**Egg roulette: Part 2**

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| Year levelStrand(s)Lesson lengthCD code | * Year 9
* Probability
* 60 mins
* [AC9M9P01](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-9/content-description?subject-identifier=MATMATY9&content-description-code=AC9M9P01&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=3&subjects-start-index=0&view=quick), [AC9M9P02](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-9/content-description?subject-identifier=MATMATY9&content-description-code=AC9M9P02&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=3&subjects-start-index=0&view=quick) and [AC9M9P03](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-9/content-description?subject-identifier=MATMATY9&content-description-code=AC9M9P03&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=3&subjects-start-index=0&view=quick)
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| Lesson summary | In Part 1, students were introduced to probability without replacement, by exploring the game of egg roulette – 12 eggs, 3 raw and 9 cooked. Contestants take it in turns to choose an egg and smash it on their head hoping to avoid the 3 raw eggs! In this lesson, students consolidate and make deeper connections using simulation to address misconceptions on this abstract concept. Note that the same resources are also available in Part 1 as Part 2. |
| **Learning intention** | * We are learning to understand probability without replacement.
* We are learning to analyse compound events visually and numerically.
* We are learning to understand the changing probabilities at each stage, including by simulation.
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| **Success criteria** | By the end of this lesson, students can: * correctly identify how probabilities change at different stages in a without replacement probability experiment
* use an array to find probabilities for two-stage compound events
* conduct repeated chance simulations
* calculate relative frequencies from collected data to estimate probabilities involving ‘and’ and ‘or’ statements.
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| **Why are we learning about this?** | Understanding probability is foundational in mathematics, aiding progression to advanced statistics topics. It intertwines with real-world scenarios, enriching our decision-making and analytical skills. Examples include in genetics, where trait inheritance follows probability patterns, and in card games where the odds may change with every card drawn. Understanding changing probabilities helps us comprehend the world and make informed decisions. |
| **Prerequisite knowledge and language** | Key language: compound event with and without replacement, simulation, two-stage compound event, relative frequency, ‘and’ and ‘or’ statements, ‘inclusive or’ and ‘exclusive or’ statements. |
| **Resources** | * Egg-cellent worksheet (Word)
* Teacher’s notes (Word)
* Egg roulette simulation (Excel with macros enabled)
* [Egg roulette video](https://www.mathematicshub.edu.au/search/egg-roulette/)
* Calculators, laptop/access to computers for each student.
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Curriculum information

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| Achievement standard | By the end of Year 9, students determine sets of outcomes for compound events and represent these in various ways, assign probabilities to the outcomes of compound events and design and conduct experiments or simulations for combined events using digital tools. |
| Content description(s) | * Students list all outcomes for compound events both with and without replacement, using lists, tree diagrams, tables or arrays; assign probabilities to outcomes. [AC9M9P01](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-9/content-description?subject-identifier=MATMATY9&content-description-code=AC9M9P01&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=3&subjects-start-index=0&view=quick)
* Students calculate relative frequencies from given or collected data to estimate probabilities of events involving ‘and’, inclusive ‘or’ and exclusive ‘or’. [AC9M9P02](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-9/content-description?subject-identifier=MATMATY9&content-description-code=AC9M9P02&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=3&subjects-start-index=0&view=quick)
* Students design and conduct repeated chance experiments and simulations, using digital tools to compare probabilities of simple events to related compound events, and describe results. [AC9M9P03](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-9/content-description?subject-identifier=MATMATY9&content-description-code=AC9M9P03&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=3&subjects-start-index=0&view=quick)
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| General capabilitiesCross-curriculum priority | Numeracy progression * Probabilistic reasoning ([Level 6](https://v9.australiancurriculum.edu.au/f-10-curriculum/general-capabilities/numeracy/slideout?code=NSUnC6&element=2&sub-element=0))

Digital literacy* Acquire and collate data ([Level 6](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-9/content-description?subject-identifier=MATMATY9&content-description-code=AC9M9P03&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=3&subjects-start-index=0&view=quick&cdref=Elaboration))
* Interpret data ([Level 6](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-9/general-capability-snapshot?subject-identifier=MATMATY9&content-description-code=AC9M9P03&general-capability-code=DL&element-code=DLI&sub-element-index=1&sub-element-code=DLIC&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=3&subjects-start-index=0&view=quick))

