Discovering triangle congruence shortcuts

Original lesson by Jessica Uy

Objective

In this lesson, students will be able to conjecture about the sets of three sides and angles that will guarantee triangle congruence.

Big Idea

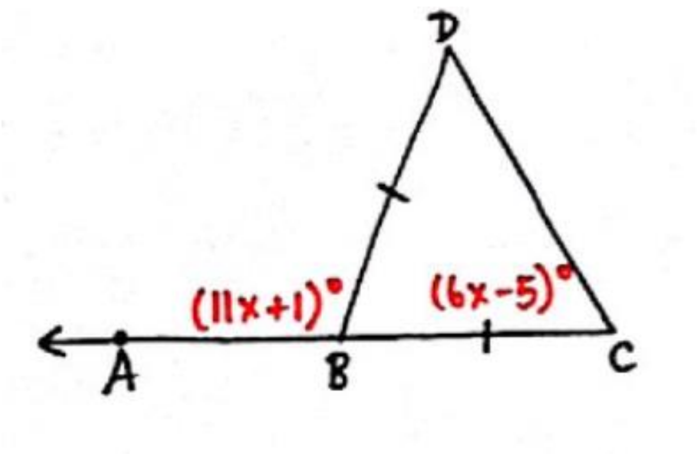
Through construction, students will work in pairs to test "shortcuts" to use when proving triangles congruent.

Warm-Up: Exterior Angles and Isosceles Triangles

20 MINUTES

I use this warm-up because it forces students to apply their understanding of the angles of isosceles triangles in an unconventional way. This problem can yield a really rich whole-class discussion since there are several ways to think and reason through the problem. What I like to do for the first problem is write out three different possible equations that represent the geometric relationships in the problem; I then ask students to take a moment to consider what ideas the equations represent and to justify how they know—this really gives students an opportunity to look for structure in the work and to defend their thinking using precise vocabulary.

1. Solve for x. Justify how you know by including vocabulary and theorems to help you explain.



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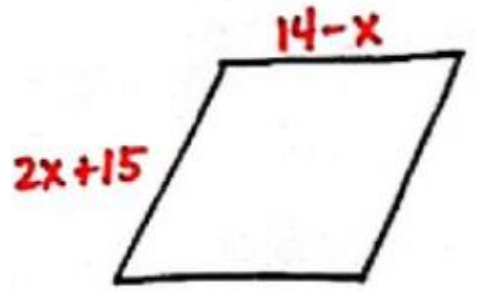
2. Find the value of x that makes this quadrilateral a rhombus.



Pair Investigation: Which "Shortcuts" Guarantee Congruent Triangles?

20 MINUTES

I launch this Investigation by leading a short whole-class discussion around triangle congruence. I offer a simple definition for congruence—all corresponding sides and angles are congruent—an idea that makes sense to students but sounds like a rather time-consuming process. This is when I plant the idea of looking for shortcuts, which motivates the investigation. The Investigation is a task to see which of the six permutations of three sides and three angles will guarantee triangle congruence.

I ask students to work in pairs and to use their constructions skills to make sense of the investigation. They are given a set of sides and angles from which they are to determine the number of triangles possible to construct—if they can construct one and only one triangle, the pair should conjecture that the shortcut might guarantee triangle congruence; if they can construct more than one triangle, the pair should conjecture that the shortcut does not guarantee triangle congruence. In this investigation, pairs will deal with SSS, SAS, ASA, SSA; they will come across AA later in the lesson, and they will prove AAS in the next lesson. Throughout this investigation, students are trying to look for and express regularity in repeated reasoning, considering which combination of three sides and angles can guarantee triangle congruence.

Are There Triangle Congruence Shortcuts?

Do we have to compare all three corresponding sides and angles of triangles to be able to conclude that triangles are congruent?

To answer this question, use the given combination of segments and/or angles to see if you can construct ONLY one triangle. If you can construct one and only one triangle, then we can conclude that the triangles are congruent.

