount of these materials or by displacing material (e.g. water) and measuring how much is displaced. *Capacity* usually refers to liquid volume (e.g. petrol in a tank) and is measured in liquid units (e.g. mL and L) while *volume* refers to solid volume (e.g. the volume of a box) and is measured in cubic units (e.g. cm^3, m^3) because it relates to length but in three directions.

Both capacity and volume refer to the space occupied by a substance; their units relate in that a cm cubed is a mL, 10 cm cubed is a L and a m cubed is a kL. The term volume can be used for both solid volume and liquid volume (capacity), e.g. volume is 55 mL. The term capacity is used here to refer to liquid volume and the units mL, L and so on.



RAMR lesson for identifying the attribute of capacity

Reality

Use relevant real-life contexts to embed the activities – use local objects or situations.

Abstraction

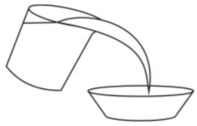
Body

Experience a lot of situations in which parts of the body hold water or rice (e.g. one or two cupped hands). Discuss the difference.

*Try to hold a lot and hold a little. Introduce the term **capacity** (i.e. the amount a container holds), and large and small capacity. Also introduce capacity as a special word for **volume** of pourable materials. Introduce pouring words like **full** and **empty**.*

Hand

Allow students to play with water, rice, sand or other material, pouring from one container to another, filling and emptying. Experience capacity in different ways – long thin containers, short fat containers, large containers, small containers, interesting containers, and so on.



Experience finding containers that hold the same amount – have the same capacity. Experience what happens when the container you are pouring from has more (capacity) than the container you are pouring into, and vice versa. Estimate how much one container will fill another before pouring.

Experience capacity without pouring – decide which container has small or large capacity by looking at it – experience containers fitting inside each other and discuss what this means for capacity.

Mind

Encourage students to imagine containers, imagine the containers being filled and emptied, and imagine the containers being made bigger and smaller to change capacity.

Mathematics

***Practice**. Use materials, computers and pictures in worksheets to experience capacity. Differentiate it in drawings from length. Experience having one container (e.g. a cup of water) and a larger container (e.g. an empty ice-cream container) and guessing how high up the water will go – repeat this activity for a variety of containers.*

***Connections**. Obtain a tall thin container and start pouring from other containers into the tall thin one. Mark on the tall thin container how high each container fills the tall thin container. Discuss how capacity here relates to length (or height).*

Reflection

***Application**. Relate understanding of capacity back to everyday life – what things in the world have capacity (e.g. cups, glasses, petrol tankers, and so on)? What has big/small capacity?*

***Flexibility.** Think of as many things as you can that have capacity. Visit a supermarket and find different containers which have the same capacity.*

***Reversing**. Up to now the students have gone from container → experiencing capacity. Reverse this – go from capacity as an experience → container. Thus, do activities where students construct containers for particular purposes.*

***Generalising**. Discuss what makes a container have large or small capacity. Discuss how we can measure this. Think back over connections – is there an idea here? Try to elicit that capacity could be measured by how high up the side a given container can be filled. (This is the beginning of the **attribute leads to instrumentation** big idea.)*

***Changing parameters**. What if the material being poured could contract or expand? What happens if the material becomes a gas as it is poured (e.g. it is only a liquid because it is under pressure and pouring releases the pressure)? What does capacity mean here? What if it freezes as it is being poured? What if a liquid turns to ice? (e.g. What happens when a full water bottle is put in the freezer?)*

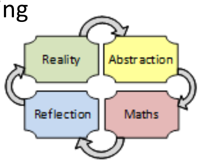
3.3.2 Comparing and ordering capacity

After experiencing objects that possess the attribute of capacity, students can begin comparing and then ordering objects according to their capacity.

Direct and indirect comparison of capacity

Capacity can be directly compared by having students pour the contents of one container directly into another container and observing the result. If the contents of the second container can fit into the first container without any overflow then the first container has a greater capacity. If the contents of the second container overflow the first container then the second container has the greater capacity.

Young students often associate the attribute of capacity with other attributes like height. They can believe that it is how tall a container is that relates to how much it can hold rather than the total amount of space inside the container that can be filled – in the case of capacity, with liquid or a liquid substitute, e.g. sand. Through direct comparison, students can be helped to understand that short containers can hold more than tall containers if they are wider. An indirect comparison can be measured using an intermediary – e.g. putting material into the same container and marking where it ends.



RAMR lesson for comparing and ordering capacity

Reality

Use relevant real-life contexts to embed the activities – use local objects or situations.

Abstraction

Body

Compare and order capacity of students' hands. Compare how much rice can be held in one or two cupped hands of students. Begin with comparing two students and then introduce a third and so on to order capacity and determine who is able to hold the greatest/smallest amount. Focus on finding the capacity between the other two.

Reinforce pouring words like full and empty and introduce comparing words such as more/less, larger/smaller capacity, greater/lesser capacity, and so on.

Hand

Use sets of containers and compare them directly by placing one inside another (works best for similar-shaped containers but different shapes will also work as long as inside each other) or beside each other. Extend to ordering three or more different containers.

Indirectly compare and order by pouring water, rice or sand (and other liquids) from one container to another and seeing which container holds more. Elicit from the students that if the first container still has material after the second is filled, the first has larger capacity; and if the first container is emptied before the second is filled, then the second has larger capacity. Extend this to order. Ensure students understand that there must be no spillage, the first container must be full, and the second container empty for the pouring to be accurate.

Continue this even more indirectly by choosing a container (tall and thin is best) that the other containers can pour into and on the side of which marks can be made to show the height of the poured material. Elicit from the students that the container whose poured material is higher up the side has the greater/greatest capacity and the one whose poured material comes lower up the side has the smaller/smallest capacity. (This continues to develop the big idea that ***attribute leads to instrumentation****.)*

Remind students that capacity is a special form of volume so that this word can also be used. Experience greater/smaller and greatest/smallest capacity using virtual means and using pictures.

Mind

Imagine, then draw and describe, a variety of objects, demonstrating comparisons and orderings of capacity. For example: an empty and full container, two identical containers with the same capacity, two different containers with the same capacity. Imagine them in the mind, and imagine them getting larger and smaller and holding more and less.

Mathematics

Practice

Use comparison and ordering experiences from Abstraction to practise being able to compare and order capacities. Continue to discuss comparative language for capacity such as large, larger, largest, small, smaller, smallest, full, empty, less, more, and so on. Present students with a variety of comparing and ordering problems, for example:

- *Find a container that will hold more water than this one.*
- *Find a container that will hold less water than this one.*
- *Find a container that will hold the same amount of water as this one.*
- *Find three containers and put them in order based on the size of their capacity.*
- *Order pictures of objects or pictures of partly filled glasses or containers.*

Make sure that students understand comparison notation (i.e. > and <, = and ≠) and the rules of comparison (i.e. nonreflexive, antisymmetrical, and transitive).

Connections

Continue to draw the connection between length and capacity through "measuring containers" as in section 3.3.1.

Reflection

Application

Relate understanding of comparing and ordering capacity back to everyday life – what things in the world have large and small capacity (e.g. a jug has greater capacity than a glass, a bucket has more capacity than a bottle, a petrol truck has less capacity than a swimming pool, and so on). Investigate (e.g. using the Internet) to find out things that you are unsure of.

Flexibility

Think of as many pairs of things as you can that have more/less capacity than each other. Visit a supermarket and find different containers which can be ordered in terms of capacity.

Reversing

Make sure teaching goes from: (a) teacher provides containers → students give comparison word, to (b) teacher gives comparison word → students provide containers.

*Also remember that comparison can be considered as a **triad**, with three parts: first container, comparison word, and second container– thus there are three "directions", or three problem types: (a) give first container and a word (e.g. larger) and require a larger capacity container; (b) give second container and a word (e.g. greater) and ask for a container that the second container is greater than; and (c) give two containers and ask for word(s) to relate them (e.g. the second container has less capacity). (This is developing the **triadic relationship** big idea.)*

Generalising

Generalise the things that have already been stated: (a) more and less capacity for a container depends on container it is being compared to; (b) comparison by pouring is only accurate if first container is full and second container is empty; (c) when pouring into a "measuring container", the container that fills the measuring container to the highest point is the largest and vice versa; and (d) for comparison, like all triadic relationships, there are three problem types.

Changing parameters

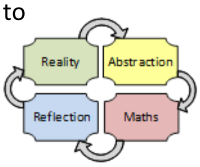
Again discuss ordering capacities for when the material changes as it is being poured.

3.3.3 Non-standard units for capacity

Background

The attribute of capacity has the ability to pour so it is through the use of smaller containers to fill larger containers that we can find non-standard units for capacity. Using buckets, cups and spoons enables students to investigate ways to quantify the amount of water, rice or sand that can fill another container. As with other attributes, students can begin to investigate quantifying the continuous attribute of capacity by using a variety of containers to fill another. When you want to compare the capacity of one container with another, the count of units becomes the focus and if the units used are different this might not result in the expected outcome.

Using uniform non-standard units for capacity is different to other measurement attributes in that you don't need multiples of the unit. Instead, the same unit can be used repeatedly. For example, the same cup can be used repeatedly to fill an ice-cream container and the capacity can be described as the count of the number of cups needed to fill the container. Care needs to be taken when using non-standard units for capacity as spillage can result in over measurement. When measuring capacity it is likely that the container being measured will not hold an exact number of the smaller units. This provides an opportunity for students to investigate part units.



RAMR lesson for non-standard capacity units

Reality

Use relevant real-life contexts to embed the activities – use local objects or situations.

Abstraction

Body

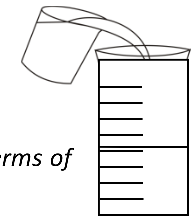
Fill containers with handfuls of materials (e.g. water, sand, rice, and so on). Count how many handfuls to fill the container (say it is 7). Say that the capacity of the container is 7 handfuls.

Hand

Pour material into containers from smaller containers and count how many small containers full are needed to fill the lager container. Use spoons, thimbles, lids, small glasses, egg cups, buckets, and so on. Ensure students are measuring with accuracy which means having the following accurate measurement processes:

- *ensuring that the unit container (e.g. the glass) is always full;*
- *ensuring that the container to be measured (e.g. the jug) starts empty; and*
- *ensuring there is no spillage of material during pouring.*

Always ***estimate*** *before pouring and calculating capacity. Always say the capacity of containers in terms of number and unit (which is the container that is being counted), for example, the jug has a capacity of 9 glasses.*



Make a homemade "measuring container". Calibrate by regularly spaced lines, from a glass or jar which has a constant diameter. Use this to measure capacity/volume of containers, by pouring material into the measuring container from various containers.

Use virtual materials and pictures to further experience measuring capacity of containers in terms of non-standard units.

Mind

Have the students imagine containers being filled from smaller containers and determining how many of the smaller container fit into the larger. Also imagine a "measuring container" being filled from other containers and the capacity being determined by how many lines up the side it fills the measuring container to. Make drawings of what is imagined.

Mathematics

Practice

Continue to provide situations for students to experience measuring capacity with non-standard units – use containers, pouring materials and also use virtual materials and worksheets with pictures. Estimate first.

Reinforce accurate measurement processes while the practice is being undertaken. Have worksheets where students have to identify inaccurate measurement processes.

Connections

*Connect non-standard measurement of capacity to division. For example, working out how many glassfuls are in a jug is the same as dividing the jug's volume by the glass's volume. This means that the rules of division apply to measurement. The most powerful of these is inverse relation – that bigger units or glasses mean fewer glassfuls to fill the jug and vice versa. (This leads into the **inverse relation** big idea.)*

Reflection

Application

Measure capacity with non-standard units in real-world situations. Set up capacity problems based on non-standard units, e.g. cupfuls, bottlefuls, etc.

Flexibility

Find capacity non-standard units in local community (e.g. measuring the capacity of a fuel tank in terms of cans of fuel, making cakes using tablespoons and cups as measuring units). Try to get students to think of all ways non-standard capacity units are used in the world, local and otherwise.

Use history and look at other units used in the past (e.g. the gallon which was the amount of wheat in a standard barrel).

Reversing

The components of a non-standard measure are object, non-standard unit and number. These form a classical triad and result in three problem types:

(a) ***Number unknown** – for example, how many jugs to fill the bucket?*

(b) ***Object unknown** – for example, find an object which is 5 jugs in capacity.*

(c) ***Unit unknown** – for example, the container has a capacity of 7, what is the unit?*

*Make sure all three directions are taught. (This leads to understanding the **triadic relationship** big idea.)*

Generalising

*Here the objective is to extend the understanding from Abstraction and Mathematics to teach **continuous vs discrete** and the **three measurement principles** as follows.*

1. ***Continuous vs discrete.** Discuss how the capacity non-standard units have broken up a continuous body of water into small parts that allow capacity to be counted. (This leads to the **continuous vs discrete** big idea – that there are two ways that number is applied: (a) to discrete objects, and (b) to continuous things such as capacity by the use of units like glassfuls to discretify the continuous volume.)*

2. ***Measurement principle 1: Common units**. Use torpedoing to show that: (a) we cannot measure accurately if we vary the container as we count containers full; and (b) we cannot know if a container is larger/smaller than another unless we use the **same unit**. Measure a container sloppily, at times only half filling the container you are pouring from. Say a large container is 5 units and a small container is 15 units and ask why this could be? (This leads to the first and second aspects of the **common units** big idea – that **units must be the same size when measuring and comparing containers**.)*

   *Then set a **common class unit** – a common size container, and measure using this. Elicit what bigger numbers mean now. (This leads to the third aspect of the **common units** big idea – that **when units are the same, the larger number specifies the larger object.**)*

   *Discuss if different units were used in different towns and countries, what would this mean? We could be paying more for a smaller bottle? (This leads to the fourth aspect of the **common units** big idea – that **there is a need for a standard**.)*

3. ***Measurement principle 2: Inverse relation**. Measure things with large and small containers and record results on a table. Study the results for patterns. This is best done with the students working in pairs. (Activities like this lead to the **inverse relation** big idea – **the larger the unit, the smaller the number and vice versa** – and to the understanding that **measuring in units is like dividing**.)*

4. ***Measurement principle 3: Accuracy vs exactness.** Three activity sets:*

   (a) *Get students to put 5 cups of water in a container and then measure it out in the same cups. There should be a difference in the second measure because of error. Discuss this error and whether we always want to be exact. Discuss how to make the measuring more accurate.*

   (b) *Measure the capacity of a container with large and small units. Discuss which is more accurate if giving whole numbers of units.*

   (c) *Give students a variety of containers as non-standard units (e.g. buckets, glasses, thimbles, and so on) and a variety of measuring activities. Have the students select the appropriate measuring container and record the capacity as number and container.*

   *Discuss situations where estimation would work. (This leads to the three consequences of the **accuracy vs exactness** big idea – **smaller units give greater accuracy**, students require **skill in being able to choose appropriate units**, and students require **skill in estimating**.)*

ACTIVITY: GUESS WHAT HOLDS MORE

1. *Take two pieces of A4 paper.*
2. *Make the round part of a cylinder longwise from one A4 sheet and the round part of a cylinder shortwise from the other.*
3. *Use more tape and paper to give each cylinder a base.*
4. *Pour rice from one cylinder to the other. Which is larger? Why?*

Changing parameters

What if the units were the standard ones, would they act the same with regard to measurement processes and principles as non-standard units? Would they need similar study of measurement processes and principles? Would this study have common points?

3.3.4 Standard units for capacity

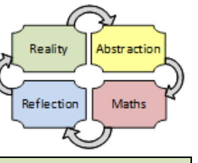
The SI base unit for capacity is the litre. The symbol used to represent litres is the capital "L" rather than the lower case "l" due to the possibility of confusion with the number "1". Capacity, like mass, is a measurement attribute where there is common usage of small to very large units.

The standard units of capacity and their relationships are:

1 litre (L) = 1 000 millilitres (mL)
1 kilolitre (kL) = 1 000 L = 1 000 000 mL

Measuring devices for capacity using standard units

As with using measuring devices for measuring mass, measuring capacity also relies on students' ability to read and interpret number lines. Depending on the number of graduations and which of the graduations are labelled, the task of measuring capacity can be made more or less complex. Measuring jugs often don't have straight sides so the variation necessary between the graduations can appear to be uneven which can confuse students when they are measuring capacity. Specific skills for the use of the measuring devices may be needed for students to be able to measure capacity accurately and therefore to understand the measurement concepts associated with measurement tasks.



RAMR lesson for standard capacity units

Reality

Use relevant real-life contexts to embed the activities – use local objects or situations.

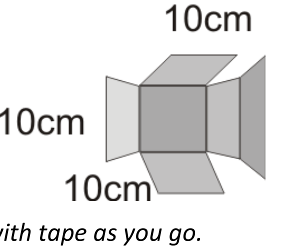
Abstraction

Common unit

After the need for a standard has been developed through the use of non-standard units in Stage 3, time can be spent measuring capacity with a class-chosen unit – e.g. a common object like a glass or cup. This can be used by students to show that with a common unit, a higher number really means a greater capacity.

Identification

Using 1 cm grid paper, draw and cut out a net for a cube of side 1 cm. Fold and tape to make the cube. Fill the 1 cm cube with rice or sand. Put this into another container. This represents 1 mL.



Use cardboard to make a net for a cube of side 10 cm. Tape and fold this cardboard to make the cube. This cube is 1 L. Check this by pouring 1 L of water or sand into it. Check that this cube holds the same as a 1 L soft-drink bottle. This is the same size as a MAB thousand block.

Calibrate a container into 100 mL levels using either of the following methods:

***Method 1** – take a glass jar and pour 100 mL amounts into it, marking the levels with tape as you go.*

***Method 2** – take a 1L milk carton, cut off the top and use a ruler to divide the height into 10 equal intervals.*

Make a cubic metre with metre-length dowelling – put a MAB thousand cube in it. Note that there are 1000 MAB blocks in the cubic metre – so a cubic metre is a kilolitre.

Internalisation

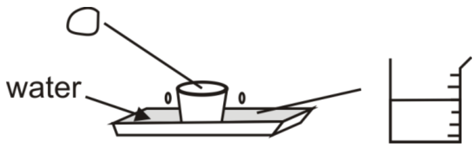
Obtain a collection of jars and jugs and pour 250 mL and 500 mL into them and note levels. Try to estimate where the levels will be before pouring.

Estimation

First estimate and then measure the capacity of objects as below (find some of your own). Estimate and measure each object before moving on to the next. ***Estimate the capacity – do not estimate length, breadth, height****. Use measuring cylinders for checking.*

OBJECT	ESTIMATE	MEASURE	DIFFERENCE
Capacity			
Cup			
Glass			
Bottle			
Plastic container			

Use an overflow tray and a measuring cylinder to find the volume of objects by immersion and overflow. Estimate first.



OBJECT	ESTIMATE	MEASURE	DIFFERENCE
Lump of plasticine			
Rock			
Your fist			

Place-value connections

Set up the place-value cards with place-metric cards as follows:

One Millions	Hundred Thousands	Ten Thousands	One Thousands	Hundred Ones	Ten Ones	One Ones	Tenths Parts of One	Hundredths Parts of One	Thousandths Parts of One
			Kilo Litre	Hecto Litre	Deka Litre	Litre	deci Litre	centi Litre	milli Litre

Analyse the meaning of milli and kilo and use relationships between place-value positions to reinforce relationships between metric units.

Mathematics

Metric expanders

Construct a larger copy of Expander B (kilolitres, litres and millilitres) in Appendix A and cut it out. Fold the expander like number expanders. Use them to relate kL, L and mL as for place-value cards.

Metric slide rule

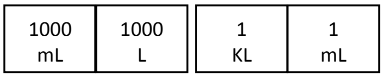
Copy the metric slide rule in Appendix A. Using scissors, cut out the slides and the scale, and slit the scale along the dotted lines. Then, using the rounded end of the slide as a tongue, thread each slide ***from the back*** *up through the slit on the left of the scale and across the front and out the slit on the right of the scale.*

Use the slide rule to relate metrics and decimal numeration.

