*If this problem is used as an introduction to modelling (Approach I), give it to the group without further advice. Ask them to work on it for a few minutes and come up with an amended recipe. They will have no knowledge of a modelling process or cycle; however, they will have made assumptions and come up with mathematical answers, which need to be interpreted in practical terms if the recipe is to be realistic. Discussing this will show the need for systematising the process (distribute the modelling diagram at this point). The boxes can be filled out as the group translates and organises what individuals have done. Approach I is useful when the attention is to move quickly to other problems – the main focus is the editing process. Approach II is useful when the construction of a report is a focus, as it illustrated aspects of report writing using a simple problem.*

# Adapting a recipe

## Describe the real-world problem

### *Chocolate mousse*

185g cooking chocolate

$\frac{1}{4}$ cup hot water

1 teaspoon vanilla essence

5 large eggs

$1\frac{1}{2}$ cups cream

Melt chocolate. Beat eggs.
Whip cream. Fold all ingredients together. Chill in fridge for 1 hour.

Serves 4–6.

## Specify the mathematical problem

Freda Nurke is planning a new cookery book where all her recipes are based around larger/extended families of 6 to 9 people. She would like advice on how she should change the chocolate mousse recipe from her previous book, as shown above.

## Pre-modelling exercise (Approach I or Approach II)

This simple problem requires a recipe to be adapted. Most people would do this on the back of an envelope or even mentally with a calculator to help. We would probably overlook the fact that we had made assumptions and used a process that was so 'obvious' that we did not realise that we had done it.

## Formulate the mathematical model

Group members should be encouraged to recognise and articulate the assumptions they made in their earlier approach to solving the real-world problem.

Introduce the mathematical modelling framework (see Figure 3) and encourage students to map out their response to the problem using this template.

Using the template means that key elements can be represented. This can be useful for structuring a report. In the formulation box, we note essential assumptions that underpin the model development. Identifying them explicitly helps to emphasise their centrality to every modelling enterprise. Similarly, in the interpretation box, real-world practicalities moderate precise mathematical results, a step that we can easily subsume without recognising it.

The three arrows out of box 6 reflect that (in the general case) evaluation may be followed by a report, but may require instead a revisiting of the problem context, the mathematical problem identified – and indeed, other stages of the modelling process.