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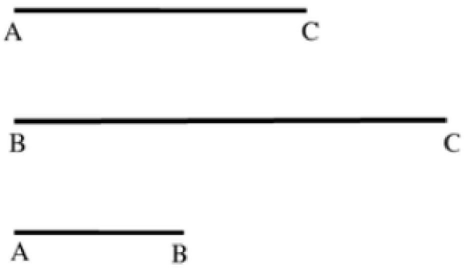
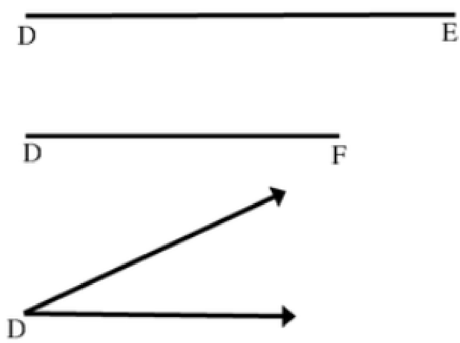
SSS Investigation

If the three sides of one triangle are congruent to the three sides of another triangle, must the triangles be congruent?



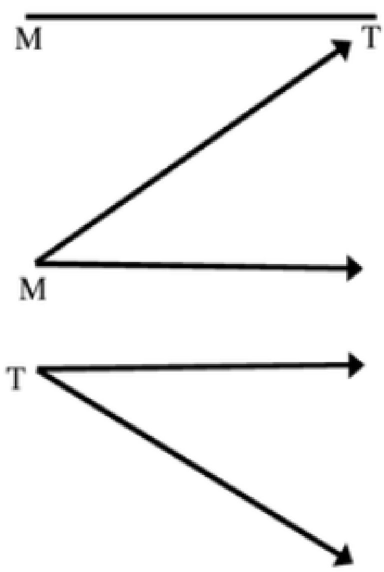
SAS Investigation

If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, must the triangles be congruent?



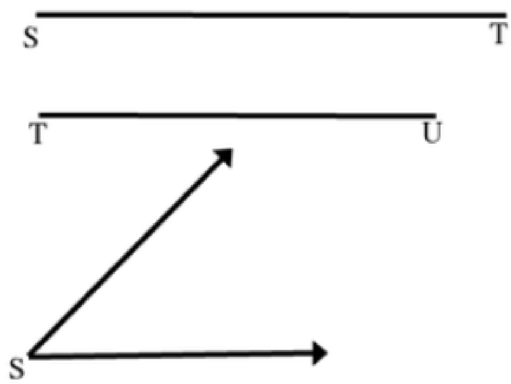
ASA Investigation: Construct Triangle *MTU*

If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, must the triangles be congruent?



SSA Investigation: Construct Triangle *STU*

If two sides and a non-included angle of one triangle are congruent to two corresponding sides and a non-included side of another triangle, must the triangles be congruent?



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Essential Question:

Do we have to compare all three corresponding sides and angles of triangles to be able to conclude that triangles are congruent?

Directions:

Below you will find Barney's response to this question. Despite all of his hard work and effort, his response is not entirely correct. Your task is to help Barney by writing him a letter. Since Barney has some good reasoning, make sure to point out what you agree with. After that, respectfully convince him of where his reasoning breaks down. Include a triangle construction for SSA and at least a sketch for AAA to support your argument.

Barney's Response:

"No, we don't have to compare all three corresponding sides and angles of triangles to conclude that they are congruent. Instead, we can compare any combination of three sides and/or angles.

For example, let's say we have two pairs of congruent sides and one pair of congruent angles (SSA). Since we know that three pairs of corresponding parts are congruent, we know the triangles are congruent! Here's another example: if we have three pairs of corresponding angles (AAA), then we know the triangles are congruent! So, all six combinations of three sides and/or angles—SSS, SAS, AAS, ASA, SSA, and AAA—will ensure that the triangles are congruent, which means they are exactly the same.

Basically, as long as we compare three parts of each triangle, whether they are sides and/or angles, we know we can conclude that the triangles are congruent!"