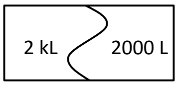
Practice

Consolidate the metric conversions through drill – some examples:

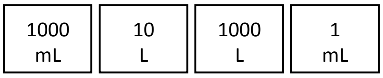
1. *Dominoes*



2. *Bingo*
3. *Mix and Match cards*



4. *Card decks (for Concentration, Gin Rummy, Snap, etc.)*



Note: Metrics should be introduced along with decimals. They apply decimal understanding and reinforce decimal concepts. For instance: two decimal places are related to money (dollars and cents) and length (m and cm); and three decimal places are related to length (km, m, cm and mm), mass (T, kg, and g) and volume (kL, L and mL).

3.3.5 Applications and formulae for capacity

There are no formulaic applications of capacity except through the conversion of units. The relationship between the attributes of mass, volume and capacity (1 g = 1 cm^3 = 1 mL) is described in section 4.4. This relationship can provide opportunities for students to solve problems involving conversion between these attributes.

Providing students with opportunities to solve real-life problems that require the measurement of capacity helps them to apply their understandings of this attribute in ways that relate to life beyond school.

Applications for capacity should be built around the idea of triad – there is an object, a unit of measure and the number of units.

Thus, applications for capacity can be built around three types of problems:

- **Number unknown** – *measure the capacity of this bowl in millilitres*.
- **Object unknown** – *find an object that is 4L*.
- **Unit unknown** – *this object is 500 units, are these units mL or L?*

4 Perimeter, Area and Volume

The attributes of perimeter, area and volume are attributes that can be measured in the same way as those described in Chapter 3 by counting units. The attributes described in this chapter are different in that they can also be calculated once particular measurements have been taken. Most often this is a more efficient way to measure these attributes. The attributes of perimeter, area and volume are all related to the attribute of length and the measurement of perimeter, area and volume can be calculated after measurements of particular lengths have been completed. Even though these attributes can be calculated, it is still important for students to work through the learning sequence for each of these attributes so they can better understand the relevance and application of the formulae related to the calculations.

4.1 Perimeter

Perimeter is a special name for length when it measures the distance around something, like around a shape, a tree trunk or a house. Perimeter is a measure of one dimension.

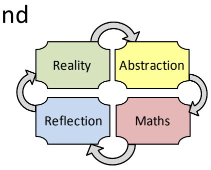
4.1.1 Identifying the attribute of perimeter

This stage focuses on identifying the attribute with the use of appropriate vocabulary.

Background

Perimeter is related to length and height but is best associated with distance around something. Because of its close relation to length, perimeter should simply be seen as an extension of the length activities. There are ways to measure perimeter directly, by walking around counting paces or using a trundle wheel, and indirectly through using an intermediary such as string. Perimeter is also the total distance found by adding the length of all sides of a shape or boundary. Perimeter for circular shapes is given a special name – circumference.

Representing the distance around objects using flexible materials, e.g. string, and then straightening the string can assist students to see that perimeter is in fact length and can be compared and measured in the same way as other lengths and by using the same measuring devices, e.g. rulers, measuring tapes and trundle wheels.



RAMR lesson for identifying the attribute of perimeter

Reality

Where possible, find real-life contexts to embed the activities in; for example, using relevant objects or situations.

Abstraction

***Body**. Use actions, mimes and dances to reinforce the notion of length: act out growing higher or growing longer and shorter (e.g. a tree growing, a fish getting longer). Act out with students' bodies all the length words – long/short, tall/short, wide/narrow, thick/thin, high/low, deep/shallow, near/far, up/down, and so on.*

*Experience "distance around" things to introduce **perimeter** – e.g. walking around a building or a shape marked on the ground, running around the oval – note that this restricts language to longer/shorter and smaller/larger. Experience distance around circular things such as a roundabout – introduce the special word **circumference**. Use words "distance around" before introducing "perimeter" or "circumference".*

Hand. *Experience length with a variety of materials – discuss whether they are long/short, thick/thin, and so on. Put out pencils of different lengths (what is different?) or put out a feather, a pencil, a strip of paper, a can, and a duster that are all the same length (what is the same?). Hold up an object, look for things that are the same length as it, look for things that are different lengths to it. Experience length in terms of distance around or* ***perimeter*** *– run finger around shapes and objects, cut strips of material to fit around head or waist, tie string around objects (how much do we need?), put tape around objects, and so on.*

Mind. *Students shut eyes and imagine walking around something that takes a long time or short time, or putting string around objects that are large and small. They describe the things they imagine – in terms of "distance around". Students shut eyes and think of a perimeter – then make it longer/shorter, and so on. Students draw the original. Students think of things that are the same perimeter.*

Mathematics

Practice. *Worksheets where students sort objects into long and short perimeters.*

Connections. *Show non-connections – find long and square and short and round, and so on.*

Reflection

Application. *Take new understanding of perimeter/circumference out into the playground and identify long/short.*

Flexibility. *Think of many perimeters that are long/short, and so on. Find a stone with the longest perimeter, and so on.*

Reversing. *Get students to draw shapes that have long/short perimeters.*

Generalising. *Look at different perimeters – what makes them long and short? Discuss what people might say in different situations – is there only one view of long and short for everything? (e.g. it's long for a piece of firewood but short for the trunk of a tree). Discuss that long/short depend on perspective or situation or experience of people. Discuss ways in which we could determine which of two perimeters is longer. (Try to elicit the beginnings of getting the* ***attribute leads to instrumentation*** *big idea.)*

Changing parameters. *Consider perimeter not being a straight line. Could we have a squiggly perimeter?*

4.1.2 Comparing and ordering perimeter

This stage focuses on comparing with two examples and ordering three or more without using a number. The change from comparing to ordering requires a focus on "between".

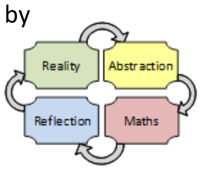
Once students understand the attribute of perimeter they can explore ways to compare and order the perimeter of different objects.

Direct comparison of perimeter

The direct comparison of perimeter is difficult due to the nature of this attribute of measurement being the distance around something. One way that two perimeters could be directly compared would be to place the edge of one object directly onto the other and to roll the second object around the edge of the first, taking care to note the starting point of the comparison on one of the objects and the end point of the shorter perimeter. For example, comparing two lids, even if they are different shapes, could be done in this way.

Indirect comparison of perimeter

To compare perimeters without referring to a count of units requires the use of intermediary measuring devices like string. Therefore comparison of perimeters is often done indirectly. For example, if you wanted to work out which of two trees had the biggest perimeter or distance around its trunk, the perimeter (girth) of each tree could be represented using ribbon and then the two ribbons could be directly compared by aligning their ends and seeing which is longer.



RAMR lesson for direct and indirect comparisons of perimeter

Reality

Where possible, find real-life contexts or use relevant objects or situations to embed the activities in.

Abstraction

Body

Indirectly compare and order distances around parts of students' bodies – use string or strips of paper or any other flexible length; use these intermediaries to compare and order – do not use numbers. Mark how long they are or cut them to length, and then directly compare and order. State which "distance around" or perimeter is longer-larger or shorter-smaller.

Hand

Indirectly compare and order distance around objects using intermediaries – use string, rope, and so on. Either mark two or more intermediaries and compare, or use one intermediary and see if more or less length is used for distance around or perimeter.

Also experience comparing and ordering distance around (circumference) circular objects and circles.

For ordering, focus on finding the "between" examples. Introduce language such as longest/shortest and largest/smallest. Note: Can compare distances around large things by running at same speed and seeing who finishes first. Complete both types of activities:

(a) *perimeter/circumference to length activities (e.g. rolling bicycle wheel one revolution along a wooden plank, wrapping a paper strip around a cylinder/can, opening out a wire rectangle to compare it with a stick);*

(b) *perimeter/circumference to perimeter/circumference activities (e.g. using string or paper as an intermediary, opening out wire or geostrip shapes and comparing the total lengths of their sides, and rolling two cans one revolution each to see which can rolls further).*

Drying glasses activity *(an idea from the "I hate mathematics" book)*

When you dry a glass with a towel, you dry up the side and around the top rim. Which is longer, the distance up the side (the height) or the distance around the top (the circumference of the circle)? Use your towel. Mark off the height of the glass on the towel. See if this distance will wrap around the top. Is this always true? Are there glasses that do not do this?

Mind

Shut eyes and imagine different distances around – which is longer/shorter? How would you determine this? Draw different examples.

Mathematics

Practice

Give students many opportunities to compare and order "distances around" (both perimeters and circumferences) indirectly – even using string on worksheets.

Connections

Connect to principles for working out the longer "distance around" – must make sure to go all the way around, and line – draw a straight line, take two objects of different length and, in turn, put one end on the start of the line and mark the end points on the line. Discuss what the positions of the end points mean [longer is further along the line].

Reflection

Application

Discuss real-world situations of longer/shorter distances around. Find things that are in between two perimeters or circumferences.

Flexibility

Think of all the situations where you would use the length words (i.e. longer/shorter, longest/shortest) with regard to distance around, perimeter or circumference.

Reversing

Make sure you do both the directions – objects → perimeter length, and perimeter length → objects. That is: "Which of these two objects has the longer perimeter?" and "Find me an object with a longer perimeter than this object".

Remember that comparison can be considered as a ***triad*** *with three parts – first object, comparison word, and second object – thus there are three "directions":*

(a) *give first object and words (e.g. longer perimeter) and require a shorter perimeter object;*

(b) *give second object and words (e.g. shorter perimeter) and ask for an object with a perimeter that the perimeter of the second object is shorter than; and*

(c) *give two objects and ask for which has the longer/shorter perimeter. (This is developing* ***the triadic relationship*** *big idea.)*

Generalising

Try to find things that hold true for all comparisons of perimeter and circumference. Obviously the circle with the smaller circumference fits inside the circle with the larger circumference. Is there something similar for perimeters? The ideas in connections can be generalisations – e.g. do not overlap. Show that a perimeter can be both longer and shorter (comparison depends on the size of the object to which it is compared). Generalise the reversing activity above.

Changing parameters

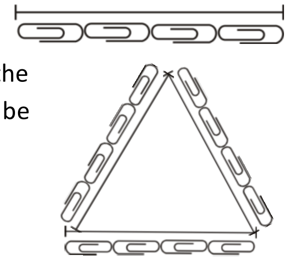
Instead of looking for, for example, longest or shortest, look for a length ***in between*** *(e.g. find something whose length is in between the board's height and the door's width). Look at length which is not straight. Use intermediaries to find which of the distance around two different shoes is longest.*

4.1.3 Non-standard units for perimeter

This stage focuses on introducing the idea of using a constant unit and developing processes and principles. It also introduces the first use of number and unit.

Background

When using objects as units for perimeter the focus needs to be on the length of the object. Perimeter is an application of the attribute of length. Because the perimeter being measured is likely to be an actual object, like a garden or a picture frame, the units used are likely to have more than one dimension. Length is one-dimensional so this could cause confusion. For example, the measurement of the perimeter of a garden could be done by counting the bricks or pavers that surround the garden. Whether these bricks are all the same length or not, the count of these bricks would be a measure of the perimeter in non-standard units.



Uniform, non-standard units are units that are the same length but are not the recognised standard measures for length (cm, m, etc.). Items such as paperclips can be used to provide a number that will represent the distance around a shape.

4.1.4 Standard units for perimeter

This stage focuses on the introduction of standard units and the conversion of one unit to another.

Background

Perimeter is an application of length, so the standard units for measuring perimeter are the same as the standard units of length, i.e. millimetres (mm), centimetres (cm), metres (m) and kilometres (km).

4.1.5 Applications and formulae for perimeter

This stage focuses on applying attributes through the use of triads and teaching formulae, if they exist. Perimeter is a major extension of length.

Applications

The approach to applications of perimeter is built around the big idea of triad – there being object, number and unit in any measure means that there are three problem types for each to be unknown:

- **Number unknown** – *measure around this room in metres.*
- **Object unknown** – *find an object with a perimeter of 16 cm.*
- **Unit unknown** – *this object has a perimeter of 35 units, are these units cm or mm?*

Formulae

There are three major formulae that can be discovered.

Perimeter of rectangle

Draw rectangular shapes on graph paper and record lengths, widths, $L + W$, and perimeters on a table as below. Discussion will lead students to discover that the **perimeter of a rectangle is $2(L + W)$.**

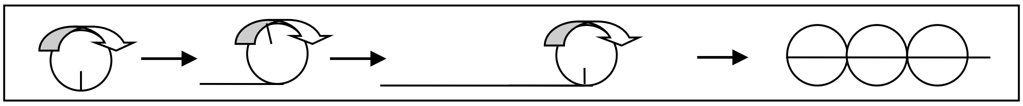
Shape	*L*	*W*	*L + W*	Perimeter

Perimeter of regular polygons

In a similar way, the students can discover that the perimeter of an n-sided regular polygon of length of side L is $n \times L$. This makes the perimeter of an equilateral triangle $3 \times L$ and the perimeter of a square $4 \times L$.

Circumference of a circle

Find a circle such as a wheel, hoop or a cylinder, and mark it on the edge; roll it so that it does one revolution and draw a line that represents this roll (this is circumference); then draw three copies of the wheel/circle on this line, with the line passing through the diameters of the three circles (see diagram below). Ask students before they place copies of the wheel along the line to guess how many diameters in the revolution (circumference).



Do this with more than one size wheel. Students will see that there are "three and a bit" diameters in the revolution. Use this for examples such as: *For a 2-metre diameter garden, how many metres of bricks around the edge?* – 6 and a reasonable bit.

When students have this, introduce **pi.** This can be done by measuring and dividing circumference by diameter. It is best if this is done by cylinders: put 10 cylinders side by side and measure across all 10 and divide by 10 for diameter; then wrap string 10 times around a cylinder, measure string and divide by 10 for circumference. Then put on a table as below for different circles/cylinders to show that there is a common ratio called *pi* (which is 3 and a bit or more accurately 3.14) which relates diameter and circumference.

Circle	*C*	*D*	*C* ÷ *D*

This will show that **circumference of a circle (C) is $\pi \times D$** where D is diameter, and **circumference of a circle (C) is $2 \times \pi \times r$** where r is radius and is half the diameter.

Three activities for introducing the circumference of a circle are given below.

Activity A – Using streamers for circumference of a circle

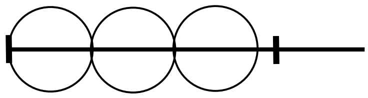
1. Obtain a variety of circular objects: lids, rolls of tape, bottles, jars, and so on.
2. Give to students with streamers – ask to wrap and tear streamer so length is once around the object.
3. Bring object out to the front and place circular object above a line and length of streamer below – look at and discuss circular objects and their length (any patterns).
4. Place objects in order of streamer length – what is noticed about circular objects?
5. Check how many diameters are in related streamer length.

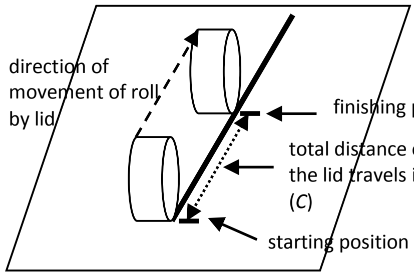
Activity B – Using rotation and diameter for circumference of a circle

To find the formula for calculating the circumference of a circle, follow the procedure below:

1. Find a sheet of paper, three different-sized lids, a ruler, and a pen. Label each lid **small**, **medium**, and **large**, according to their relative sizes.
2. Use a ruler to draw three straight lines across a sheet of paper.
3. For each of the three lids in turn, mark a line on it in one place on its side (as shown in the figure at right). Turn the lid on its side, and position the lid at one end of one of the lines you drew on the paper so that the line you drew on the lid touches the paper. Roll the lid one time along the paper (as shown in figure on right below). Mark the position where it stopped on the paper.

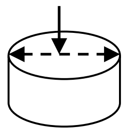
4. For each of the three lids, lay the lid flat, and see how many fit on the line (as in figure immediately below). Note there are 3-and-a-bit circles in the line.





5. Is it the same 3-and-a-bit for all the lids? What does this mean?

6. For each lid in turn, measure this distance called C on the line (see figure above right) and measure the distance called D directly across the centre of the lid from edge to edge (as shown in the figure at right). Make sure you label the lid for which C and D are calculated.



7. Write the data you collected in Step 6 on the table below (remember to include the units of length that you are using) and then divide C by D for the third column:

Lid	***C***	***D***	***C ÷ D***
Small			
Medium			
Large			

8. What did you get in the third column for each of the lids? Was it close to 3-and-a-bit? What does this mean?

Activity C – Using formulae for circumference of a circle

1. From Activity A, you should have observed that the value for $C \div D$ is equal to 3-and-a-bit. This value represents the ratio of the **circumference** (C) of a circle to the **diameter** (D) of the circle. A better approximation of this ratio is known to be 3.14 and is more formally referred to as **pi**, symbolised using the Greek letter π.

2. So, we have developed the formula $C \div D = \pi$. Rearranging the formula, we obtain $C = \pi \times D$. Hence, the formula for calculating the circumference of a circle is $C = \pi \times D$. That is, to find the circumference of a circle, we multiply the diameter of the circle by π. Your calculator may have a π button. If so, then you can use this button to calculate the circumference of a circle. If not, then you can just use 3.14 in place of π.

3. Match each of the expressions on the left with its approximated calculation on the right. Draw a line to connect the expression with its calculation.

1.	2π	0.52
2.	$\pi + 2$	1.27
3.	$\pi \div 6$	1.86
4.	$5 - \pi$	2.37
5.	$3\pi \div 4$	3.57
6.	$4\pi - 9$	5.14
7.	π^2	6.28
8.	$4 \div \pi$	9.87

4. Use the formulae $D = C \div \pi$ and $C = \pi \times D$ to complete the tables below. Round your answers to the same number of places as given. What will the formulae be for the radius r?

C	***D***
48 cm	
3.5 m	
	718 mm
86 cm	
	0.19 km

C	***D***
	2.75 m
	41 cm
777 mm	
	21 m
34.8 cm	

4.2 Area

Area is the amount of space enclosed within a plane or flat shape.

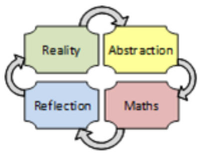
4.2.1 Identifying the attribute of area

This stage focuses on identifying the attribute with the use of appropriate vocabulary.

Background

When students refer to the size of an object like a table as being "big" or "bigger" they are often referring to the attribute of area. Area is a planar or two-dimensional concept related to the geometric notion of region. It is a measure of the amount of **coverage** of that region (it is related to length usually in two directions). In the earlier primary years, students can investigate the attribute of area by covering surfaces in a number of ways. Painting a surface, covering a book with coloured paper or sticking lots of smaller pieces of paper onto a picture to fill a space are all area examples. Completing jigsaw puzzles where the pieces fill a frame or a particular area are also examples of investigating the attribute of area.

RAMR lesson for identifying the attribute of area



Reality

Have a discussion relating to real-life examples of the area attribute, e.g. painting a wall, tiling a floor, paving a path.

Abstraction

Body

*Start with experiencing area – wrapping packages, wrapping strangely shaped (or hard to wrap) parcels, painting and colouring in. Introduce the term area as **"coverage"** (and large and small area). Use bodies to "cover" the floor, or hands to cover the desk.*

Hand

Repeat body activities with hands – covering desktops with paper, covering play areas with paper, cutting and pasting one shape to cover another, and using stamps or paint on hands to cover a space with colour.

Experience area in different ways – long thin shapes, short fat shapes, large shapes, small shapes, interesting shapes. Discuss that area requires things to be closed. ***Discuss that area does not have to be flat*** *and provide experiences covering non-flat things.*

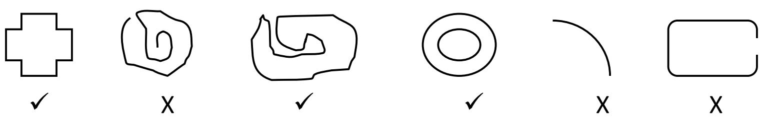
Mind

Encourage students to imagine surfaces and the covering of surfaces.