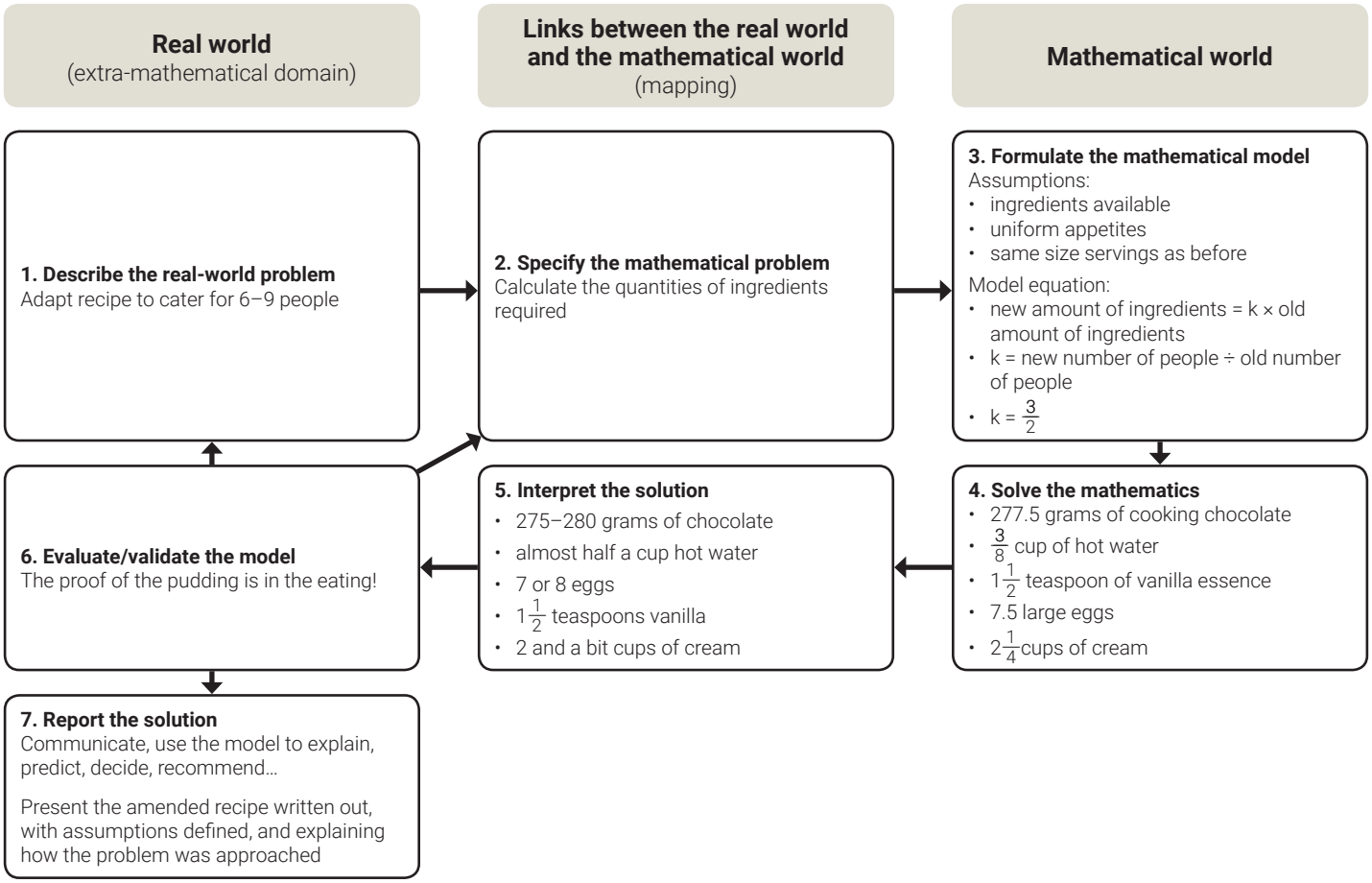
### Situational assumptions

Facilitate students' discussion of what assumptions they will need to make before creating a model. These might include:

- availability of all ingredients
- all the people being served have uniform appetites
- serving size will be the same for the adapted recipe as it was for the original recipe.

Emphasise that the framework is cyclical, not strictly linear. Real-world modelling will often require that students return to earlier steps in the framework to consider new variables, source new data, reassess assumptions and test solutions.

## Figure 3: Adapting a recipe using the mathematical modellling framework




### Mathematical formulation

k = new amount of people ÷ old amount of people

new amount of ingredients = k × old amount of ingredients

k = 6 ÷ 4 or k = 9 ÷ 6

k = $\frac{3}{2}$ or k = 1.5

## Solve the mathematics

277.5 grams of cooking chocolate

$\frac{3}{8}$ cup of hot water

$1\frac{1}{2}$ teaspoon of vanilla essence

7.5 large eggs

$2\frac{1}{4}$ cups of cream

## Interpret the solution

Consider the solution in real-world terms. How would you cook with half an egg? Rewrite the ingredients list, for example:

275 or 280 grams of cooking chocolate

almost half a cup of hot water

$1\frac{1}{2}$ teaspoons of vanilla essence

7 or 8 eggs

2 and a bit cups of cream

## Evaluate the model

The exercise can be extended to apply the model to related problems.

What if there are two people coming to dinner? A dozen?

Or, working backwards: If I have half a carton of eggs with which to make mousse, how many people can I invite for dessert?

Will the larger volume of mousse in the amended recipe take longer to chill? If I have less time to prepare, how might I reduce chill time? (For example, use several smaller serving dishes instead of one large dish.)