Debrief/Notes: Triangle Congruence Shortcuts

10 MINUTES

Thoughts on How to Jump into the Debrief

During the debrief of this activity, I choose four different students' constructions for SSS, SAS, ASA, and SSA to share using the document camera. The rest of the class checks their work against the constructions shown with the goal of showing a counterexample, which would disprove the conjectures students had written during the investigation. After we go through this process, we formally debrief on our note takers, which is a routine for our class.

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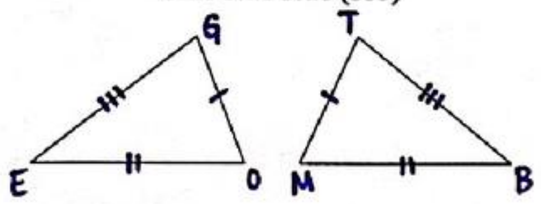
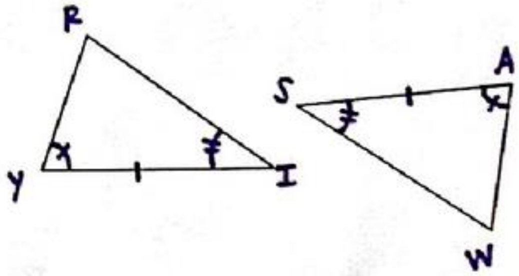
Side Side Side (SSS)



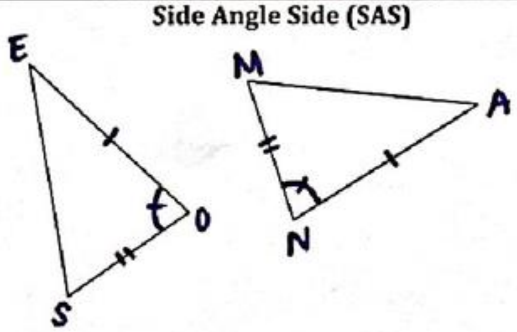
Congruence Statement:

Angle Side Angle (ASA)



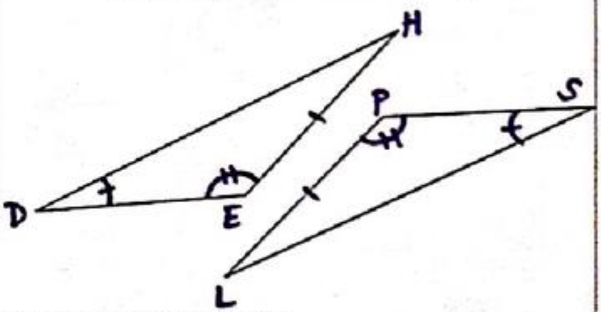
Congruence Statement:

Side Angle Side (SAS)



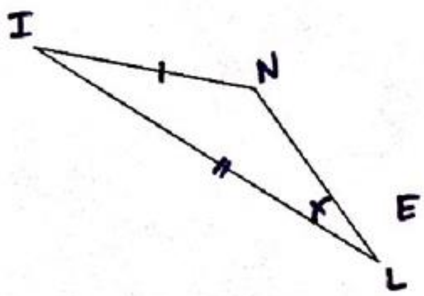
Congruence Statement:

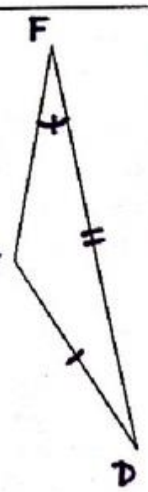
Side Angle Angle (SAA or AAS)



Congruence Statement:

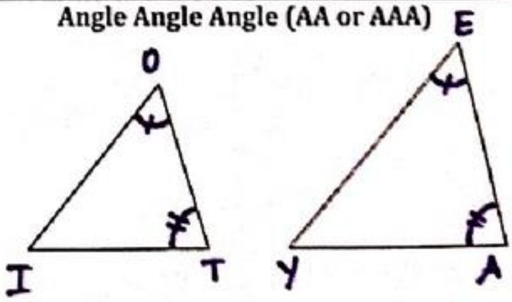
Side Side Angle (SSA)