Mathematics

Practice

Use materials, computers and pictures in worksheets to experience area. Get students to colour shapes that have area and do not have area – hexagon yes, hexagon with part of a side missing no, rolled up long thin rectangle yes, spiral no (see below). Use pictures of surfaces as well.



Connections

Discuss how area relates to length – look at a rectangle. What is it about the lengths of the sides that will affect area? Make length and width longer – discuss how this may relate to area.

Reflection

Application

Relate understanding of area back to everyday life – what things in the world have area (e.g. plates, tables, walls, footballs, and so on) – what has big/small area? Introduce surface area – the term means "area of the surface".

Flexibility

Think of as many things as you can that have an area. Walk around the school. Find things with the same or similar area.

Reversing

Up to now the students have gone from shape/surface → experiencing area. Reverse this – go from area as an experience → shape/surface. Thus, do activities where students construct areas for particular purposes, such as wrapping a present or covering a book.

Generalising

*Discuss what makes a surface/shape have large or small area. Discuss how we can measure this. (This is the beginning of the **attribute leads to instrumentation** big idea.)*

Changing parameters

How can we change area – what kind of materials will do this, e.g. blowing up a balloon?

4.2.2 Comparing and ordering area

This stage focuses on two examples and ordering three or more without using number. The change from comparing to ordering requires a focus on "between".

After experiencing the attribute of area through covering surfaces, students can begin comparing and then ordering objects according to their area.

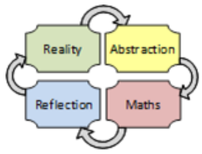
Direct comparison of area

Area can be directly compared by overlaying objects to see which has the greatest surface. This is more complex than the other direct comparisons due to area being a two-dimensional measure. There could be overlap of one of the dimensions but not the other, e.g. one item is longer but not as wide. Making a judgement about which has a larger area is not directly obvious.

Indirect comparison of area

The use of an intermediary object for area can allow two surfaces to be compared when they are not able to be directly compared. Again the complication of the comparison needing to consider the two dimensions of area can make this a complex task. In many ways the comparison of area is easier if units are involved.

RAMR lesson for comparing and ordering area



Reality

Where possible, find real-life contexts for area to embed the activities in; for example, you could compare the area of different sporting fields – football, netball, tennis.

Abstraction

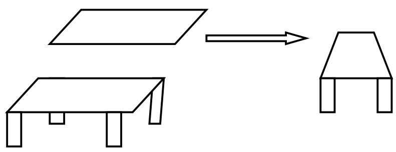
Body

Compare and order things related to body – e.g. compare and order area of students' hands, feet, etc. One idea is to lie down and trace around the body and compare with other bodies. Begin with comparing two and then introduce a third and so on to order area and determine who has greater/smaller and then greatest/smallest amount. Focus on finding the area ***between*** *the other two. Reinforce words associated with area.*

Hand

Directly compare *and order areas by placing one area on top of another area – a book on top of a seat, a pencil case on top of a sheet of paper, etc. Draw around something and compare this with the same thing of different size but same shape – e.g. pencil cases or bags. First compare two, then order more than two.*

Indirectly compare *and order area by covering one area with paper and then transferring this paper to another area. This is very good for areas of differing shape where the paper has to be cut and rejoined to fit on the other area. You can use other material to do the same thing – cloth, plastic, etc. If there is not enough material, then first surface is smaller – if too much material, then second surface is smaller.*



Practise with dissections – jigsaws and shape puzzles – before doing this to get idea of cutting and re-forming. Use tangrams (and other shape puzzles) to cover shapes/spaces/surfaces, and then break the dissections apart and rejoin differently to cover other shapes.

Experience greater/smaller and greatest/smallest area using objects, virtual means and pictures.

Mind

Imagine, then draw and describe a variety of surfaces, demonstrating comparisons and orderings of area. For example: different desktops or pinboards. Imagine them in the mind, and imagine them getting larger and smaller and having more and less area.

Mathematics

Practice

Use comparison and ordering experiences from the examples developed above to practise being able to compare and order areas. Continue to discuss comparative language for area such as large, larger, largest, small, smaller, smallest, and so on. Present students with a variety of comparing and ordering problems, for example: find a surface larger/smaller than this one. Order pictures of objects or pictures of shapes by area.

Make sure that students understand comparison notation (i.e. > and <, = and ≠) and the rules of comparison (i.e. nonreflexive, antisymmetrical, and transitive).

Connection

Continue to draw the connection between length and area.

Reflection

Application

Relate understanding of comparing and ordering area back to everyday life – what things in the world have large and small area (e.g. playgrounds, ovals, desks, books). For example, what is the largest park in your local area? Investigate (e.g. Google Maps on the Internet) to find out. Discuss where we need area – e.g. painting a house, building a wall, paving a floor or courtyard.

Flexibility

Think of as many pairs of things as you can that have more/less area than each other. Visit a hardware shop or look around your school to find different pavers which can be ordered in terms of area.

Reversing

*Make sure teaching goes from the teacher providing surfaces and students giving comparison words, to teachers giving comparison words and students providing surfaces. Also remember that comparison can be considered as a **triad** with three parts – first surface, comparison word, and second surface– thus there are three "directions", or three problem types:*

(a) *give the first surface and word (e.g. larger) and require a larger area surface;*

(b) *give the second surface and a word (e.g. greater) and ask for a surface that is larger in area than the second surface; and*

(c) *give two surfaces and ask for word(s) to relate them (e.g. the second surface has less area). (This is developing the **triadic relationship** big idea.)*

Generalising

Generalise the three things that have already been stated:

(a) *more or less area of a surface depends on the surface it is being compared to;*

(b) *comparison by covering is only accurate if surfaces are fully covered; and*

(c) *for comparison, like all triadic relationships, there are three problem types as in reversing. (Again this is developing the **triadic relationship** big idea.)*

Changing parameters

Discuss ordering areas for when the material can be stretched.

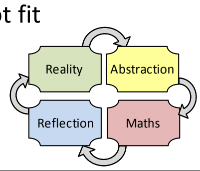
4.2.3 Non-standard units for area

This stage focuses on introducing the idea of using a constant unit and developing processes and principles. It also introduces the first use of number and unit.

Background

By using different objects to cover surfaces, students can be led to understand that some shapes are better for covering than others. The better shapes leave no gaps and do not need to be overlapped. Through using a variety of shapes, students can discover that the shapes that will tessellate, either with themselves or with other shapes, make the best units for measuring area. Then the area of the shape can be found by counting the units. When a variety of shapes are used it could be misleading to use the count of shapes as the measurement. Students will come to understand that using the same unit allows for comparison of the area of different shapes.

Another important concept that relates to the use of units for measuring area is the concept of part units. Because area is coverage, students work at fitting units into the shape. It can be a problem when students believe that the units need to fit into the shape being measured. When shapes are narrower than the unit they can think the area is zero. Being able to **partition** units into smaller part units where a whole unit will not fit and then being able to add all the parts to make whole units is a complex process.



RAMR lesson for non-standard units of area

Reality

Where possible, find real-life contexts for area to embed the activities in; for example, using relevant objects or situations.

Abstraction

Body

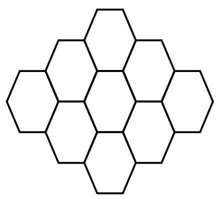
Use hands to cover surfaces – put paint or water on hands to see coverage – say that the area is 7 hands; that is, give both number and unit.

Hand

Use a variety of materials as units, e.g. blocks, cubes, tiles, sheets of paper, hands. Particularly interesting are stamps on paper, wet hands on coloured paper, etc. Use geoboards, both square and isometric (triangles). Ensure students are measuring with accuracy which means having these accurate measurement processes:

- *ensuring that the shapes cover without gaps and overlaps (or with regular gaps and overlaps);*
- *counting part units or counting those over half a shape and not counting those less than half a shape;*
- *ensuring that the surface is covered to edges (in some cases, this means choosing the unit to match the shape of the surface).*

Always ***estimate*** *before covering and calculating area. Always say the area of surfaces in* ***terms of number and unit****.*



Use tessellations as non-standard units, e.g. triangles, quadrilaterals, hexagons, Escher-type drawings. Make clear plastic overlays of these tessellations so they can be placed easily over shapes. Otherwise cut out shapes and place on top of tessellating grids. (Note: Do not be afraid to use non-tessellating shapes to cover and count.) Use tangrams to cover shapes and assign unit value to one of the smallest tangram pieces – then use this to assign a number to the other pieces. Use virtual materials and pictures to further experience measuring area in terms of non-standard units (e.g. containers).

Discuss the following:

(a) *how different units give different numbers for area [smaller units give larger numbers and vice versa];*

(b) *which size units are most accurate [small units] but discuss why we cannot use these all the time [it takes too long];*

(c) *what type/size of unit would be better for large/small surfaces and why [depends on how accurate you want to be and how quickly you have to do the job];*

(d) *what shape of unit is the most effective [squares as these form arrays which can be quickly calculated by multiplication]; and*

(e) *what is needed if you are giving an area verbally such as over the phone [need a common or standard unit].*

Mind

Have the students imagine surfaces being filled with smaller shapes and determining how many of the smaller shapes fit into the larger. Make drawings of what is imagined.

Mathematics

Practice

*Continue to provide situations for students to experience measuring area with non-standard units – use grids, overlays, and tessellating shapes to measure area of shapes on paper/worksheets. **Always estimate first**.*

Reinforce accurate measurement processes while the practice is being undertaken. Even have worksheets where students have to identify inaccurate measurement processes.

Connections

*Connect non-standard measurement of area to division. For example, encourage students to realise that working out how many hexagons cover a surface is the same as dividing the surface into equal parts (i.e. area is related to fractions and division). This means that the rules of division apply to measurement. The most powerful of these is inverse relation – that bigger units mean fewer units are needed and vice versa. (This leads into the **inverse relation** big idea.)*

Reflection

Application

Measure area with non-standard units in real-world situations. Set up area problems based on non-standard units.

Flexibility

Find area non-standard units in the local community (e.g. measuring the area of paving in terms of large pavers). Try to get students to think of all the ways non-standard units are used in the world, local and otherwise, for area. Use history and look at other units used in the past (e.g. acres).

Reversing

The components of a non-standard measure are object, non-standard unit and number. These form a classical triad and result in three problem types:

(a) ***Number unknown** – for example, how many hexagons cover the surface?*

(b) ***Object unknown** – for example, find a surface which is 12 hexagons in area.*

(c) ***Unit unknown** – for example, the surface has an area of 7, what is the unit/tessellating shape?*

*Make sure all three directions are taught. (This leads to understanding the **triadic relationship** big idea.)*

Generalising

*Here the objective is to extend the understanding from Abstraction and Mathematics to teach **continuous vs discrete** and the **three measurement principles** as follows.*

1. ***Continuous vs discrete**. Discuss how the area non-standard units have broken up a continuous surface into small parts that allow area to be counted. (This leads to the **continuous vs discrete** big idea – that **there are two ways that number is applied**: (a) to **discrete** objects, and (b) to **continuous** things such as area by the use of units like hexagons to discretify the continuous area.)*
2. ***Measurement principle 1: Common units.** Use torpedoing to show that: (a) we cannot measure accurately if we vary the unit, and (b) we cannot know if a surface is larger/smaller than another unless we use the **same unit**. Measure a surface sloppily, allowing units to have gaps/overlaps, and ask what is wrong. Say that John measured a large surface as 6 units and Jack measured the same surface as 15 units and ask why this could be. (This leads to the first and second aspects of the **common units** big idea – that **units must be the same size when measuring/comparing containers.)***

*Then set a **common class unit** – a common size tessellating shape and measure using this. Elicit what bigger numbers mean now. (This leads to the third aspect of the **common units** big idea – that **when units are the same, the larger number specifies the larger object.**)*

*Discuss if different units were used in different towns and countries, what would this mean? We could be paying more for paving? (This leads to the fourth aspect of the **common units** big idea – that **there is a need for a standard**.)*

Look particularly at the best area unit in terms of shape – why square? [use of arrays and multiplication]

3. ***Measurement principle 2: Inverse relation.** Measure things with large and small units and record results on a table. Study the results for patterns. This is best done with the students working in pairs. (Activities like this lead to the **inverse relation** big idea – **the larger the unit, the smaller the number and vice versa** – and to the understanding that **measuring in units is like dividing**.)*

4. ***Measurement principle 3: Accuracy vs exactness.** Get students to measure a surface with three different-sized tessellating shapes. Which is most exact/least exact? Repeat for other surfaces. Which unit is best for the surface? Why do we want to be accurate? Are there cases where this is not needed? Discuss situations where estimation would work. (This leads to the three consequences of the **accuracy vs exactness** big idea – **smaller units give greater accuracy**, students require **skill in being able to choose appropriate units**, and students require **skill in estimating**.)*

Changing parameters

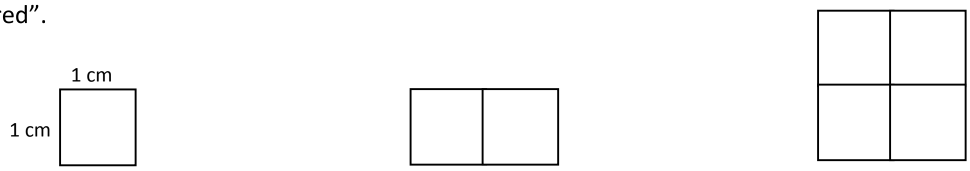
What if the units were the standard ones, would they act the same with regard to big ideas as non-standard units? Would they need similar study of measurement processes and principles? Would this study have common points?

4.2.4 Standard units for area

This stage focuses on the introduction of standard units and the conversion of one unit to another.

Background

The standard units of area are squares where the length of the side is one metric unit that relates to the size of the area being measured. Care needs to be taken when describing individual units for area. A square that is 1 cm × 1 cm is one square centimetre. Two square centimetres will be the area of two of these units. However "two centimetres squared" is referring to the area of a square that has a length of 2 cm. This square would have an area of 4 cm^2 (see figure below). These two terms (square units and units squared) are often used interchangeably and this can be linked to the way the unit is written (cm^2) which, if read from left to right, reads as "cm squared".



1 square centimetre (1 cm^2) 2 square centimetres (2 cm^2) 2 centimetres squared (4 cm^2)

Square centimetres vs centimetres squared

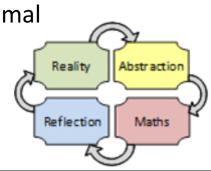
The standard units of area and their relationships are:

1 square centimetre (cm^2) = 1 cm × 1 cm = 10 mm × 10 mm = 100 square millimetres (mm^2)
1 square metre (m^2) = 1 m × 1 m = 100 cm × 100 cm = 10 000 cm^2
1 hectare (h) = 100 m × 100 m = 10 000 m^2
1 square kilometre (km^2) = 1 km × 1 km = 1 000 m × 1 000 m = 1 000 000 m^2 = 100 h

Care must be taken when converting between units of area to consider the two-dimensional nature of area. Use of place-value cards and understanding of the multiplicative nature of area can help students to manage conversions of area units.

Trillions			Billions			Millions			Thousands			Ones		
H	T	O	H	T	O	H	T	O	H	T	O	H	T	O
		km^2		h				m^2				cm^2		mm^2

Use multiplicative relationships between PV positions to look at the conversion rate between the area units, mm^2, cm^2, m^2, h, and km^2: 100 mm^2 = 1 cm^2; 10 000 cm^2 = 1 m^2; 10 000 m^2 = 1 h; 100 h = 1 km^2. By being flexible with the decimal point, students can see the connections, e.g. 2.45 m^2 = 24 500 cm^2 (place decimal point after m^2, place numerals in PV positions for 2.45, move decimal point to after cm^2).



RAMR lesson for standard area units

Reality

Where possible, find real-life contexts for area to embed the activities in; for example, using relevant objects or situations.

Abstraction

Common unit

After the need for a standard has been developed through the use of non-standard units in Stage 3, time can be spent measuring area with a class-chosen unit – e.g. a square piece of paper. This can be used by students to show that with a common unit, a higher number really means a greater area.

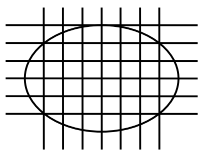
Identification

Use paper and cardboard to construct a square mm, cm and m (square m can also be made by connecting four one-metre lengths of dowel or metre rulers into a square). Place the square mm inside the square cm inside the square m – discuss the differences in size and how many of the smaller fit into the larger (100 mm^2 in 1 cm^2, 10 000 cm^2 in 1 m^2, 10 000 m^2 in 1 hectare, and 100 hectares in 1 km^2 – there is also 1 000 000 mm^2 in 1 m^2 and 1 000 000 m^2 in 1 km^2).

Attempt to experience a hectare (a square 100 m by 100 m) and a square km, by walking around such spaces.

Internalisation

Relate the units mm^2, cm^2, m^2, h and km^2 to something in students' world. Find something on their body which is 1 mm^2 and 1 cm^2, find something in their everyday world that is 1 mm^2 and 1 cm^2 (e.g. end of a paperclip and end of a pencil). Draw around students' bodies – cut out a square metre from newspaper taped together and then cut this up to cover the drawing around the body – how much is covered by a square metre? Look for square metres in the classroom – part or all of a door, a window, a blackboard, the teacher's desktop, and so on. Get a map of the local area – draw around areas that are a hectare or a square km. How many house blocks fit into the hectare? How many road blocks fit into a square km? Google Maps is a good resource for exploring area in the physical landscape.



Make up plastic grids for 1 cm squares and a grid with 2 mm squares (4 mm^2) – overlay these on shapes and determine the area by counting. Make up m^2, h and km^2 grids for various maps and overlay these to work out areas for different maps (this activity will require a discussion of scale). Use the scale on the map to work out the grid size. Use cm graph paper (preferably in the form of plastic overlays) to measure area of hand, A4 paper, bag, etc. (anything that is *an everyday item for the student). Use centicubes or MAB units and pack into spaces (these are 1 cm²). Use geoboards (1 cm squares) and rubber bands to make and calculate areas. Use MAB flats – these are ¹⁄₁₀₀ of a m²).*

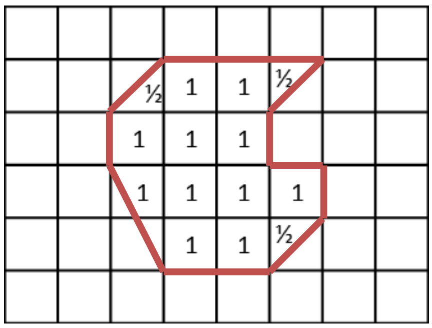
Realisation

Some realisation questions to explore with the students are as follows.

1. *Would you use square metres or square centimetres to measure:*
   - (a) *the cover of a book?*
   - (b) *a sports oval?*
   - (c) *a DVD cover?*
   - (d) *the classroom floor?*
   - (e) *a floor rug?*
2. *Name four things that are:*
   - (a) *smaller than 1 m^2*
   - (b) *about 1 m^2*
   - (c) *larger than 1 m^2.*

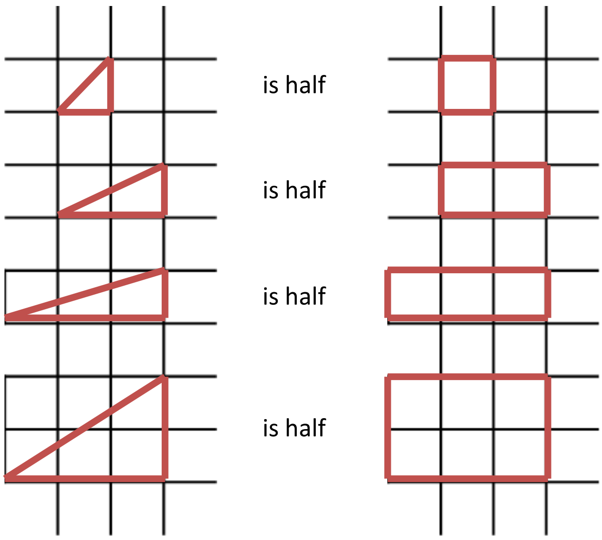
Learn the two techniques, "breaking into parts" and "enclosing with a rectangle", plus the technique of finding the area of a triangle by halving the appropriate rectangle. Use these techniques to calculate the areas of polygons (any type). Calculate the area inside curved shapes by counting the squares that are all or more than halfway inside the shape.