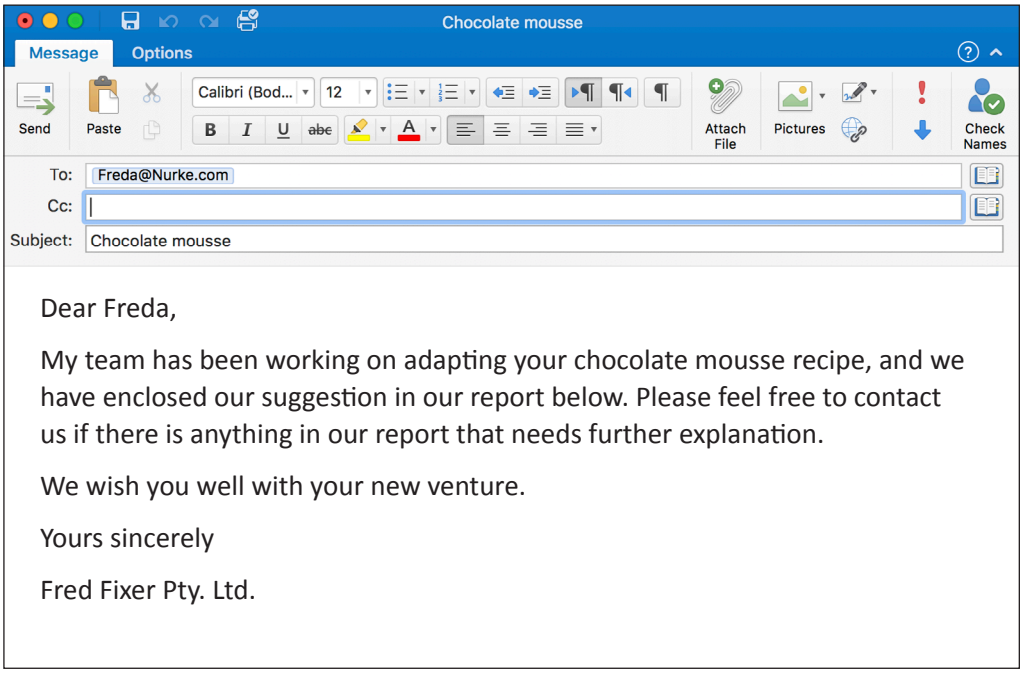
## Report the solution

Write out a report summary or full report. Examples are shown on the following page.

## Possible extensions

Research real-world shopping limitations. Can I buy exactly 275 grams of chocolate, 7 eggs, 2 cups of cream? How many blocks of chocolate, cartons of eggs, containers of cream would this recipe need? Visit supermarket websites to research product sizes, and write out a shopping list.

Follow up this piece of modelling by a statistical exercise. Make two different lots of mousse and then have a blind testing to see which version people prefer. This could be complicated by using a different recipe for mousse from a different cookbook.




## Summary

Using the recipe that you sent us that serves 4 to 6 people, we have produced a mathematical model in which our main assumption is that the new amount of each ingredient will be equal to the old amount of that ingredient, multiplied by the ratio of the new number of people to the number of people in the original recipe. As a consequence of this, we have made a suggestion for the new recipe ingredients to create a dessert to serve 6 to 9 people. This suggestion can be found at the end of our full report.

It is not clear to us, though, how long the new mixture needs to be chilled before it will set satisfactorily. It would be worth your while experimenting in your kitchen to determine an optimal chilling time for your recipe.

In the full report we detail our approach to the problem and discuss other matters that might be considered.

## Full report

Situational assumptions: all ingredients are available; all the people being served have uniform appetites; serving size will be the same for the adapted recipe as it was for the original recipe.

Mathematical assumption: new amount of each ingredient (now denoted k) = old amount of ingredient × new number of people ÷ recipe number of people.

Mathematical model: k = new amount of people ÷ old amount of people

k = 6 ÷ 4 or k = 9 ÷ 6 therefore k = 3/2 or k = 1.5

There are elements in the recipe that may introduce inaccuracies in the adaptation for 6 to 9 people.

For example, there is no international standard for 'cup'. Around the world the volume of a cup varies from 200ml up to nearly 290 ml (see https://en.wikipedia.org/wiki/Cup_(unit)). Further, it is subjective as to what a 'large egg' actually is. If the chef has only small eggs, will 6 be sufficient in the original recipe given? The amount of cooking chocolate to use in the adapted recipe may be unrealistically precise for most kitchen cooks; however, the resulting mousse may taste just as nice or even sweeter if more, rather than less, chocolate is used. And it may be that 1 teaspoon of vanilla essence may do just as well for 4 to 6 people as 6 to 9.