Congruence Statement:

Angle Angle Angle (AA or AAA)



Congruence Statement:

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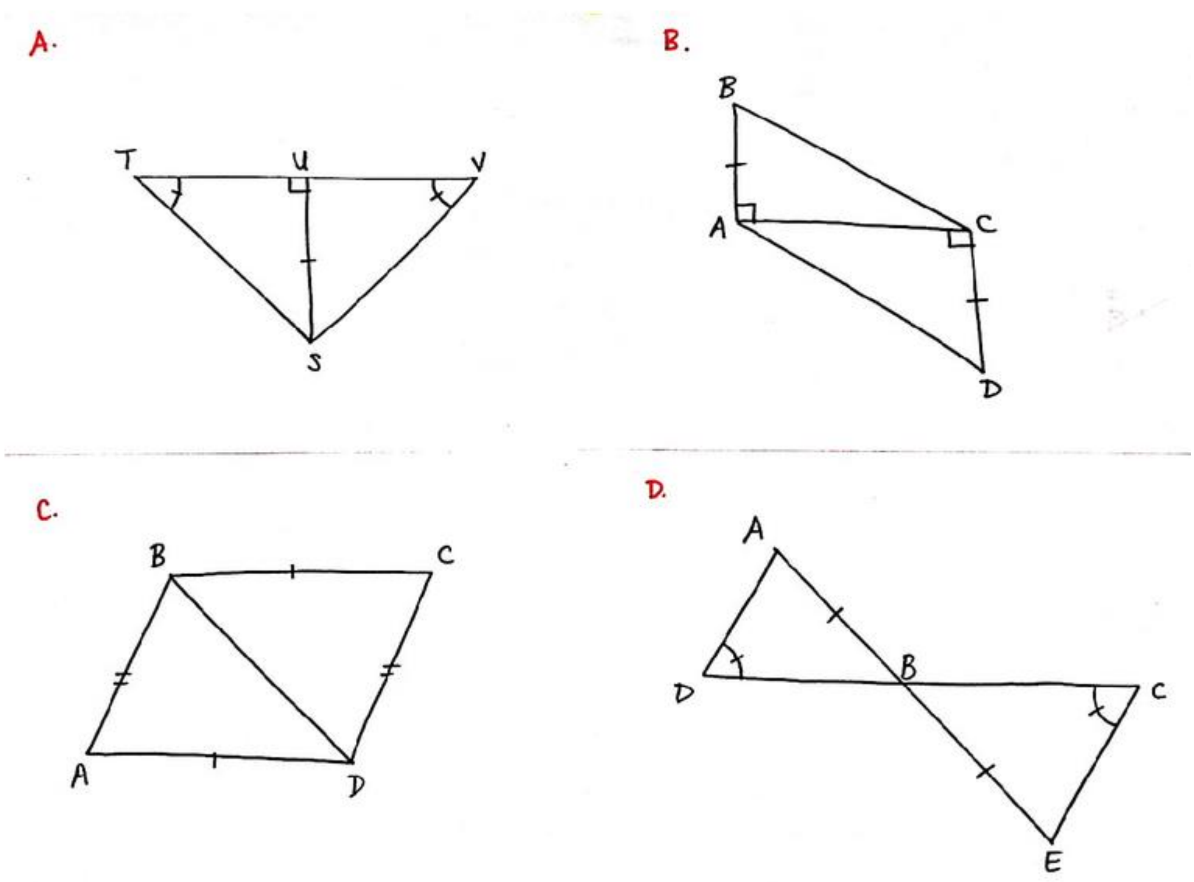
Exit Ticket

10 MINUTES

In this Exit Ticket, I display four pairs of triangles from the station work. Students choose any one of the pairs of triangles to prove congruent. This exit ticket gives me a way to formatively assess my students' understanding about triangle congruence criteria and whether they can prove triangles are congruent.



Acknowledgement

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https://teaching.betterlesson.com/lesson/538680/discovering-triangle-congruence-shortcuts

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