1. ***Breaking into parts***



The area of the shape is $12\frac{1}{2}$ cm²

This relies on techniques as below:



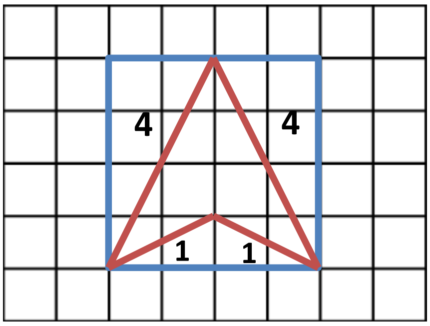
and therefore is $\frac{1}{2}$ cm²

and therefore is 1 cm²

and therefore is $1\frac{1}{2}$ cm²

and therefore is 3 cm²

2. ***Enclosing with a square or rectangle***



Area is 16 – 4 – 4 – 1 – 1 = 6 cm^2

Mathematics

Estimation

First estimate and then measure the area of objects, and record the information in a table like the one below. Estimate and measure each object before moving onto the next. ***Ensure the students are estimating the area – and not estimating the length and width.*** *Use 2 mm and 1 cm grid overlays. Use MAB units or centicubes. Use paper copies of a square metre with paper copies of 10 cm × 10 cm squares or MAB flats (1⁄100 of a square metre) and 10 cm × 100 cm rectangles (1⁄10 of a square metre).*

OBJECT	ESTIMATE	MEASURE	DIFFERENCE
Book			
Paperclip box			
Pencil case			
Student desk			
Door			
Whiteboard			

Repeat the above on maps for hectares and square km using grid overlays from map scales.

Repeat the above for very small things and 2 mm grid paper.

Practice

Continue to provide situations for students to experience measuring area with standard units – use ***grids and overlays*** *to measure area of shapes on paper/worksheets.* ***Always estimate first****. Reinforce accurate measurement processes while the practice is being undertaken. Have worksheets where students have to identify inaccurate measurement processes.*

Connections

Connect standard measurement of area to division as for non-standard units – connect to the ***inverse relation*** *big idea.*

Students can make 1 m^2 squares of paper and use them to measure the area of a basketball court. (If you have a small class, perhaps select a handball court or something smaller – students might get bored with making a large number of 1 m^2 pieces of paper.) Let students devise their own method for working out the area coverage.

Most students will begin to lay down squares in a grid of rows and columns, and count them all individually. Others may lay them around the perimeter and fill in the middle, then count them individually. Large spaces are particularly good to find area. Students need to be made aware that the area of rectangles and squares is related to the units along the rows and columns of the shape/area.

Students will see that the area is in fact an array, and work out how many in the first row, and then how many columns, and then either multiply them or use repeated addition. Some students may not need to use the "squares" and simply want to measure the sides and multiply the measurements. This is the beginning stage of developing the formula for the area of a rectangle.

Reflection

Application

Measure area with standard units in real-world situations. Set up area problems based on standard units.

Flexibility

Find standard units of area in the local community (e.g. measuring the area of a house – how many square metres?) Try to get students to think of ways standard area units are used in the world, local and otherwise.

Reversing

The components of a standard measure are object, non-standard unit and number. These form a classical triad and result in three problem types: (a) ***number unknown*** *– for example, how many m^2 cover the surface?; (b)* ***object unknown*** *– for example, find a surface which is 12 m^2 in area; and (c)* ***unit unknown*** *– for example, the surface has an area of 70, what is the unit? Make sure all three directions are taught. (This leads to understanding the* ***triadic relationship*** *big idea.)*

Generalising

As in section 4.2.3 for non-standard units, again emphasise continuous vs discrete and the three measurement principles as follows.

1. ***Continuous vs discrete.*** *Discuss how the area standard units have broken up a continuous surface into small parts that allow the area to be counted.*

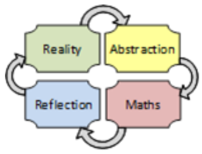
2. ***Common units.*** *Show that you must use the same unit for the square units and the length units that give the shape. For example, you cannot work out the area of a 40 cm by 2 m rectangle without changing all length units to the same unit – i.e. 40 × 200 or 0.4 × 2. (This leads to the first and second aspects of the common units big idea – that* ***units must be the same size when measuring/comparing area.****)*

   Then set a common class unit to lead to ***when units are the same, the larger number specifies the larger object****, and discuss what happens if different units are used to lead to the fourth aspect of the common units big idea – that* ***there is a need for a standard****.*

3. ***Inverse relation.*** *Measure things with large and small units to lead to the inverse relation big idea –* ***the larger the unit, the smaller the number and vice versa****.*

4. ***Accuracy vs exactness.*** *Further discuss the use of large and small units, the reasons why we want to measure area, and when we can use estimates, to introduce the consequences of the accuracy vs exactness big idea –* ***smaller units give greater accuracy****, students require* ***skills in being able to choose appropriate units****, and students require* ***skills in estimating****.*

RAMR lesson for conversion of metric units for area

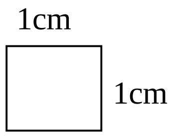


Reality

Where possible, find real-life contexts to embed the activities in; for example, using relevant objects or situations.

Abstraction

Revise length conversions: *1 cm = 10 mm*
1 m = 100 cm
1 km = 1000 m



Make a 1 cm square. How long is each side? Use 1 mm grid paper.

How many 1 mm^2 fit inside 1 cm^2?

Students can count, multiply rows or devise their own method to determine 1 cm^2 = 100 mm^2. For some students you will need to emphasise the 10 rows of 10 × 1 mm squares.

Repeat this process for 1 m^2 = 10 000 cm^2.

Mathematics

Use patterns to develop ways to calculate:

1 m^2 = 10 000 cm^2 (students must know this conversion)
2 m^2 = __?___ (20 000 cm^2)
3 m^2 = __?___ (30 000 cm^2)

11 m^2 = __?___ (110 000 cm^2) ← Some students will see the pattern. How do you do it?

Try saying the conversion aloud with the students – "For every one square metre there are ten thousand square centimetres".

2 m^2 = 2 × 10 000 = 20 000 cm^2
8 m^2 = 8 × 10 000 = 80 000 cm^2
2.3 m^2 = 2.3 × 10 000 = 23 000 cm^2

Repeat for cm^2 to mm^2.

The emphasis is on the students knowing and understanding the relationship between the units.

Place-value connections

Set up the place-value cards as follows:

H	T	O	H	T	O	H	T	O	H	T	O	H	T	O
	Trillions			Billions			Millions			Thousands			Ones	
		km^2		h				m^2				cm^2		mm^2

Use multiplicative relationships between PV positions to look at the conversion rate between the area units, mm^2, cm^2, m^2, h, and km^2: 100 mm^2 = 1 cm^2; 10 000 cm^2 = 1 m^2; 10 000 m^2 = 1 h; 100 h = 1 km^2. Be flexible with the decimal point to show, e.g. 2.45 m^2 = 24 500 cm^2 (place decimal point after m^2, place numerals in PV positions for 2.45, move decimal point to after cm^2).

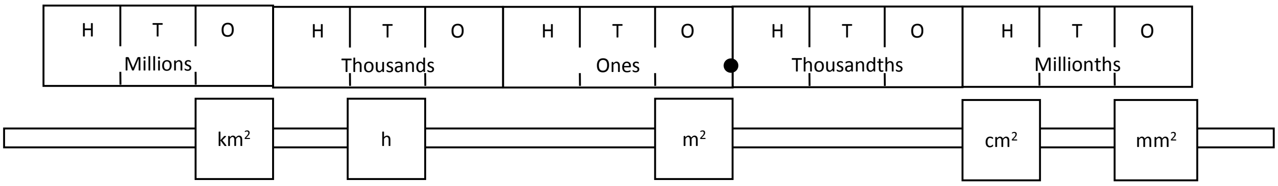
Practice/Formality

Use metric expanders and metric slide rules (see Appendix A for examples) to relate/connect metrics to PV and to each other (conversions). The metric expander folds like ordinary number expanders with the pleat fold at the units.

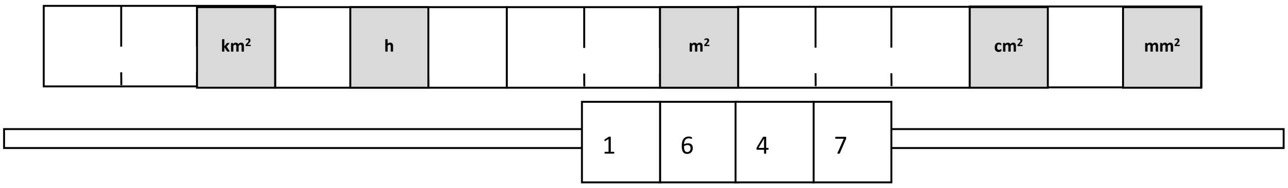
	km²			h					m²					cm²			mm²

Metric slide rules can be two types:

1. *PV chart going from millions to millionths and the metric units on the slide as follows. The slide can move across and back placing any metric area unit at the ones (to see where the other units are). It is also possible to have the PV on the slide.*



2. *PV chart having metric area units where the number PVs normally go, and numbers on slides to move across and back.*



Give students experience with specialist measuring techniques such as plastic overlays of cm grid paper. Undertake outdoor activities using maps and scales. Consolidate the metric conversions through drill – dominoes, bingo, mix and match cards, card decks for snap and concentration.

Connections

Metrics should be introduced along with decimals. They apply decimal understanding and reinforce decimal concepts. Area can be seen to have very large conversions, therefore we have to use a large number of PV positions to get from mm² to km².

Reflection

Look at metric conversions in the world. Think of where they are needed. Look at applications, flexibility, reversing, generalising and changing parameters. Show how non-standard principles such as inverse relation extend to metrics.

4.2.5 Applications and formulae for area

This stage focuses on applying attributes through using triads and teaching formulae, if they exist.

Applications

The approach to applications of area is built around the big idea of triad – there being object, number and unit in any measure means that there are three problem types for each to be unknown:

- **Number unknown** – *measure the area of the wall in m².*
- **Object unknown** – *find an object with an area of 16 m².*
- **Unit unknown** – *this object has an area of 35 square units, are these units cm² or mm²?*

Formulae

The formula for area of a rectangle can be discovered. Three other formulae can be introduced by relating them to this area. It is crucial to **maintain the same symbols**; for example, not to have area of rectangle = $L \times W$ and area of triangle = $\frac{B \times H}{2}$. Choose one set of symbols (we have chosen L and W), then:

- Area of rectangle = $L \times W$; and
- Area of triangle = $\frac{L \times W}{2}$.

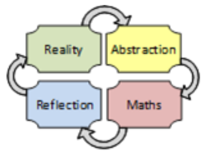
This strengthens students' abilities to see connections between formulae and to remember formulae (connections are the best way to remember).

Area formula development needs students to have a sound understanding of the array model of multiplication. Once students see that multiplication can be used to find the number of squares when they are arranged in rows, and that this is more efficient than repeated addition, they can apply this understanding to the formula for the area of a rectangle as multiplying the length by the width. Thus the progression is as shown in the diagram below.

3 fives (3 × 5) 15 counters	3 fives (3 × 5) 15 squares	3 cm × 5 cm = 15 cm^2	3 cm × 5 cm = 15 cm^2

Progression of development of the formula for area of a rectangle

Then, the formulae for area of a parallelogram, area of a triangle and area of a circle can be found by relating these areas to the area of a rectangle, $A = L \times W$.



RAMR lesson for applications and formulae of area

Reality

Discuss different size "areas" in local neighbourhood.

Abstraction

Students may have missed the link from grids to formulae. Have students calculate the area of a classroom, basketball court or similar, using square metres. Lay out the first "row". How many squares fit along this row?

Formulae building

Verbalise the row and column array – "there are 14 rows of 16 square metres in each row. The area is 14 rows of 16 square metres".

Area = 14 rows of 16 m^2
Area = measure of width × measure of length
Area = Width × Length
Area = W × L

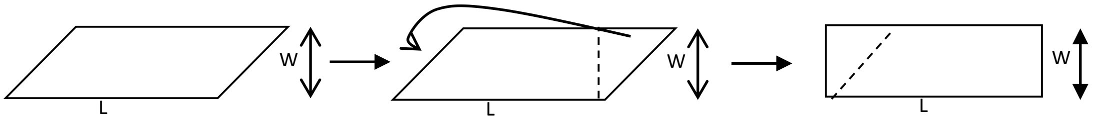
Area of a rectangle by discovery

*Draw rectangular shapes on graph paper and record lengths, widths, and area as number of squares on a table as follows. Discussion will lead students to discover that the **area of a rectangle is $L \times W$**.*

Shape	*L*	*W*	Area

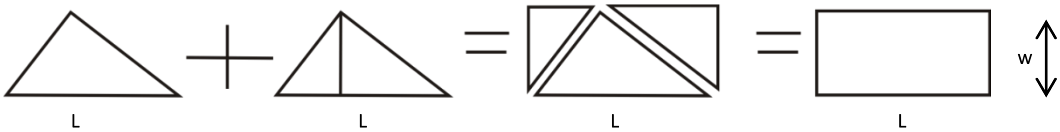
Area of a parallelogram by connection

*Cutting and re-forming shows that a parallelogram is a rectangle with the same length (L) and perpendicular width (W), thus the **area of a parallelogram is also** $\boldsymbol{L \times W}$. Also, you might need to have students draw parallelograms and cut off the triangular ends to turn and attach to the other end of the parallelogram to create a rectangle.*



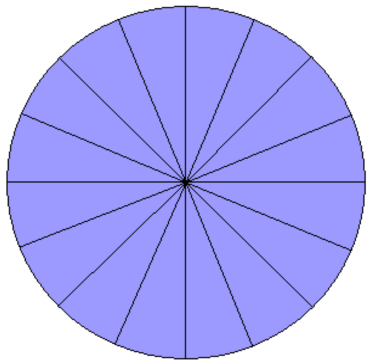
Area of a triangle by connection

*Cutting and re-forming shows that two triangles are a rectangle with the same base length (L) and perpendicular width (W). Thus a triangle is half a rectangle and the **area of a triangle is*** $\boldsymbol{\frac{1}{2}(L \times W)}$.

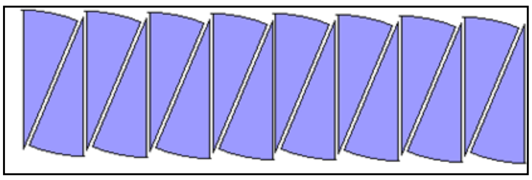


Area of a circle by connection

*Cutting and re-forming shows that a circle can be considered as a rectangle of length $\pi \times r$ and width r and thus the **area of a circle is** $\boldsymbol{\pi \times r \times r}$ **or** $\boldsymbol{\pi r^2}$. To do this, we have to remember that the circumference is $2 \times \pi \times r$. The cutting and re-forming is shown in the following activity.*



1. *Find a piece of paper, a large lid (at least 12 cm in diameter), a ruler, and scissors.*
2. *Measure the diameter of the lid, and calculate the radius (we shall call this r).*
3. *Place the lid firmly on the piece of the paper, and draw a circle around it. Calculate the circumference ($C = 2 \times \pi \times r$) and the area ($A = \pi \times r^2$).*
4. *Draw lines across the circle to form 16 sectors that are approximately equal in area, as shown in the figure at right. (Hint: keep halving.)*
5. *Cut out the 16 sectors, as shown at right. Arrange the sectors in the configuration shown below. Note that this configuration looks very much like a rectangle. Cutting one of the sectors in half and putting one half at the start and end of the "rectangle" makes it even more like a rectangle.*



6. *What is the height? What is the length? Can you see how these relate to radius and circumference?*
7. *Measure the height and length of the rectangle. Calculate the area of the rectangle. Divide the length of the rectangle by the radius. Did you get an answer that is close to π? Remember that π is approximately equal to 3.14.*
8. *Did you recognise that the height of the rectangle is the same as the radius of the lid? What does this mean for length?*

Mathematics

Language, formulae and practice

Through the Abstraction phase of finding and calculating area, students will be moving to the formal mathematical recording and calculating. Continue practising the calculations of area, using the correct mathematical terms and representations. Remember to reverse – shape to area and area to shape.

Connections

These have already been made in Abstraction.

Reflection

Validate formulae by applying and problem solving in local neighbourhood. Explore the relationships below to build flexibility, reversing and generalisation.

Relationship perimeter ←→ area

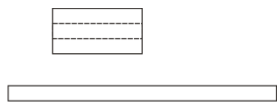
The idea here is to investigate and discover relationships between length, perimeter and area. For length and area, the relationship is based on squaring, that is, if the lengths of sides of a rectangle or the radius of a circle increase by 2 or 3 times, the area increases by 4 or 9 times (the square of the increase in length). This is important because it means a pipe of double width will carry four times the water. The relationship between perimeter and area is more varied – the perimeter can vary for the same area, however there are some relationships: (a) the largest area for a given perimeter is the circle, and (b) if restricting shapes to rectangles, the largest area for a given perimeter is a square. Overall, the circle for any shape and square for rectangles give the biggest area for the least perimeter.

Length ←→ area

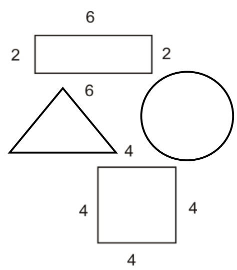
Make rectangles and circles where lengths and radii are doubled – find areas – see that area is four times as large. For example, rectangle 2 cm × 3 cm = 6 cm^2 → rectangle 4 cm × 6 cm = 24 cm^2.

Perimeter ←→ area

Find ways to show that perimeter can be very long for a small area, and investigate what shapes give the largest area for a given perimeter (see below for some activities).



1. *Cut up "fat" rectangles of paper to cover long thin rectangles as on right – we can see that the long thin rectangle has the same area but a greater perimeter, and we can see that, if we keep slicing, we can end up with any length of side we want for the same area.*



2. *Start with cm graph paper and make a circle of string which is, say, 16 cm long; make a variety of shapes with this string (circles, ellipses, rectangles, triangles, etc.) on the cm grid paper and count squares for area. Make a table with headings shape and area, and see which shape has the largest area – it will be the circle. This activity can also be done using geoboards and the Maths Mat.*
3. *Start with cm graph paper and draw a variety of rectangles with the same perimeter to show that the square has greatest area; for example, for a perimeter of 16, a square (4 × 4) is 16 cm^2 and a rectangle (2 × 6) is 12 cm^2.*
4. *Another way is to start with an area, and use a fixed number of blocks to make a variety of shapes of the same area and compare their perimeters.*
5. *Finally, this leads to an investigation of why square houses are the cheapest. Get students to make a square, rectangular (ranch style) and L-shaped house of same area on graph paper – ask students to put in walls to make three bedrooms, kitchen, lounge, bathroom, etc. Then measure the lengths of the walls. The square will have the shortest length of walls and so is the cheapest to make – concrete base and roof are the same for all houses in terms of area.*

4.3 Volume

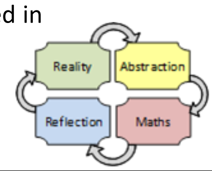
Volume is the amount of space inside a shape, measured in cubic units.

4.3.1 Identifying the attribute of volume

This stage focuses on identifying the attribute with the use of appropriate vocabulary.