Critical and creative thinking: * Draw conclusions and provide reasons ([Level 6](https://v9.australiancurriculum.edu.au/f-10-curriculum/learning-areas/mathematics/year-9/general-capability-snapshot?subject-identifier=MATMATY9&content-description-code=AC9M9P03&general-capability-code=DL&element-code=DLI&sub-element-index=1&sub-element-code=DLIC&detailed-content-descriptions=0&hide-ccp=0&hide-gc=0&side-by-side=1&strands-start-index=3&subjects-start-index=0&view=quick))
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| Areas of challenge | * Students may believe that the probability of an event occurring in a two or three-stage probability experiment is the same as the probability of the event occurring in a one-stage experiment. The impact of an experiment without replacement is Illustrated via egg roulette, where it is clear that eggs are ‘used up’ and no longer appear in the sample space in later stages. This misconception is addressed through detailed modelling of the sample space using a table approach for the first two stages.
* Students may assume that outcomes are equally likely. The distribution of cooked / raw eggs has been deliberately chosen to be heavily skewed towards cooked eggs. Students can be asked questions such as, ‘Are there more cooked or raw eggs?’ ‘Is it more likely [person] gets a cooked or raw egg? Explain why.’
* Students may be confused with the discrepancies between theoretical and experimental probabilities. An example such as tossing a coin once could be used where it is impossible to get half a head. Using the simulation to increase to a large number of trials allows for illustration of the Law of Large Numbers, reducing the discrepancy between the two.
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| Strategies | [Mathematics investigation](https://www.mathematicshub.edu.au/plan-teach-and-assess/teaching/teaching-strategies/mathematics-investigation/)[Using games](https://www.mathematicshub.edu.au/plan-teach-and-assess/teaching/teaching-strategies/using-games-and-storybooks/)[Concrete, Representational, Abstract (CRA)](https://www.mathematicshub.edu.au/plan-teach-and-assess/teaching/teaching-strategies/concrete-representational-abstract-cra/)[Questioning](https://www.mathematicshub.edu.au/plan-teach-and-assess/teaching/teaching-strategies/questioning/) |

Lesson structure

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| **Lesson hook**20 mins | This lesson follows from the previous lesson Egg roulette: Part 1. Access the [video](https://www.mathematicshub.edu.au/search/egg-roulette/) here that was introduced in Part 1 if a review is required or refer to the online version of the lesson where the video is embedded in the learning hook.Refer students to the Egg-cellent worksheet and pair students together. If you had not reached the Egg roulette: exploring the probabilitiesactivity last lesson, students can continue to complete the questions on the worksheet. Otherwise come together as a class and use the following questions to ensure a deep connection with the content has been made and to provide further opportunity to clear up misconceptions.Refer to the completed table in the Egg-cellent worksheet and ask the class to consider the scenario **before the game started**. Ask the following questions. * What is the probability that both Denise **and** Miriam choose raw eggs?
	+ $\frac{6}{132}≈0.045≈4.5\%$ very unlikely.
* What is the probability that Miriam chooses a raw egg on her first pick?
	+ This could be calculated simply as $\frac{3}{12}=\frac{1}{4}=0.25=25\% $ or using the table counting all the cells starting with R as $\frac{33}{132}=\frac{1}{4}$.
* What is the probability that Denise chooses a raw egg on her first pick? What do you notice? Why?’
	+ Counting all the cells ending in R gives $33$, therefore $\frac{33}{132}=\frac{1}{4}$. The initial probability that Miriam and Denise both choose a raw egg on their first pick is the same, as the game is fair. Initially both Miriam and Denise are equally likely to get the same number of raw / cooked eggs (though this changes as the game progresses).
* What is the probability that the first egg is raw (Miriam) **and** the second egg is cooked (Denise)?
	+ Counting all the cells with RC gives $27$, therefore $\frac{27}{132}=\frac{9}{44}≈0.204≈20.4\%$.
* What is the probability that **exactly one** teacher chooses a raw egg on their first pick?
	+ Counting all the cells with RC plus CR gives $27+27=54$, therefore $\frac{54}{132}=\frac{9}{22}≈0.409≈40.9\%$. ‘Exactly one’ is an ‘**exclusive or**’ phrase and does not include the possibility that they both picked raw eggs. This question could be rephrased as ‘Miriam **or** Denise, **but not both**, chooses a raw egg on her first pick’.
* What is the probability that **at least** **one** of the teachers chooses a raw egg on their first pick?’
	+ Counting all the cells with RC plus CR plus RR gives $27+27+6=60$, therefore $\frac{60}{132}=\frac{5}{11}≈0.454≈45.4\%$. **At least one** is an ‘**inclusive or**’ phrase and does include the possibility that they both picked raw eggs. This question could be rephrased as ‘Miriam **or** Denise **or both**, choose a raw egg on their first pick’
* It is important to distinguish between the language of **OR** (one of the two) versus **AND** (both of the two).
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| **Explore /investigate**20 mins | Draw up the table below on the whiteboard showing the theoretical probabilities of the four different combinations that you have just covered and also from the activity completed in the Egg-cellent worksheet.