Importantly, it is not clear that the simple ratio method for calculating ingredients is suitable for cooking, and you may want to look further into this. It might be better to make enough for 12 to 18 people and halve this when chilling is complete. The remainder could then be put into the freezer for another day.

Finally, we recommend that you use the amounts proposed below.

Chocolate mousse recipe to serve 6–9 people:

275–280 grams of cooking chocolate

almost half a cup of hot water

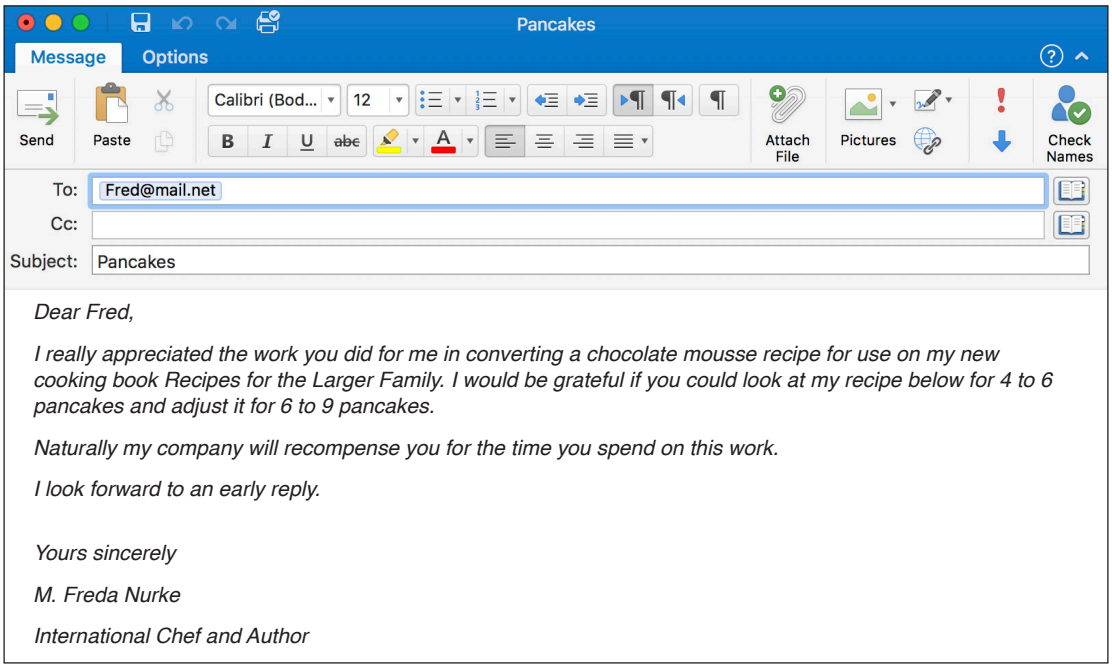
$1\frac{1}{2}$ teaspoons of vanilla essence

7 or 8 eggs

2 and a bit cups of cream

# Pancakes

## Describe the real-world problem




*Dear Fred,*

*I really appreciated the work you did for me in converting a chocolate mousse recipe for use on my new cooking book Recipes for the Larger Family. I would be grateful if you could look at my recipe below for 4 to 6 pancakes and adjust it for 6 to 9 pancakes.*

*Naturally my company will recompense you for the time you spend on this work.*

*I look forward to an early reply.*

*Yours sincerely*

*M. Freda Nurke*

*International Chef and Author*

### *Recipe for 4 to 6 pancakes*

1 egg

1 cup self-raising flour

200 ml milk

Mix all ingredients thoroughly so that there are no lumps.

Heat pan until a drop of water dances on the pan.

Add oil to pan.

Add pancake mixture to pan.

Cook pancake on one side till brown, then turn over and cook the other side.

Remove pancake from pan and put on a plate covered by paper towel.

Put into warm oven and add subsequent pancakes to the plate.

## Model the mathematical problem

The modelling activity here is generated by a letter from a satisfied customer, Freda Nurke.

The modelling goal is to produce a recipe for 6 to 9 pancakes from the original recipe for 4 to 6 pancakes. It might be good if the new menu could be tested against the old one for taste.

This example problem is presented without further comment, but can be solved in a similar way to the 'Adapting a recipe' example problem, discussed on the previous pages.

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