Background

Capacity and volume are three-dimensional concepts related to the geometric notion of a solid. Both are measures of the amount of space enclosed by a solid shape. Both volume and capacity can be measured by filling the interior of solid shapes with liquid (e.g. water), liquid substitutes (e.g. rice, sand) or solid materials (e.g. blocks) and measuring the amount of these materials, or by displacing material (e.g. water) and measuring how much is displaced. *Capacity* usually refers to liquid volume (e.g. petrol in a tank) and is measured in liquid units (e.g. mL and L) while *volume* refers to solid volume (e.g. the volume of a box) and is measured in cubic units (e.g. mm^3, cm^3, m^3, and km^3) because it relates to length but in three directions.



RAMR lesson for identifying the attribute of volume

Reality

Where possible, find real-life contexts to embed the activities in; for example, using relevant objects or situations.

Abstraction

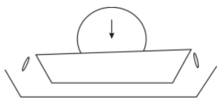
Body

Start with re-experiencing length and area – how tall people are, the size of the area of their hands or body. Now look at how big/small people are. Act out being big (standing tall and spreading arms and legs) and small (bend over, crouch and pull in arms and legs so become a "ball"). Say big objects have large volume and small objects have small volume. Discuss the difference. In this way, introduce volume as a special word for size.

Hand

Allow students to play with different-sized objects or to make different-sized constructions. Do the following: building sandcastles, building with blocks and other material, making things with plasticine, and so on. Continue with a variety of size activities, e.g. packing things away, filling a box or carton with material, enclosing space, building a house or a fort, stacking materials, blowing up a balloon, making cakes (or any type of cooking), acting out or miming big and small with students' bodies, enclosing space with students' bodies.

Experience volume as the amount of space an object takes up → immerse objects in water and watch the level rise for larger objects. Experience volume situations (as the amount held in a container) with virtual materials and pictures. For virtual materials, use "click and drag" to change volume.



Mind

Encourage students to imagine containers and objects. Imagine the containers being filled and emptied, and the objects displacing space. Imagine containers/objects being made bigger and smaller (to change volume) – draw big and small things (e.g. mouse and elephant).

Mathematics

Practice

Use materials, computers and pictures in worksheets to experience volume. Differentiate it in drawings from length and area. Experience different-sized objects – small and large cars, etc. – be varied. Experience drawings of things that have volume and things that do not (or at least cannot be filled or cannot displace volume).

Connections

Look at long, narrow and thin objects and short, wide and thick objects. Does volume relate to length and area of surface? Discuss this!

Reflection

Application

Relate understanding of volume back to everyday life – what things in the world have volume, what do not (e.g. boxes, cars, houses, boats, petrol tankers, and so on) – what has large/small volume?

Flexibility

Think of as many things as you can that have volume. Visit a supermarket and find different containers which have the same volume.

Reversing

Up to now the students have gone from container → experiencing volume. Reverse this – go from volume as an experience → container. Thus, do activities where students construct containers for particular purposes or build things to certain sizes.

Generalising

Discuss what makes an object have large or small volume. Discuss how we can measure this. Think back over connections – is there an idea here? Try to elicit how volume could be measured. (This is the beginning of the ***attribute leads to instrumentation*** *big idea.)*

Changing parameters

What if material made into a 3D shape could contract and expand? What does volume mean here?

4.3.2 Comparing and ordering volume

Once students understand the attribute of volume they can explore ways to compare and order the volume of different objects.

Direct comparison of volume

The direct comparison of volume is difficult due to the nature of the attribute. Containers can be placed one inside the other but as with area where two dimensions had to be considered and could vary, volume has three dimensions – length, width and depth. So a container could be longer and wider than another but not as deep. Another container could be shorter but wider and deeper. The tendency would be to fill each container with items and compare the count. This is further down the learning sequence due to the use of units.

Indirect comparison of volume

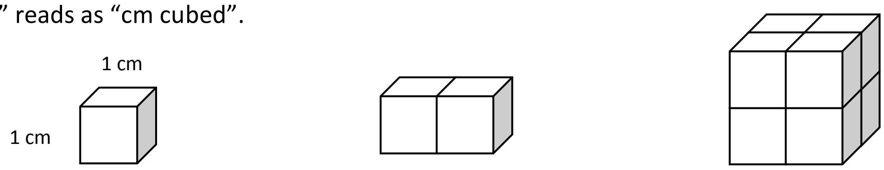
The use of an intermediary object for volume can allow two containers to be compared when they are not able to be directly compared. Again, the complication of the comparison needing to consider all three dimensions can make this a complex task. In many ways the comparison of volume is easier if units are involved.

4.3.3 Non-standard units for volume

Any objects that can have volume are able to be used to measure volume. By using objects that are different, students can be helped to see the need to use a standard unit or at least the same non-standard units if the volume of different containers needs to be compared. Using uniform non-standard units, e.g. marbles, allows students to fill containers with things that they can count and then the count can be used to compare volumes. Marbles will leave gaps between the units and students can be assisted to see that objects that are cube-shaped or rectangular-prism-shaped fit together better. By using large objects to fill containers, students can also be helped to understand that it is possible to consider part units when measuring volume.

4.3.4 Standard units for volume

The standard units of volume are cubes where the length of the side is one metric unit that relates to the size of the area being measured. The same complication that was described in section 4.2.4 for area applies to volume. A cube that is 1 cm × 1 cm × 1 cm is 1 cubic centimetre. Two cubic centimetres will be the volume of two of these units. However "two centimetres cubed" is referring to the volume of a cube that has a length, width and depth of 2 cm. This cube would have a volume of 8 cm^3 (see figure below.) These two terms (cubic units and units cubed) are often used interchangeably and this can be linked to the way the unit is written (cm^3) which if read "left to right" reads as "cm cubed".



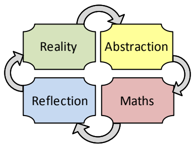
1 cubic centimetre (1 cm^3) 2 cubic centimetres (2 cm^3) 2 centimetres cubed (8 cm^3)

Cubic centimetres vs centimetres cubed

The standard units of volume and their relationships are:

1 cubic centimetre (1 cm^3) = 1 cm × 1 cm × 1 cm = 10 mm × 10 mm × 10 mm = 1 000 cubic millimetres (mm^3)
1 cubic metre (1 m^3) = 1 m × 1 m × 1 m = 100 cm × 100 cm × 100 cm = 1 000 000 cm^3

RAMR lesson for standard volume units



Reality

Where possible, find real-life contexts to embed the activities in; for example, using relevant objects or situations.

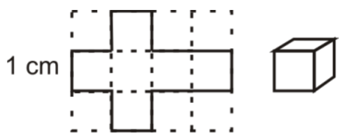
Abstraction

Common unit

After the need for a standard has been developed through the use of non-standard units, time can be spent measuring volume with a class-chosen unit – e.g. a common object like a box or block. This can be used by students to show that with a common unit, a higher number really means a greater volume.

Discuss what is the most efficient volume unit – the cube can be argued to be the best because it packs well, is based on square area units, and leads to multiplication.

Identification



Construct or experience mm^3, cm^3, m^3, and km^3. For 1 mm^3, try to find something that is 1 mm long, 1 mm wide and 1 mm high. For 1 cm^3, use 1 cm grid paper, draw and cut out a net for a cube of side 1 cm, fold and tape to make the cube, and compare with a centicube and a MAB unit. For 1 m^3, make a cubic metre out of 12 pieces of 1 m long dowelling (or use a commercial construction kit). For km^3, look on the Internet for something that is as large as this – the capacity of a large dam may be large enough.

Internalisation

Find things in everyday life that are 1 mm^3, 1 cm^3, 1 m^3, and 1 km^3 – so that students can refer back to them. Some ideas are: (a) obtain a collection of small rectangular prisms (e.g. matchbox, tetra pack) and pack these with MAB units to find their volume; (b) look at the classroom and work out how many cubic metres of air are in it; and (c) find out how many cubic km of water are in Wivenhoe dam (convert from megalitres to cubic km).

Mathematics

Estimation/Practice

The first point to note here is that it is too difficult to pack every volume and count the units – so we need formulae. The answer lies with packing small boxes with small cubes, or building larger rectangular prisms with small cubes, and seeing that the number of cubes is the same as:

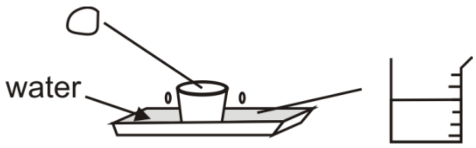
the number of cubes for length × the number of cubes for width × the number of cubes for height

This means that volume is length × width × height ($L \times W \times H$) and if measured in cm, then volume is cm^3 (remember that there are 1 000 000 cm^3 in 1 m^3, if we wish to change to m^3).

First estimate and then measure the volume of objects as below (find some of your own). Measure L, W and H with tape and use calculator to find volume and check your estimates. Estimate and measure each object before moving on to the next. ***Estimate the volume – do not estimate length, width, height.***

OBJECT	ESTIMATE	MEASURE	DIFFERENCE
Chalk box			
Shoe box			
Cupboard			
Under the table			
...... and so on			

Use an overflow tray and a measuring cylinder to find the volume of objects by immersion and overflow. Estimate first.



OBJECT	ESTIMATE	MEASURE	DIFFERENCE
Lump of plasticine			
Rock			
Your fist			
...... and so on			

Connections

Build the connection between metric units for volume and PV – see section 4.4.

Reflection

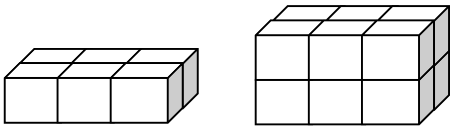
Reflection ideas

Look at metrics for volume in the world. Think of where these are needed. Look at applications, flexibility, reversing, generalising and changing parameters. Show how measurement principles such as inverse relation extend from non-standard to standard units.

4.3.5 Applications and formulae for volume

The formulae for volume build on the formulae for area where rows of squares made the link to the multiplicative nature of these measurements. With volume, the two-dimensional measure of area is extended by including the third dimension of depth. The development of volume formulae is best started by focusing on prisms. Prisms are shapes where the size and shape of the base is consistent throughout the entire shape.

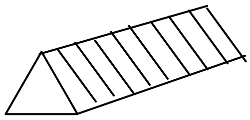
Formula for volume of a rectangular prism



The easiest shape to work with is the rectangular prism. Once the base layer of cubes in a rectangular prism can be calculated using an area formula, this can be multiplied by the number of layers that are present. In the figure on right the base layer of this rectangular prism is shown to be two rows of three cubes. When another layer is added, it can be seen that the volume of this rectangular prism is two layers of six cubes or $2 \times 3 \times 2$.

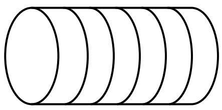
The formula for volume of a rectangular prism is therefore $L \times W \times H$. Once this formula is understood the formulae for other prisms can be investigated as long as the area of the base can be calculated.

Formula for volume of prisms



The formula for the volume of any prism is the area of its base × height. For example, to find the volume of a triangular-based prism the formula would be $\frac{1}{2}L \times W \times H$. The base of the prism is a triangle so the area of the base is $\frac{1}{2}L \times W$ which is effectively the same as cutting slices that are the same shape and size as the base and one unit wide along the whole length of the shape.

Formula for volume of a cylinder



Geometrically, a cylinder is not a prism as it does not have flat faces other than its bases. However, in relation to volume, the formula for volume of a prism applies because the shape of the base (a circle) is consistent throughout the whole length and can be used to multiply by the height to find the volume. The formula for volume of a cylinder is therefore $V = \pi r^2 H$, that is, the area of the circle base (πr^2) multiplied by the height of the cylinder.

4.4 Relationship between attributes – volume, capacity and mass

There is a particular relationship between the attributes of volume, capacity and mass. This relationship is unique to metric measures and does not apply in a similar way to imperial measures. A cube that is 1 cm × 1 cm × 1 cm has a volume of 1 cm^3. If this cube was filled with water (at 4° C) it would weigh 1 gram (g) and the capacity would be 1 millilitre (mL). So the relationship is 1 cm^3 = 1 mL = 1 g.

This relationship is seen in car engine sizes where the volume of the cylinders in the engine is sometimes described as cc (cubic centimetres) or as litres (L). A vehicle could have a 4000 cc motor which could also be describe as a 4 L engine.

The relationships between units are:

1 cubic cm (cm^3) = 1 cm × 1 cm × 1 cm = 1 millilitre (mL) = 1 gram (g)
1 cubic decimetre (dm^3) = 10 cm × 10 cm × 10 cm = 1 litre (L) = 1000 mL = 1 kilogram (kg) = 1000 g
1 cubic metre (m^3) = 100 cm × 100 cm × 100 cm = 1 kilolitre (kL) = 1000 L = 1 tonne (t) = 1000 kg.

5 Time and Angle

The measurement attributes of time and angle are both measured in base 60.

5.1 Time

The first important point to note about time is that there are three perspectives: (a) **point of time** (e.g. reading a clock); (b) **sequence of time** (e.g. knowing the days of a week); and (c) **duration of time** (e.g. how long is a minute). As a measurement, the most appropriate perspective is duration of time. This section will look at the three perspectives and give the five stages for duration of time.

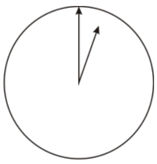
The second important point in relation to time is that it is not a base 10 attribute – it has a base of 60 – being a construction of the Persian or Babylonian Empire which considered 60 an important number.

5.1.1 Point of time

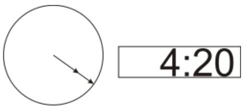
Point of time is crucial for employment as it enables the worker to tell time, keep time logs and calculate the amount of time on a job. Three aspects will be looked at: (a) the sequence for acquiring time-telling skill, (b) kinaesthetic activities for time-telling, and (c) a RAMR lesson plan for time-telling.

Sequence for acquiring time-telling skill

Begin with full turns, part turns, quarter and a half turns. Study the clock face and the two hands, working with non-geared and geared clocks, looking at clockwise and anti-clockwise turns. Relate full turn of large hand to movement of small hand. Relate full turn to an hour and o'clock; relate part turns to half past, quarter past and quarter to. Use **geared clock** activities to discover relation between small hand and large hand movements.



Relate part turns to fractions. Use angle wheels, rotagrams, and Rotascan clock (see below). Look at:



- counting by fives, telling time in five-minute intervals (5 past, 10 past, etc.);
- introducing sixtieths in relation to clock face;
- introducing the notion of a minute;
- reading time on digital clocks;
- reading 24-hour clocks; and
- relating, for example, 5:42 to 18 minutes to 6.

Use worksheets with clock faces, relating drawings of time on face with language and digital time. Relate different ways to use language to tell time. Introduce the second, using stopwatches and apply timing to sport and navigation.

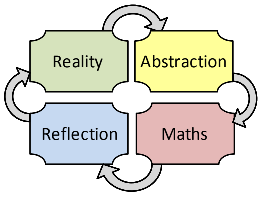
Look at point of time and sequence of time. Discuss when things happen in each day. Ask students, aides, parents, elders for local ways of telling the time. Introduce the calendar – days, weeks, months, years. Look at the Torres Strait Islander calendar – how does it differ? Look at movement of stars and moon.

Kinaesthetic activities for time-telling

Turn the classroom into a clock – number 12 at front and 1 to 11 around the walls. Turn bodies, counting o'clocks as students point at numbers; turn bodies, counting minutes by five. Act out times with students' hands as hours and minutes; go both directions (reverse) – teacher calls a time → students act as the hands of a clock, and teacher acts as hands of a clock → students call a time (both directions also for o'clock and minutes by fives).

Relate time to movement of the sun. Act out pointing at the sun as it moves across the sky while calling out hours of the day and times when things are done (e.g. start of school, lunch, and so on). Connect point of time to these well-known times of the day (also TV programs).

RAMR lesson for time-telling



Materials. *Two ropes and a post. A timer. A5 numbers 1–12 (or 24). Two paper plates, two hairpins, scissors and glue. An analogue clock.*

Reality

Students discuss and list events that take small and large amounts of time, e.g. washing up, getting to school, sleeping, time between breaths, eating morning tea, running 100 m, becoming a teenager.

Students describe the different units we use to record time, especially second, minute, hour, day, week, fortnight, month, year.

- *Discuss the mechanics of an analogue clock.*
- *Discuss the mechanics of a digital clock. Discuss as many different locations where digital clocks are found in the home, and in the school.*
- *Discuss where students might have seen a timetable (bus stops, train station, TV guide, classroom wall for break times, etc.).*

Abstraction

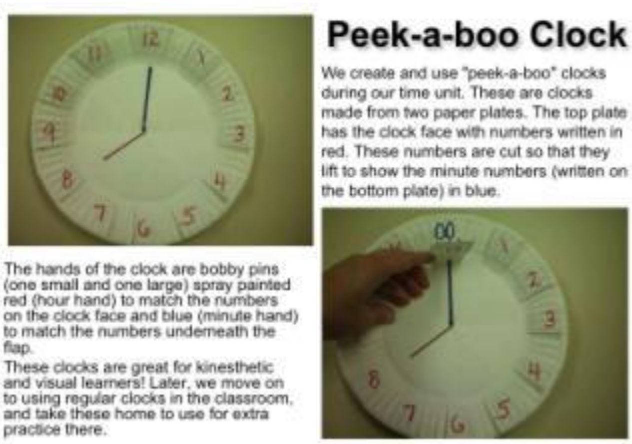
Students go outside *and form a human clock by making a circle and using a post and two ropes/ribbons.*

Students discuss and help their peers with the minute and hour hands to move to different times of the day, keeping track of both hour and minute hand movement and that it is co-ordinated.

Teacher calls out new times e.g. "Go ahead in time 1 hour and 20 minutes"; "Go backwards in time 75 minutes".

Back in the classroom *students create a giant analogue clock around the walls of the classroom. Print off large A4 numbers 1–12 (and 13–24 if your students are ready for 24-hour time). Stick the numbers around the walls starting with 12 in the centre of wall above the whiteboard (or centre of wall where most students see it from their desk). Place the 6 opposite the 12 on the back wall. Have students place the remaining numbers. By standing a student in the centre of the room, you can have the student model any particular time, or change in time. This can be made into a game.*

A second abstraction classroom activity is where you have the students make a peek-a-boo clock using two paper plates and hairpins (see right).



Mathematics

Practice/Notation

Students come back to the classroom (if outside) or return to their desks (if inside) and record some of the times they saw demonstrated as well as a changed time and explain how far each hand moved to arrive at the new time. They record how many hours and minutes and the correct pronunciation of each time.

It is essential students have an understanding of counting in fives as they explore changes in time around the analogue clock face. This is difficult to do with digital clocks.

Students work in pairs with their partner and make two different times on their peek-a-boo clocks and then each calculates the difference between the clocks. Students draw and record several examples.

Students can make up an analogue clock with their "own numbers" for a little maths problem e.g. for the 1 o'clock, they might like to write "½ of 2", at 2 o'clock they might put 5 – 3, and so on. When completed, students can set a time on their clock, and swap clocks with another student. The class then has to solve each little maths problem and tell the time.

Connection *activities would focus on relation between fractions and movement of the hands of a clock.*

Reflection

Application/Validation

Students create a story using six clock times and illustrate their story with representations of the clock times. The six clock times should describe events in the story.

An extension of this might be to use students' "own number" clock at the front of the room. Set a time on it, and have each student create a time story using the little maths problem and the time as shown on the clock.

Extension

Activities would be to extend the 12-hour clock to 24-hour time. The peek-a-boo clock would help here. Also acting out two turns of the classroom clock – 1 o'clock, 2 o'clock, ... , 12 o'clock; 13 o'clock, ... , 24 o'clock.

5.1.2 Sequence of time

This is a crucial area – knowing day, date, month and year. It requires familiarity with hours across a day, am and pm, days in a week, days (and weeks) in a month, months in a year, decades and centuries, AD and BC, and so on. The history of many of the names is interesting; for example, the names of the days relate to the names of the Norse gods (Saturn, Sun, Moon, Tuor, Wodin or Odin, Thor and Fria). The sources of the names of the months include the god Janus, the Caesars Julius and Augustus, and the Latin names for 7, 8, 9 and 10.

A kinaesthetic activity is to relate seasons to actions that are typical of those seasons – e.g. summer and swimming for the south-east.

There are also many interesting ideas using calendars – e.g. the difference between the sum of opposite corners of days forming a square or rectangle on a calendar within one month is always zero (Why?)

Tuesday	Wednesday
1	2
8	9
15	16

(16 + 1) – (15 + 2) = 17 – 17 = 0

Look at point of time and sequence of time – integrate the two. Discuss when things happen in each day. Ask students, aides, parents, local leaders and elders for local ways of telling the time. Introduce the calendar – days, weeks, months, years. Look at local calendars – Aboriginal and Torres Strait Islander calendars – how do they differ? Look at the movement of the stars and the moon (full, new, etc.). Connect this work to astronomy.

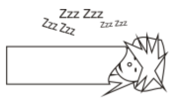
5.1.3 Duration of time: Identifying the attribute of time

The perspective of duration of time will be outlined in full using the five measurement stages.

Background

Time is often called the fourth dimension. It is the duration we spend undertaking an activity. We are fixed within the flow of time. We are unable to move through time of our own volition or at our own speed and direction. Students are continually experiencing time. We do not have to set up special "changes" for them to experience. They only need to be directed to reflect on their own experience. But, of course, we cannot set up any contrasting experience (such as no change in time or twice as fast a change in time) with which to compare. Time is also a very subjective experience. Duration is the component that relates to the learning sequence because it is the measurement of an amount of time.

Activities



Arrange pictures of events in the order in which they happened and hang the pictures with pegs on a clothes line in this order (with the size of the pictures related to the length of the event and the closeness of the pictures related to how closely they occur next to each other in time).

Associate events with the time of the day, e.g. when do we come to school? Look at how cycles repeat themselves (e.g. out of bed, get dressed, eat, clean teeth, etc.).

Experience activities that take a short or a long time. Wait while others do things (hop 10 times, run around oval, etc.). Note when things start and finish. Experience a variety of activities which all take the same time. Relate the passing of time to another activity (e.g. shading in a column or row on a piece of paper).

5.1.4 Duration of time: Comparing and ordering time

Once students understand the attribute of time they can explore ways to compare and order the time of different activities and events.

Direct and indirect comparison of duration of time

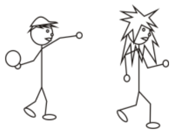
The direct comparison of time is quite simple. It aligns with the concept explored in relation to length where a direct comparison can only be done effectively with a common baseline. When directly comparing the duration of activities it is important to start them at the same time. Once two or more activities have been started at the same time the duration can be compared and described. The activity that continues when the others finish is the activity that took the longest time. The activity that finished first is the one that took the smallest or shortest time. The others are in between.

The indirect comparison of time, without using units, requires an understanding of the transitivity principle. This principle was described in Chapter 3 in relation to length. The transitivity principle holds that:

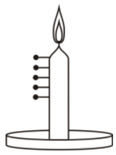
- if event A takes a longer amount of time than event B, and
- if event B takes a longer amount of time than event C, then
- event A will take a longer amount of time than event C.

One event is used as an intermediary (in this case event B). The intermediary event will need to be something familiar to the student. For example you could compare the amount of time taken to read a paragraph out loud with how long it takes to brush your teeth (a familiar activity that you have a good feeling for how long it takes), then compare the reading of another paragraph also with the event of brushing your teeth. Then the time taken for reading the two paragraphs can be compared after consideration of the intermediary event (cleaning your teeth).

Activities



Directly compare which activity takes longer by "having a race" (e.g. time for a ball to stop bouncing compared with walking across a room; running around building compared with writing name and address, and so on). Ensure students start both activities at the same time.



Indirectly compare activities by **timing both events with the same technique** (e.g. the amount of sand to run through an egg timer, the length of a taper candle that is burnt down, the distance a ball rolls down an incline, the depth a holed container sinks to, and so on).

Relate common events (e.g. eating breakfast is longer than a short cartoon on TV but dressing takes less time). Label strips of paper with events; the length of each strip of paper is determined by the length of time the event took.

Warm up: Guess how long? The teacher sets a timer for 20 seconds and the girl students sit with eyes closed and raise their hands when they think 20 seconds is up and the boys judge. Swap roles and the boys try to guess. Teacher secretly sets the timer for a particular time, e.g. 28 seconds, and students listen until buzzer rings, try to guess and write down how long it was. Check to see who was the closest.

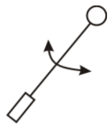
5.1.5 Duration of time: Non-standard units for time

Background

Non-standard units of time need to be activities that have duration. Generally units will be of a short duration but longer activities can be used to time longer events. The focus with these units is that the measure of time will be the count of the number of times the unit activity is repeated to time the event in focus. For example, the time it takes one student to run around the school oval could be measured in star jumps done by another student. Keeping these non-standard units uniform is likely to be difficult so these sorts of activities are good to introduce the idea of a count of units as a measure but also to introduce the need for standard units.

The duration of activities measured in non-standard units is a good way for young students to gain an experience of the length of time in each standard unit. For example, counting the number of times a student can write their first name in one minute helps the students develop an understanding of the length of a minute. Students can create non-standard clocks that can be used to measure time by marking equal spaces on a candle or by pouring sand from one container into another that has marked graduations. Students can also observe the passage of time related to the movement of the sun by observing shadows or making a sundial.

Activities



Use a regular action to time events (e.g. counting, pulse, a pendulum's swing, hopping, the roll of a ball bearing in a curved groove). Look at history for interesting methods.

Calibrate the regular change in something to time events (e.g. a candle burning down is marked with pins at regular distances – thin taper candles are best); water or sand pouring out a small hole into another container marked into height intervals; a container marked at intervals down its side sinking in water; a ball rolling slowly down an incline (with obstructions) marked into distance intervals; and so on.

Relate time to the movement of the sun. Construct a sundial. Discuss historical time measurers. Discuss terms such as midday, morning, afternoon, evening (use worksheets and relate times to drawings). Develop the measurement principles.

5.1.6 Duration of time: Standard units for time

Background

The standard units of time are not metric. Time is base 60. The relationships between units of time need careful consideration and exploration to be understood. While the smallest unit of time is the second, when times less than one second needs to be considered it is fractions of a second that are the focus. The fractions used are decimal fractions. So time is measured in seconds, tenths of seconds, hundredths of seconds and so on. The metric prefixes can be used to refer to fractions of a second just as they are used with other metric measurement units, e.g. $^1/_{1000}$ of a gram is a milligram. However, there is a tendency for any fraction of a second to be referred to as a millisecond which is incorrect.

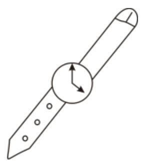
The standard units of time and their relationships are:

1 minute (min) = 60 seconds (secs)
1 hour (hr) = 60 mins = 3 600 secs
1 day = 24 hours = 1 440 mins = 86 400 secs
1 week = 7 days = 168 hours = 10 080 mins = 604 800 secs
1 fortnight = 14 days = 336 hours = 20 160 mins = 1 209 600 secs
1 month = 28/29/30/31 days (depending on which month)
1 year = 365.242 days
1 decade = 10 years
1 century = 100 years

Activities

Introduce minutes, hours, seconds, clock faces, digital time, and 24-hour clock using the four steps of common unit, identification, internalisation and estimation. Determine what students can do in a minute. Work out the length of a favourite TV program.

Have students experience a minute and other time intervals by doing something for this time. Relate these times to everyday events like brushing teeth or eating, or school lessons, etc. Turn back and call out when 20 seconds is up. Do this while doing something else (e.g. hopping, holding breath, etc.). Choose actions that may change perception of time.



Use stopwatches and clocks to time events. Calibrate a sun dial in hours (e.g. put a stick in the ground and mark where shadow points when each hour is up).

Relate daily events to the times they commonly occur (use worksheets and join pictures to times). Work out a roster (daily or for a special activity). Relate digital time to clock faces. Introduce 24-hour time.

Measuring devices for time using standard units

Clocks and stopwatches are devices used to measure the passage of time and the duration of events. As time is constantly moving on and it can't be stopped, the measurement of the amount of time an activity or event takes requires noting of the start time and the finish time and then a calculation if standard clocks are used as the measuring devices. Students can be cued to start their event when the second hand of the clock is on the 12 (indicating the start of a minute) and then noting where on the clock this hand is when the activity is finished. Then a calculation will be needed to determine the duration. If the activity takes longer than one minute the number of minutes will need to be remembered or recorded as well as where the second hand was when the activity ended. This provides an added complication to the use of standard clocks for measuring duration.

The other option for measuring duration is to use a stopwatch. Stopwatches can be either digital or analogue. Stopwatches allow duration to be measured by being started at the same time as the event. These devices keep time in seconds, minutes and fractions of seconds and when stopped record the length of time of the event. Analogue stopwatches generally record seconds and minutes only. Digital stopwatches are quite complex as they record not only minutes and seconds but also the tenths and hundredths of seconds. For young students all the numbers on a digital stopwatch can be confusing. The figure below shows an analogue and a digital stopwatch.

An analogue and a digital stopwatch for measuring duration of time

5.1.7 Duration of time: Applications and formulae

Because time does not stand still it is necessary to complete calculations to measure the duration of an activity or event. The start time and finish time are noted and the duration is calculated as the difference between these two times. As described above the use of stopwatches to measure duration requires the reading and interpreting of a scale (analogue) or digital total rather than a calculation.

There are no formulaic applications of time except through the conversion of units. Time can be measured using clocks or timing devices and once known the duration of an activity or event can be described using any of the related units of time. The length of time taken to run a marathon can be described in hours; hours and minutes; hours, minutes and seconds; hours, minutes, seconds and fractions of a second, depending on the accuracy required. All these conversions rely on calculation.

Providing students with opportunities to solve real-life problems that require the measurement of time helps them to apply their understandings of this attribute in ways that relate to life beyond school. Many of these are related to rate (e.g. water wastage per day, speed – km/hr, and so on).

The following outlines four different time applications students may have experienced – especially if they have travelled.

24-hour time

Discuss where students might have seen 24-hour time used. Responses might include on digital clocks, on some TV guides, train or airline ticketing information. Some students may have heard it on TV shows from America, or in use by the military.

Discuss why 12-hour time might have been invented. What are the advantages? What are the disadvantages?

Present a clock face that shows both 12 and 24-hour time. Have the students create a table of corresponding times (e.g. 1 pm = 1300 hrs, 2 pm = 1400 hrs and so on). Discuss strategies (other than memorising) the relationships between 12 and 24-hour time. Get students to create a story using 24-hour times and illustrate their story with representations of the clock times. The clock times should describe events in the story.

Consider Indigenous time and tidal change. Have students research the significance of the tides to coastal Indigenous peoples in the past, and in the present.

Timetables (not to be confused with times tables)

Discuss the purpose of a timetable in everyday life. Consider this from multiple points of view – the students', the adults in their community, and perhaps people living in a very busy city.

Have a number of timetables available for the students to practise reading. Make up some stories telling the students, for example, if they need to be at a sport game by 2 pm, what bus or ferry should they catch from the bus stop or jetty near their house? Try to use authentic timetables that are specific to the local environment. When students are comfortable with these, use a timetable for a local but larger town or city (like Townsville or Brisbane). As an extension activity, consider a tube timetable for London, having the students pretend they are travelling to visit one of London's well-known attractions like Buckingham Palace or the London Eye. Let the students select the attraction and locate the timetable on the Internet.

Time zones

Print out a map of Australia and its time zones. Choose a map that clearly illustrates the time zones, as well as the different states. Photocopy the map – one for each pair of students. Discuss the concept of time zones with students. Explain that time zones exist because the sun rises and sets at different times in different parts of Australia and the world. Discuss the implications of this in terms of TV coverage of big sporting events like the Olympics (events might take place in the daytime in the country where they are being held, but viewed "live" during the night-time in Australia).

Shine a torch and hold it over a globe. Rotate the globe and explain that the torch represents the sun. As you rotate the globe, students will see how the sun illuminates different parts of the earth at different times. It is important to rotate the globe correctly, from left to right. (e.g. Australia has time generally 10 hours ahead of London, so we need to receive the torchlight before London). Inform students that you will be discussing different time zones in Australia. Have them look at the map and explain the regions of the different time zones as follows:

Australia has three time zones: Eastern Standard Time (EST) for the eastern states, Central Standard Time (CST) for the Northern Territory and South Australia, and Western Standard Time (WST) for Western Australia. CST is half an hour behind EST and WST is two hours behind EST.

Daylight Saving Time: Most Australian states wind their clocks forward an hour during the Daylight Saving period. New South Wales, Australian Capital Territory, Victoria, Tasmania and South Australia do this from the beginning of October to the beginning of April. Western Australia, the Northern Territory and Queensland do not observe the practice of Daylight Saving.

Explain that there is a time difference between the time zones. For example, when it is noon in the Eastern time zone, it is 11.30 am in the Central time zone, and 10.00 am in the Western time zone.

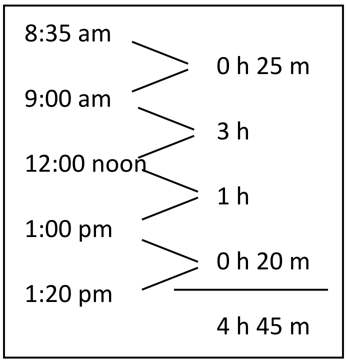
Ask students to determine the times in different time zones. State different times in different time zones and ask them what time it would be in another time zone. For instance, you could ask, "If it's 3 o'clock in the Western time zone, what time is it in the Eastern time zone?" or "If it's midnight in Brisbane, what time is it in Perth?"

Time logs

Get students to fill in time logs – time in and time out. Ask them to calculate the time spent on each job. Try to get examples of actual logs so students can see relevance in filling them in. If students have trouble working out how long spent on a job, try the following:

1. **Number line**. Develop a number line for 7 am to 4 pm – mark in hours, ¼ hours. Get students to mark in starting and ending times on number line. Use the line to work out how long was taken. Look at the hours and the minutes between start and finish. To assist getting over the 12 noon restart of the numbers – do the difference in two steps – up to the 12 noon and after the 12 noon, or translate the number line to 24-hour time.

2. **Additive subtraction.** Do subtractions using the "shopkeeper's algorithm", that is, starting from smaller and building to larger. For the example of starting at 8:35 am and finishing at 1:20 pm, make a series of jumps from 8:35 to 1:20 – first go to the next hour, then to 12, then to the finish hour and finally to the finish minutes. Then add up all the jumps – this is the subtraction (the difference).



5.2 Angle

Angle is the amount of turn between two directions (arms) represented as two arrows from a common starting point. The amount of angle is the amount of turn, not the length of the arrows. Angle is also the corner of 2D or 3D shapes hence the link between this topic and geometry.

The measurement attribute of angle is measured in terms of full turn and this is 360 degrees. It is 360 degrees because the Babylonians visualised a full turn as six equilateral triangles meeting at a point and gave each corner of the equilateral triangles an angle of 60 degrees (thus angle, like time, has a base 60). Small angles are also measured in minutes (1/60th of a degree) and seconds (1/60th of a minute).

5.2.1 Identifying the attribute of angle

Background

The attribute of angle relates very closely to the mathematics topic of geometry. When two lines radiate from a common point they form an angle. Angles are the corners of 2D and 3D shapes and the study of trigonometry combines measurement of angles and sides in right-angle triangles. For young students the attribute of angle is best experienced as a measure of the amount of turn. They can turn their bodies, observe the movement of the hands of clocks and investigate the properties of shapes by comparing and measuring the corners. The measurement of the amount of turn or size of angles follows the same teaching sequence as other measurement attributes.

Angles in real life will be either static or dynamic. A static angle does not change and a dynamic one does. The corner of a room or corner of a window is an example of a static angle. The hands of a clock are an example of a dynamic angle which changes during the day. Discuss with students other examples of static and dynamic angles. If you have swinging doors in your classroom, explore different-sized dynamic angles created by opening the doors wider or narrower. Have students create static angles and dynamic angles using their bodies.

A range of words are used to describe angles in particular size ranges. An **acute** angle is an angle between 0° and 90°. An **obtuse** angle is an angle between 90° and 180°. A **reflex** angle is an angle between 180° and 360°. A **revolution** angle is a full 360° circle.

Activities

Discuss things that turn, experience turning with the students' bodies, follow the turns of another (e.g. "follow the leader", "Simon says", etc.). Relate the turns to the directions to which the body points at the start and end of the turns. Notice that a turn begins and ends with a direction, so define angle as the amount of turn from one direction to another. Use the turn activities in the YDM Geometry book. Use **rotagram** and **angle wheel** activities. Turn the pages of a book, turn geostrips in relation to each other. Give directions to a blindfolded partner (e.g. "turn right, turn left, turn further, ...", etc.).

Try to use activities that students find real. For example, skateboards doing 360° jumps. Try to connect these to the formal work; for example, once the skateboard is experienced in reality, a small toy skateboard could be used to translate the movements onto paper and then replaced with a drawing of an angle – joining reality to mathematics.

5.2.2 Comparing and ordering angles

Once students understand the attribute of angle they can explore ways to compare and order the size of different angles.

Direct and indirect comparison of angles

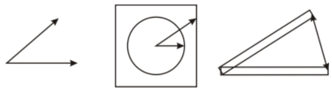
To directly compare angles, representations of different angles can be place directly on top of each other and the size of the angles can be compared. Students need to focus on the angle and not be distracted by the length of the lines or sides of the angle. Many students mistakenly believe that changing the length of the arms of an angle will change the size of the angle. These students are focusing on the area or space between the arms rather than the amount of turn between the two arms. The two angles on right are the same size but the right-hand angle has longer arms.

To indirectly compare angles requires the use of an intermediary angle. Often when angles are being compared students use the size of known angles, e.g. right angle, as the intermediary. Each angle to be compared is compared to a right angle and this knowledge can be used to compare the initial angles.

Activities

Continue turning with body from one direction to another. Experience small and large turns. Compare angles directly by taking a copy of one angle as a sector of a circle and placing this copy over the other angle. The copy can be made on tracing paper and cut out.

Compare angles indirectly by using an angle wheel or rotagram (see on right) or by using **two hinged strips** (see on right) – these can be joined by string a set distance from the hinge to hold the angle. In this angle copier, the length of the string determines the size of the angle. A compass and a ruler can also copy an angle by measuring the distance between the two rays (or directions) with the ruler along an arc a fixed radius from the vertex (arc is drawn by compass held at a fixed radius).



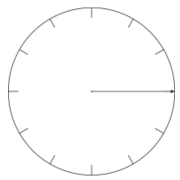
Construct a **right-angle measurer** by folding paper and comparing with angles to classify them as acute or obtuse (see on right for fold). Open out right-angle measurer to get a straight-line angle. Use this to show when an angle is reflex. Show that four of the angles is a full turn – relate to compass directions N, E, S and W.

5.2.3 Non-standard units for angle

Non-standard units for angle can be made from sectors and by making own non-standard "protractor". The use of language to describe whole and part turns, particularly in the giving and following of directions, can be considered as a non-standard unit for angle. These units are uniform non-standard because they are likely to be the same each time as they relate to a fraction of a full turn or full circle. Students can be instructed to make a quarter or half or full turn to the left or right and the result will be a change in their direction. (Do not use degrees.) Use angle activities to develop the **big ideas** of measurement as they pertain to angle.

Activities

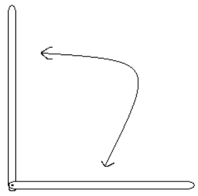
Make non-standard units by cutting a circle into sectors (or pie pieces). Measure angles by filling in space between two directions with small **sectors**. Measure extent of turn by how many small sectors will fit in the angle. Construct a non-standard protractor by cutting out a plastic circle with marks on edge (see right). This can be done on computer by using Excel and constructing a pie chart of, say, 12, 40, 60 or 100 equal numbers, and printing this on plastic



transparency. A calibrated homemade protractor can also be constructed from a rotagram or an angle wheel. The angle is read from the intervals marked along the arc (it is good to have a starting point).

Approximate angle measures can be found by counting how many of an object can be fitted into the angle at a fixed distance from the vertex (e.g. fingers, pencils). Once again, these can be used to develop the measurement principles.

5.2.4 Standard units for angle

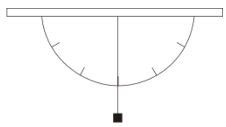


The standard units for angle are base 60 and relate to the 360° that are in a circle or one full turn. When a finer measurement than one degree is needed fractions of a degree are referred to but they are not decimal fractions. The fractions of a degree are 60ths of an angle and are called minutes and seconds (which are other base 60 measures). With angle, particular sized angles are commonly used as referents and so are like standard units for this attribute, e.g. a right angle is an angle measuring 90° or ¼ of a circle (or full turn).

The standard units for angle and their relationships are:

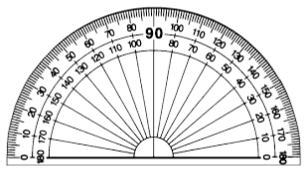
1 degree (°) = 1/360th of a full turn or of a circle
A right angle (∟) = 90°
A straight angle (or line) = 180°
A full turn = 360°

Activities



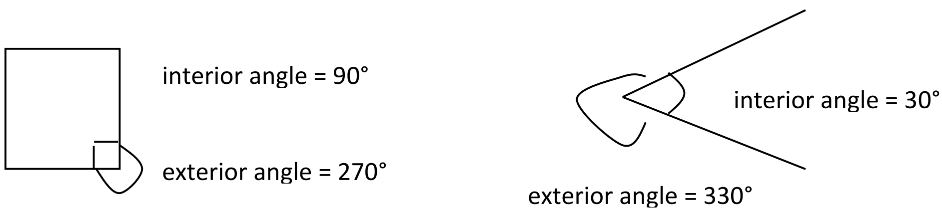
Construct a sector equal to 1 or 2 degrees. Experience the protractor. Use the four steps – common unit, identification, internalisation and estimation. Measure things on your body (e.g. angle between thumb and forefinger).

Measuring devices for measuring angle in standard units

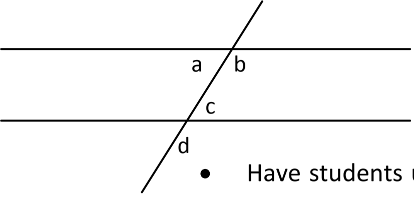


The measuring device most commonly used to measure angle is the protractor. Protractors are generally a full or half circle and have individual degrees marked usually in multiples of ten. They measure in degrees. Measuring angles with a protractor is quite complex. Generally the full circle protractors are easier for students to use and understand. It also helps if the protractors are see-through so they can be aligned with the arms of the angle being measured.

Every pair of lines that share a common point can represent two different angles that will add to 360°. When these angles form the corner of a 2D shape these angles are referred to as interior and exterior angles. The figure below shows the interior and exterior angles of one corner of a square as well as the same idea for an angle that is not the corner of a 2D shape. This causes difficulty when using a protractor for many students as they need to identify which of the two angles they are measuring.



It is important for students to recognise the names of the parts of an angle – arms and vertex – and that the size of an angle is the amount of turn required by one arm in relation to the other arm.



- Have students use “angle strips” and protractors to measure angles around the room. Have students create angle strips from pieces of cardboard, and use these to replicate angles in classroom, then measure the angle using a protractor. The angle strip is held together with a split pin.
- Have students draw a variety of angles and estimate their size, then check using the protractor. Once students are familiar with the use of a protractor, have them explore a flower garden and find examples of where angles appear in nature in fixed patterns (e.g. a daisy with five petals has each petal at an angle of approximately 72°). Students can be sent out into the garden to find an example of a 35° angle in a bush (many leaves grow at 35° from the stem).
- Most of the angles found in nature are the result of rotation and symmetry. So it is not surprising that many diagrammatic representations of the Central and Western Desert people represent such symmetry in their descriptions of kinship. Have students explore such representations on the Internet, and perhaps create their own kinship representations.

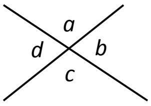
5.2.5 Applications and formulae for angle

There are many applications for angle in the sense of using formulae to calculate other measures once the size of particular angles is known. This aspect of mathematics is usually described and taught in relation to geometry. These aspects of angle geometry are based on there being 360° in a full circle or full turn and that parallel lines and lines that cross parallel lines create angles with particular equivalences.

Angle properties

Intersecting lines

Vertically opposite angles are angles formed by any two intersecting lines. The two pairs of opposite angles will be equal in measure.



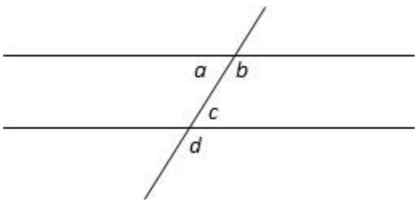
Angles ***a*** and ***c*** are vertically opposite and equal in measure
Angles ***b*** and ***d*** are vertically opposite and equal in measure

Parallel lines

Alternate angles are formed when a straight line traverses (crosses) a pair of parallel lines. Alternate angles are on opposite sides of the transversal and will be equal in measure. Alternate angles form a Z shape.

Co-interior angles are formed when a straight line traverses a pair of parallel lines. Co-interior angles are on the same side of the transversal and add to 180° (are supplementary). Co-interior angles form a C shape.

Corresponding angles are formed when a straight line traverses a pair of parallel lines. Corresponding angles are on the same side of the traversal. Corresponding angles form an F shape.



Angles ***a*** and ***c*** are alternate angles and equal in measure
Angles ***b*** and ***c*** are co-interior angles and add to 180°
Angles ***b*** and ***d*** are corresponding angles and equal in measure

Shape-angle properties

Angle sums of polygons are as follows: (a) the interior angle sum of an n-sided polygon is $(n-2) \times 180°$; and (b) the exterior angle sum of an n-sided polygon is always 360°.

Other shape-angle properties are as follows:

- The angle within a circle with the diameter as its base is always 90°.

- The diagonals of a square and rhombus meet at 90°; the diagonals of a rectangle and parallelogram do not.
- Shapes that tessellate have angles that are factors of 360° or add to 360° or 180°.

Teaching activities

Tear off the corners of polygons to determine the interior angle sum rules for these polygons (as on right). Triangulate polygons into triangles for the same purpose. “Walk” around regular polygons to find relation of number of sides to the exterior angle (the amount that has to be turned).

Use the activities in the YDM Geometry book. Relate angles in regular polygons; relate interior and exterior angles in triangles (and determine the number of triangles in any polygon); relate angles in circles (particularly those from diameters); relate angles in parallel line situations; relate angles between diagonals to the various quadrilaterals; relate angles to tessellation; relate angles in line and rotational symmetry, and so on.

Use angles on a sphere to introduce nautical mile as the distance along the equator of one minute of angle at the centre of the earth.

5.3 Time and angle rich task

In this task, students create a machine that measures a single minute. There is clearly no correct design for the machine. The creativity is limitless. What is interesting is how students tackle the problem, how they organise their thinking, and what levels of complexity they can imagine and construct.

The results can be revealing in a number of ways. Students will need to troubleshoot their machine designs as they strive to reach the desired one-minute run time for the marble. Students may make designs that are more sophisticated than you expected.

The mathematical ideas are:

- understanding measurement
- understanding time periods
- using angles to increase or decrease speed.

Presenting the problem

You may need to discuss what a ball run is. Consider showing the students video examples from the Internet.

Help students recognise that they will have to work in a team of three for best results (many hands to hold and tape joints etc.)

Assessment criteria

The major emphasis of the task is recognising the single minute as a unit of time. You may find one minute is too long a period of time for your students, so suggest the combination of machines (joining of two groups – will the time be a simple addition of the individual times?). To a lesser degree, you are interested in their representation and understanding of angles and gradients in the real world.

“I’ll do it in a minute!” machine

Ever been asked to do something and you say “I’ll do it in a minute”?

Here is your chance to build yourself a machine that will let you know when your minute is up so you have to go and do something.

Build a minute machine made from junk paper and cardboard and masking tape.

The contraption needs to send a rolling marble through tubes and funnels, across tracks and bumpers, to be caught EXACTLY 1 minute later in a disposable cup.

In the design of your minute machine, make sure you have at least one reflex angle and one acute angle between the tracks.

6 Temperature and Money

The attributes of temperature and money are often included in curriculum documents in relation to measurement. Both these attributes are metric in a sense as they work with values between 0 and 100 for their most common units. Temperature (in Celsius) ranges from 0–100 and money works on multiples of 10 with 100 cents in a dollar.

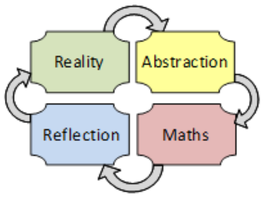
6.1 Temperature

Temperature is a measure of how hot or cold objects are – a very subjective experience without measuring instruments. The most important reference points are the freezing point and boiling point of water.

6.1.1 Identifying the attribute of temperature

This stage builds meaning for the attribute temperature.

RAMR lesson for identifying the attribute of temperature



Materials. *Pen, paper, heat sources (e.g. sun, shade, ice, and so on), a variety of clothing for hot and cold situations, and other materials as mentioned in activities.*

Abstraction

Body

Experience different temperatures by touching warm and cold objects. Use the environment to experience these temperature changes. Go outside and walk in a sunny area and a shady area. Discuss differences experienced and reasons why the temperature feels different. If able, organise for students to experience cold rooms, freezers, hot houses, and rooms that are very hot.

Use clothes to experience the difference between feeling warmer and cooler. Have students describe the temperature. Use this as an opportunity to build a rich vocabulary for describing temperature (hot, cold, freezing, warm, and so on). Organise students to walk (in bare feet) along surfaces with different temperatures (concrete, grass, sand, mud, water, and so on). Talk to students how each day feels and relate to temperature chart.

Hand

Experience many different temperatures. Examples of activities include: (a) moving different objects to experience varying degrees of warmth from sunlight and coolness from shade; (b) touching ice and warm water; and (c) feeling containers where the contents affect the temperature (water tanks, coffee cups, and so on). Develop the notion that sometimes senses can't be relied upon by heating up hand then placing quickly on a cool surface and vice versa (i.e. cool hand and place on a warm surface).

Mind

Shut eyes and think of different temperatures. Draw situations and use different colours to represent different temperatures. Act out how it feels on a hot day, cold day, and so on (e.g. acting out shivering and sweating).

Mathematics

Practice

Continue to experience, describe and visually represent different temperatures. Even worksheets with pictures can represent different temperatures. Use a daily weather chart to record students' observations about the weather. Discuss daily temperature and the variation that can occur within a day.

Connections

Construct a homemade temperature gauge made up of a long thin tube connected to a reservoir or just a long thin tube with water in it. As it heats up, the liquid should expand and rise up the tube. In this way, there is a connection between temperature and length.

Reflection

Application

Look at temperatures in local environment, and places where the temperature differs (e.g. fireplace, refrigerator).

Flexibility

Think of all the situations that are hot and cold in the world (ice and snow, active volcanoes, and so on). Look for places (e.g. deserts) that are both hot and cold.

Reversing

Make sure that teaching goes from: (a) teacher provides a situation which has a certain temperature (e.g. a refrigerator) → student says whether hot and cold; to (b) teacher chooses whether hot, cold, warm, and so on → students think up situation.

Generalising

Spend time on activities that reinforce three generalities: (a) feeling temperature is objective (i.e. relates to the person and their previous experience – a cold hand chilled from a refrigerator will find an object is hotter to touch than a hand that has been in hot water which will find the object cold); (b) big idea: attribute leads to instrumentation (e.g. look at what happens to things as they get hotter and colder and try to work out something that will provide a measure – in particular, the activity in "Connections" will lead to the thermometer); and (c) the connection between length and temperature.

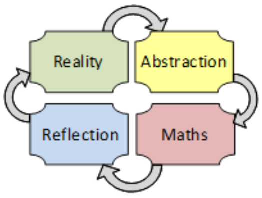
Changing parameters

Instead of experiencing temperature, look at how change of temperature affects us.

6.1.2 Comparing and ordering temperature

This stage compares and orders temperature without use of numbers.

RAMR lesson for comparing and ordering temperature



Materials. *Pen, paper, heat sources (e.g. sun, shade, ice, and so on), a variety of pictures showing various temperatures (hot, freezing, roasting, and so on), and other materials as mentioned in activities.*

Abstraction

Body

It is not possible to directly compare temperature – we have to use indirect methods like touching or feeling both objects to see which is hotter. These activities involve comparing temperatures directly with the hand, moving hand from one instance to another – the hand will feel the change. There is also relating temperatures by comparison with or to known things (e.g. that one is hotter than my hand feels yet this one is not, and so on).

Comparison has to move on to ordering by temperature. Activities include filling cups with water at different temperatures and letting students order the cups from warmest to coldest. Need also to introduce comparison temperature language (e.g. warmer, colder, hotter, warmest, hottest, coldest, and so on).

Hand

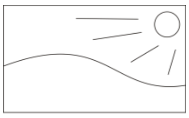
Students should experience a variety of temperature comparisons and orderings. The following are some examples.

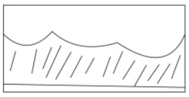
Use touch to compare and order a variety of things at different temperatures, using out in the sun, in a cold place, refrigerators, freezers, ovens, and so on, as sources of these objects and their different temperatures.

Use materials and their melting and boiling points to signify temperature changes. For example, provide students with a bowl of ice cubes to take outside on a sunny day. Encourage them to decide where they want to put the ice cubes. Have students predict what will happen to the ice cubes that are left in the different places.

Students should observe that ice melts at different rates in the shade and in the sun. They will also learn that solid ice will turn to liquid when heated. Discuss how the change in temperature from freezer to sunlight outside causes the ice to melt. Thus we have freezing temperatures and melting temperatures. Other materials can also be used to allow students to indirectly relate temperatures by what they do to other things (e.g. at this temperature, the paraffin is runny but at this colder temperature, it remains solid, and so on). Construct a simple temperature measurer with a thin tube and coloured water in a small container. Discover how the coloured water goes up the tube as the temperature rises.

Use experiences and drawings to consider comparison and order of temperatures. Discuss hot days and cold days, order pictures that show hot and cold. Discuss summer and winter, what is common to wear, what is commonly eaten and where is the common vacation area. Keep a weather chart, drawing and describing the weather on each day. Relate this work to other subjects (e.g. science). Look at how heat and cold affect humans, animals, food and other materials. Look at the environment and consider how changes of temperature affect things such as butter, care of pets, local vegetation, wild animals, fish, and so on. Discuss the effect of hot and cold. Use language to talk about feelings and observations concerning different types of weather. Discuss health effects (e.g. survival in hot and cold, influenza, and so on).





SPECIAL ACTIVITIES: "HOW DOES IT FEEL!"

Have students place their hands on their cheeks to feel how warm or cool their hands are, then have the students rub their hands together briskly for about 30 seconds, and put them against their cheeks again. Ask the students if their hands are warmer or cooler after they rubbed them together.

Arbitrarily label several spots in the room which have different temperatures for the students to touch; for example, window glass, metal shelf, spot in the sunlight, spot in the shade, and so on. Let the students try to determine which spot has more heat and which spot has less heat. Let the students make statements such as, "The metal shelf has more heat than the window glass." Let the students attempt to order the spots by the amount of heat they feel. Point out to the students that the things that have more heat than their hands feel warm or hot to them and vice versa.

Obtain two glasses of water from the tap, and let the students feel them to see that they are both cool. Set one in the shade and one in the sun. Thirty minutes later, let the students feel them again. Discuss why one glass now feels warm, whereas the other is still cool.

Let two student "judges" stand in front of the room. Have six or so students file past and lay their hands on the judges' cheeks. The judges try to decide who has the coldest/warmest hands.

Ensure students are introduced to comparing and ordering language. Use virtual situations and pictures to also compare and order by temperature.

Note: If in an Indigenous community, get an elder to describe the changes in temperature across a year and how this affects country.

Mind

Imagine hot and cold and all temperatures in between and either side.

Mathematics

Practice

Give students the opportunity to continue to compare temperatures through activities, virtual materials and pictures in worksheets. Ensure all language relating to comparing and ordering temperature is known. Keep a daily temperature chart with visual representations of temperature over extended time periods, say one term or month. Make comparisons of temperature based on daily, weekly, and seasonal differences.

Connections

Use the homemade temperature measurer to draw connections between temperature and length.

Reflection

Validation/Application

Discuss real-world situations for hotter/colder. Find things that are between the two ends of a comparison (e.g. hot water and an oven).

Flexibility

Brainstorm hot and cold situations in the world. Mix things up – where is it very cold and very hot at the same time? Where is it the same temperature all the time?

Reversing

Make sure teaching goes from: (a) teacher provides two temperature situations → students use feel and give temperature comparison word, to (b) teachers give temperature comparison word → students provide examples that meet that word. Also remember that comparison can be considered as a ***triad*** *with three parts – first situation, comparison word, and second situation – thus there are three "directions", or three problem types: (a) give first situation and word (e.g. hotter) and students find a cooler situation; (b) give second situation and a word (e.g. hotter) and students give a hotter situation; and (c) give two situations and ask for word(s) to relate them (e.g. "the freezer is cooler than the refrigerator").*

Generalising

Generalise that (a) need a better measuring system than feel to accurately order situations by temperature; and (b) there are three problem types in comparison as in reversing.

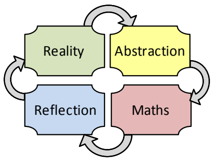
Changing parameters

Look beyond temperature to how different temperatures affect life (ours and the world around us).

6.1.3 Non-standard units for temperature

This stage introduces the notion of a unit and the measurement processes and principles. Because measuring instruments for temperature are highly technical (other than by feel), there is not much scope for non-standard units.

RAMR lesson for non-standard temperature units



Materials. *Pen, paper, heat sources (e.g. sun, shade, ice, and so on), large thermometers without scale, a variety of pictures showing various temperatures (hot, freezing, roasting, and so on), and other materials as mentioned in activities.*

Abstraction

Body

Use your feelings to make your body a rough non-standard instrument as follows: freezing (1), cold (2), lukewarm (3), body temperature (4), warm (5), hot (6), and hotter than you can touch (7). Use this to give a number to temperatures.

Hand

There are only a few examples here as follows:

1. *Make up a checklist for temperature based on effect on materials – freezes water, ice melts but paraffin does not, and so on. This will require scientific knowledge and special materials.*
2. *Construct a homemade thermometer with a thin tube and red coloured water and attach to cardboard with regular lines on it, numbered from 0 to whatever.*
3. *Obtain a large thermometer and stick this on cardboard with made up regular lines on it, numbered from 0 to whatever.*
4. *Make up a series of pictures or virtual slides showing different temperatures (e.g. ice blocks, sunny day, and so on) and give a number to each from cold to hot – use number to measure temperature roughly. These can be used to give a number to temperature.*

Mind

Imagine a thermometer with red line going up and down with temperature.

Mathematics

Formality/Practice

Give students the opportunity to continue to compare temperatures through one of the above methods. Continue with the daily temperature chart with visual representations of temperature over extended time periods, say one term or month, making comparisons of temperature based on daily, weekly, and seasonal differences.

Connections

The important one is relation between temperature, land, flora, fauna and our behaviour.

Reflection

Validation/Application

Look at measuring temperature in real-world situations. Set up temperature problems based on non-standard units in everyday life situations.

Flexibility

Find use of non-standard temperature units in local community activity. Discuss with elders.

Reversing

Consider the non-standard triad for temperature: situation, temperature as a number, and temperature unit. Give the three types of activities in lessons – where situation, number and unit are unknown.

Generalising

Here the objective is to extend the understanding to teach the following:

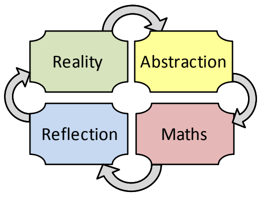
1. ***Continuous vs discrete** big idea – how units make continuous temperature discrete steps.*
2. ***Common units** big idea – how steps in scales must be common/same for measuring and comparison of temperatures, how we must set a common unit after a while in sequence of activities, and the bigger number is hotter when units are common/same. All this should lead to the need for a standard.*
3. ***Inverse relation** big idea – comparing non-standard units (particularly the distance between lines on thermometer) will lead to the larger the unit, the fewer the number of units needed to measure.*
4. ***Accuracy vs exactness** big idea – discuss units to see that smaller gaps between lines is more accurate, that accuracy is not always needed and so skills in choosing appropriate units and estimating are useful. Because there is only one unit (°C), there is not much need for this.*

Changing parameters

Once again, the main extension here is to relate temperature and environment.

6.1.4 Standard units for temperature

This stage focuses on introducing the degrees centigrade or Celsius (°C). Temperature is a measure of how hot or cold objects are – a very subjective experience without measuring instruments. The most important reference points are the freezing point and boiling point of water. There is only one unit here as there is only one temperature unit (°C).



RAMR lesson for standard temperature units

Materials. *Thermometer, pen, paper, ice, water for boiling.*

Reality

Find real-life contexts to embed the activities in; for example, using relevant objects or situations.

Abstraction

Common unit

Although difficult for temperature, after the need for a standard has been developed through the use of non-standard units in Stage 3, time can be spent measuring temperature against a class-chosen unit – e.g. temperature of body.

Identification

Use a thermometer to measure temperature of ice and boiling water and some common things in between.

Internalisation

Use a thermometer to measure body temperature, fridge temperature, tap water temperature, and so on. Look up recipes and find common cooking temperatures.

Mathematics

Estimation

Estimate first and then measure the temperature of a variety of objects (find some interesting things) inside, in the sun, slightly heated, etc. Complete estimates and measures of object before moving onto the next.

OBJECT	ESTIMATE	MEASURE	DIFFERENCE

Research the effect of altitude on boiling and freezing points of water. Estimate first. Research other interesting temperatures – estimate first – e.g. magma from a volcano, temperature at which iron turns to liquid, and so on.

6.1.5 Applications for temperature

There are no formulae for temperature. And, since there is only one unit, applications can only be built around two of the three triad types. The two types left are:

- **Number unknown** – *what is the temperature of this material?*
- **Object unknown** – *find a material with temperature 62° Celsius*.

6.2 Money

Money as a measurement is the measure of the value or cost of something. There are two components – money handling skill, and money as measuring value.

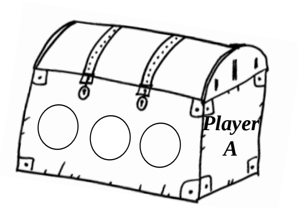
6.2.1 Money handling skill

This involves being able to recognise and use money for everyday activities such as shopping, buying, selling, getting change, calculating amounts, and so on. Some activities are as follows.

1. Recognising coins through coin rubbings, drawing coins, coin posters, etc. Understanding the value of coins and relating coins to each other. Recognising what can be bought for certain coins, add and subtract coins, and make change.
2. Setting up a "shop" – have things to buy and sell, use play money, etc. Look at computation and word problems in money situations.
3. Organising a party or a trip somewhere – working out costs of things (travel, accommodation, drinks, food, entertainment and so on) and making a plan and keeping to a budget.
4. Setting up a family budget – identifying all costs for a family and then relating these to income and working out how much in each area (rent, petrol, food, entertainment, and so on) can be spent.

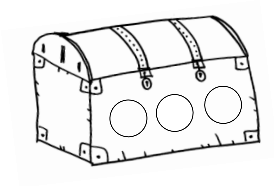
Game: Change the treasure

Instructions. Use play money if real money is not advisable.



- Student throws two dice, and adds the numbers.
- Round to the nearest 5c (does it come closer to 5c or 10c?)
- Pick up that coin and add it to your treasure stash.
- If you can combine the value to make a different coin, then exchange it. For example, 5c + 5c → 10c or 10c + 10 c → 20c.
- The winner is the person who first makes a total of 60c (3 × 20c coins or 50c + 10c).

Extension. Vary the dice and coins. For example, use 10c and 20c coins to make 50c (using dice with numbers 1–10).



- Have students read an amount of money on a price tag, and then calculate the exact amount using coins and or notes. This can be played as a shopkeeper's game.
- Discuss the notion of rounding (Australia does not use 1c or 2c coins, there is a discussion about whether to discontinue the 5c piece – what impact would that have on small items like lollies still sold "2 for 5c" in some corner stores?).
- Practise rounding prices up and down for the sale of single items. Practise rounding items' prices up or down for multiple items purchased at once.
- Take the students to the local shop and have the shopkeeper show students how a cash register (till) works. Explore what maths the register person needs to help a shopper make a purchase. If possible, visit a shop with a very old cash register that required the worker to calculate the correct change for the customer.
- Have students check their change after making a purchase from a pretend shop.
- Have students explore money transactions in different currencies. Pretend the students are buying a "McDonald's Happy Meal" in three different countries (Australia, Japan and America). What currency would they use? What money (notes and coins) would they need to pay an exact amount? What change would they need if they had to overpay? An interesting discussion point is what do they actually purchase as a Happy Meal in Asian countries? (Cabbage is often used instead of a pickle on a cheese burger, the sauce is often fish sauce, and sometimes seaweed is used.)

Extra activities. Introduce the coins and notes used in Australia, using where possible identifying, internalising and estimating activities. Play "shop", determine everyday items that can be bought for the coin/note under discussion or multiples of that coin/note. If possible, involve students in actual buying and selling, actually having to use or plan to use money (e.g. organising lunches and lunch money, buying, cooking and making and selling food on days the tuckshop is closed; organising an outing and determining the money needed, etc.)

6.2.2 Money as value: Identifying the attribute of money (value)

Background

As a measure, money is not as absolute as length but varies according to demand. It may seem strange to place money within the measurement context but measurement teaching approaches can be useful in instructing students about money. The measure of value is subjective in that an object could hold great value to one person but not to another. While this value might not be monetary, discussions about what students value may give them an idea of the concept of trying to measure this value.

The cost or value of an object or experience is not tangible and needs to be represented either by using symbols, (e.g. $) or by using coins and notes that can represent the equivalence in value or cost. Therefore before students can work on the measurement aspects of this attribute, they need experiences with number and quantity and they need to have experienced the recording of quantities using numbers and other symbols. Students will not be able to compare or order the cost or value of items without the associated number concepts to compare coins and notes or symbolic representations of amounts of money, including the use of a decimal point to separate the whole dollars from the part dollars (cents).

Activities

Discuss what things are the most valuable for students (e.g. "what one thing would you take to a desert island"). Discuss why they value it, and how they would show it had value. Choose among alternatives. Discuss things that have little value. Make a collage of pictures of expensive and cheap things.

6.2.3 Money as value: Comparing and ordering money

Once students understand the attribute of money, value and cost they can explore ways to compare and order the cost and value of different objects. The difficulty with comparing money is that the measure is not visible or tangible. The comparison must be done to representations of the value or cost.

Activities

Subjectively compare objects to see which one is valued more highly. Indirectly compare by relating to a common object (e.g. "I like my toy more than this pen but the trip to get hamburgers is better than the toy"). Discuss how many of one object is a fair trade for another. Organise the class into groups all with a different product and discuss how the groups would barter one thing for another (introduces supply and demand). Look at the history of the students to whom you are teaching – was bartering common?

Students can be encouraged to barter and swap items or activities for other things. To make the swap or barter they will need to compare the value of their items. This comparison could be done directly or indirectly. A novel way to discuss the value of items, or a barter system, would be to explore the swapping of food that may occur at morning tea or lunch time, e.g. a meat and salad sandwich (pretty tasty and filling) swapped with a small cup cake or biscuit (junk food).

Direct and indirect comparison of money/value

Students can **directly compare** their valuations of particular objects to see which object is valued more highly or which student values the object more highly. These comparisons are not absolute and are subjective but serve to focus on the attribute of value and the differing amounts of the attribute that different objects possess.

Students can **indirectly compare** the value of items or activities by comparing them to other intermediary objects or activities by considering how much each is liked comparatively. To enable comparison of different activities according to which is liked more or less, the liking is equated with having greater value for that student. A student could say that they like playing soccer better than watching television and they like going to the movies better than playing soccer. The transitive relationship between these activities means that it can be deduced that this student likes going to the movies better than watching television.

6.2.4 Money as value: Non-standard units for money

When students are encouraged to barter or swap skills or items for other activities or objects, they consider the value of what they have and what they would accept in return for this. By introducing a count of particular items that are considered to equal the value of an object or activity, students are working with non-standard units of value. If the objects being used as non-standard units are all the same they can be described as being uniform non-standard units. The benefit of using uniform non-standard units is that the count of these units for one item can be compared to the count of the same units for another item. Through the use of non-standard units students develop an understanding of the need for a standard unit.

An interesting topic to discuss with students is how early civilisations worked before they had money as we know it. Bartering was one method but in many cultures a particular object was valued and was used as a non-standard unit of currency. For example, cowrie shells or pigs were highly valued in some island cultures and particular tasks were considered to be equivalent to a particular number of these items. Discuss the use of a common or valued object as a medium of exchange (e.g. cowrie shells, oxen or pigs or horses or other animals, marbles, etc.). Discuss the historical development of money. Again look at the history of the local people. Discuss the *talent*, an iron ring that had the mass of the gold needed to buy an ox in biblical times – this became a basis for money and mass.

6.2.5 Money as value: Standard units for money

The activities here are similar to those under money handling skill (6.2.1).

One activity is to look at different currencies. The standard units of money in different countries vary. In Australia our monetary system is decimal and the units are related by multiples of 10. This means that money provides a close link to the study of our number system which is also decimal.

By using money for actual purchases and shopping experiences including the giving of change, students can be helped to understand the value and relative value of the money in our society. Young students should start with $1 and $2 coins and combinations of these rather than 5c and 10c coins as they can be counted relatively easily. Activities involving students making particular values with coins are quite complex. Counting money to find the total value when there are a number of different coins is difficult due to the need to change the count.

The standard units of money in Australia are:

1 dollar ($) = 100 cents

The standard coins and notes in our currency are:

Coins: 5c, 10c, 20c, 50c, $1 and $2
Notes: $5, $10, $20, $50 and $100

- Have students explore the different countries that use the terms *dollar* and *cents* as their currency. What do our Asian neighbours use? Is it always base 100?
- Have students identify equivalent values for coins or notes (two 10c pieces = 20c). This can be done virtually with pictures in Word or PowerPoint.

Comparison of standard units

Comparison can be made of the actual coins in our system of money with the focus being the comparison of their monetary value. Due to the nature of the coins and notes in our money, students can focus their comparisons not on the associated value but on the physical characteristics of the coins and notes themselves. This can lead to misunderstandings. For example, the 50c coin is larger in size than both the $1 and $2 coins although the latter coins are of greater value.

- Compare this with the New Zealand monetary system. Have the students find out why the sizes of the Australian and New Zealand coins do not follow the same pattern.
- Have students compare coins from neighbouring Asian countries and investigate the relationship between size of coin and its value.

Comparison of amounts of money written symbolically requires a good understanding of number concepts including place value so that students know to compare the place with the greatest value when comparing written amounts of money. For example, $34.10 is of greater value than $29.65 even though each number in the second price is larger than those in the first price, except the first number.

6.2.6 Money as value: Applications for money

There are no formulaic applications of money except through the conversion of units. Providing students with opportunities to solve real-life problems that require the use of money to measure the value or cost of items or activities helps them to apply their understandings of this attribute in ways that relate to life beyond school. As students progress through school the financial aspects of mathematics focusing on shopping, giving change, designing budgets and working with percentage discounts and interest provide many opportunities for problem solving in real-life contexts.

- The Goods and Services Tax (GST) of 10% is a relatively recent phenomenon in Australia's financial history (introduced on 1 July 2000), but most students do not know it exists because it is automatically included in the regular price of an item. Compare this with the added tax applied in America. If you were to hand over the exact amount printed on a price tag in Target in America, they would ask you for the additional sales tax of between 1% and 10% (depending upon which US state you are in).
- Assist students to set up a business in the classroom with a purpose (e.g. to raise money for a class excursion). The business is to do a sausage sizzle one lunch time. The class needs to establish a budget, purchase items, and calculate what GST is required, what items are exempt, how much to charge per sausage on bread and so on. Provide students with information on which shops are giving discounts on bread, onions, sausages, tomato sauce etc. Calculate the percentage discount and overall savings. If you can, take the students to the supermarket and look at the price tags to determine best value for money (cost per 100 g).
- Explore the methods various shops use to express discounts (e.g. sales tags: "From $10", "Nothing over $5", "30% off lowest marked price").

7 Teaching Framework for Measurement

The teaching framework organises the content for measurement into a framework of 10 topics, namely, length, mass, capacity, perimeter, area, volume, time, angle, money, and temperature. Each of these topics is partitioned into sub-topics. Each sub-topic is described and any concepts or strategies used in the teaching framework are listed. They are also related to big ideas. Topics and sub-topics are chosen so as to represent ideas that recur across all year levels. The resulting framework is given in Table 1. This overall framework can be compared to the Australian Curriculum to produce year-level frameworks.

Table 1. *Framework for teaching measurement*

TOPIC	SUB-TOPIC	DESCRIPTION AND CONCEPTS/STRATEGIES/WAYS	BIG IDEAS
	Attribute	Length, width, thickness, depth, tallness	Attribute→instrument
	Comparison (no number)	Direct and indirect comparison (intermediary, e.g. string); order (between-ness); language (longer, longest, etc.)	
Length	Non-standard units	Body parts (e.g. hand spans), common lengths (e.g. pencils, sticks)	Common units, inverse relation, acc. vs exact.
	Standard units	mm, cm, m, km; metric conversions	Place value
	Formulae/applications	Object, unit, length	
	Attribute	Heft (press down on hand), heavy-light	Attribute→instrument
	Comparison (no number)	Direct (balance) and indirect comparison (intermediary, e.g. rubber band/stocking); order; language (heavier, heaviest, etc.)	
Mass	Non-standard units	Equal mass objects (e.g. marbles, small cans), lengths stockings stretch or boats sink	Common units, inverse relation, acc. vs exact.
	Standard units	µg, mg, g, kg, t; metric conversions	Place value
	Applications/Formulae	Object, unit, amount	
	Attribute	Amount of space, filling with liquid or material that flows	Attribute→instrument
	Comparison (no number)	Direct comparison (pouring from one container to other); indirect comparison (pouring both into common container); order (between-ness); language (larger, smallest, etc.)	
Capacity	Non-standard units	Common containers (spoonsful, cupsful, jugsful, etc.)	Common units, inverse relation, Acc. vs exact.
	Standard units	mL, L, kL; metric conversions	Place value
	Applications/Formulae	Object, unit, amount	
	Attribute	Distance around	Attribute→instrument
	Comparison (no number)	Mainly indirect comparison (intermediary – e.g. string); order; language (longer, longest, etc.)	
Perimeter	Non-standard units	Any material from length but distance around	Common units, inverse relation, acc. vs exact.
	Standard units	mm, cm, m, km	Place value
	Applications/Formulae	Object, unit, amount; perimeter formulae (rect-circle)	Formulae
	Attribute	Coverage (painting, collage, gardening); array-multiplication	Attribute→instrument
	Comparison (no number)	Direct comparison (placing on top) and indirect comparison (cover one, cut up and cover other); order; language (more, most, etc.)	
Area	Non-standard units	Hands, A4 paper, any tessellating object repeated, focus on rows and columns	Common units, inverse relation, acc. vs exact.
	Standard units	mm^2, cm^2, m^2, hectare, km^2; complex conversions	Place value
	Applications/Formulae	Object, unit, amount; formulae, area of 2D shapes and 3D shapes – all related to area rectangle; area-perimeter relationships	Formulae

TOPIC	SUB-TOPIC	DESCRIPTION AND CONCEPTS/STRATEGIES/WAYS	BIG IDEAS
Volume	Attribute	Amount of space	Attribute→instrument
	Comparison (no number)	Direct comparison (fits in) and indirect comparison (fill with something and pour into other; order; language (more, most, etc.)	
	Non-standard units	Any repeated objects, tessellating 3D shapes	Common units, inverse relation, acc. vs exact.
	Standard units	mm^3, cm^3, m^3, km^3, complex conversions	Place value
	Applications/Formulae	Object, unit, amount; volume formulae; area-volume relationships	Formulae
Time	Attribute	Point (time telling), sequence (e.g. months) and duration of time (measure)	Attribute→instrument
	Comparison (no number)	Direct and indirect comparison (intermediary – e.g. candles, water or sand running out); order; language (longer, longest, quick, slow, etc.)	
	Non-standard units	Pulse, pendulum, counting, and so on; sinking objects and candles with lines marked	Common units, inverse relation, acc. vs exact.
	Standard units	Seconds, minutes, hours, days, weeks, months and so on	Other bases
	Applications/Formulae	Object, unit, amount; applications in time computation	
Angle	Attribute	Amount of turn	Attribute→instrument
	Comparison (no number)	Direct comparison (copy of one angle on top of other); indirect comparison (intermediary – mark on unnumbered protractor); order; language (larger, smaller, etc.)	
	Non-standard units	Small sectors inside angle; homemade protractor	Common units, inverse relation, acc. vs exact.
	Standard units	Protractor	Place value
	Applications/Formulae	Object, unit, amount; direction and movement (e.g. orienteering, navigation); angle formulae in geometry	Formulae
Money	Attribute	Money handling (coins, notes); money as value (measure)	Attribute→instrument
	Comparison (no number)	Discussion of value or what worth more (what would you take to a desert island?); order; language (rich, richer, richest, etc.)	
	Non-standard units	Assigning numbers for value, using money or cost as value	Common units, inverse relation, acc. vs exact.
	Standard units	Dollars and cents (coins and notes)	Place value
	Applications/Formulae	Object, unit, amount; applications with money; interest etc.	Formulae for interest
Temperature	Attribute	Hot/cold	Attribute→instrument
	Comparison (no number)	Direct comparison (feel) and indirect comparison by using a base heat (is it hotter than this, colder than this, etc.); order; language (hotter, colder, hottest, etc.)	
	Non-standard units	A classification system to compare with; homemade thermometer	Common units, inverse relation, acc. vs exact.
	Standard units	Thermometer (0–100° Celsius)	Place value
	Applications/Formulae	Object, unit, amount; temperature applications (negative numbers)	

Metric Expanders

Fold shaded part so it is only shown as expanders are opened.

Expander A

			Kilometres					Metres					Millimetres	

Expander B

			Kilolitres					Litres					Millilitres	

Expander C

			Tonnes					Kilograms					Grams	

Metric Slide Rule

✂ slit

Whole Numbers				Decimal Fractions		
TH	H	T	O	t	h	th
1000	100	10	1	0.1	0.01	0.001

✂ slit

How does it work?

		3	0	0				What is my new number?

✄ slit

Whole Numbers				Decimal Fractions		
TH	H	T	O	t	h	th
	3	0	0			
1000	100	10	1	0.1	0.01	0.001

✄ slit

Pull the slider one place to the left to multiply by 10; Pull the slider two places to the left to multiply by 100 etc.
Pull the slider one place to the right to divide by 10; Pull the slider two places to the right to divide by 100 etc.
3 in the Hundreds place – how many ones is that? etc.

Place Value (PV) Chart

Whole number PVs				Decimal PVs		
TH	H	T	O	t	h	th
1000	100	10	1	0.1	0.01	0.001

Cut out PV chart and slides.

Cut along dotted lines

and insert slides.

Slides

			km			m		cm	mm				

						L			mL				

			t			kg			g				

Prep to Year 9: Resource 6 – **Measurement**

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