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| Outcome | Theoretical probability |
| Cooked – Cooked | $$\frac{72}{132}≈54.5\%$$ |
| Cooked – Raw | $$\frac{27}{132}≈0.204≈20.4\%$$ |
| Raw – Cooked | $$\frac{27}{132}≈0.204≈20.4\%$$ |
| Raw – Raw | $$\frac{6}{132}≈0.045≈4.5\%$$ |

**Differentiation** (support): amend the activity to include fewer eggs: for example, 6 eggs, with 2 raw and 4 cooked.**Differentiation** (extension): various options are given below.* Introduce a probability tree diagram to model the different outcomes for the first two picks.
* Explore the product rule for multi-stage probability events.
* Ask students if there were 6 raw and 6 cooked eggs would each of the outcomes above be equally likely? The perhaps surprising answer is no, as Cooked–Cooked and Raw–Raw each have a $\frac{6}{12}×\frac{5}{11}=\frac{30}{132}≈22.7\%$ chance of occurring, whereas Cooked–Raw and Raw–Cooked have a higher chance of occurring, calculated as $\frac{6}{12}×\frac{6}{11}=\frac{36}{132}≈27.3\%$.

**Egg roulette – simulation**(Note that the Excel spreadsheet requires you to enable macros. Download the Excel spreadsheet from the What you need section. A window may pop-up asking whether to make file a trusted document. Select Yes, if not you can change setting in the Trust Center, located in File>Options>Trust Center>Trust Centre Settings.) In pairs, students conduct probability simulations using 50 trials to gain data on: * the likelihood of each of the outcomes above for the result of the first two picks and
* the number of raw eggs they would expect to get in a full game

Refer to the activity and Teacher’s notes. Distribute the Egg roulette simulation Excel spreadsheet to students and demonstrate on-screen how to run the simulation and answer any questions about the questions on the sheet. Give students time to complete the activities, monitoring their progress and offering guidance using the Teacher’s notes.**Differentiation** (extension): introduce expected value and ask students how many times they would expect to get 0, 1, 2 or 3 eggs in 100 trials or 500 trials. **Differentiation** (support):amend the activity to include fewer eggs, for example, 6 eggs, with 2 raw and 4 cooked. |
| **Summary and reflection**20 mins | Conduct a class discussion reviewing the results of the simulation and the overall learnings from the lesson and reserving time for students to complete the exit ticket described in the Assessment section.Key points are as follows.* In experiments without replacement, probabilities change at each stage and both the denominator and sometimes numerators need to change (depending on the previous pick and which outcome you are looking at).
* Tables / arrays are a good way to help visualise two-step probability experiments (like the first two picks).
* The game is initially fair, but as it progresses probabilities change owing to the without replacement property.
* Owing to chance, experimental probabilities tend not to match theoretical probabilities.
* The law of large numbers: the larger the number of trials, the closer experimental probabilities are to theoretical probabilities, which could be demonstrated if time permits by combining simulation data to give a class result
* The Gambler’s Fallacy: random events are not impacted by past events.
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| **Assessment** | **Exit ticket**The exit ticket is included in the downloadable Teacher’s notes. Note that there are three exit tickets per A4 landscape page to pre-cut before distributing. A box of chocolates contains four chocolates that look identical but three are caramel (yum) and one is a peanut (yuk). Use your knowledge of probability, with a diagram, to help me decide whether I should eat one or two chocolates.Some possible answers (in order of increasing sophistication) include the following.* If you are anaphylactic you should have no chocolates. If you have only 1 there is still a $\frac{1}{4}$ chance you get a peanut and could have an allergic reaction! It’s too big a chance.
* If you only have one chocolate there is a $\frac{1}{4}$ chance you get a peanut and $\frac{3}{4}$ caramel, so it seems like quite a good chance you get a chocolate you will like, so worth taking.
* The table shows the outcomes if you have two chocolates. Of the 12 outcomes, in 6 of them get two caramels (CC); that is, there is a $\frac{6}{12}=50\%$ chance you get two caramels but also a 50% chance you get a caramel and a peanut, so you may not want to do this if you really hate peanuts. Just having one chocolate might be a better choice as there is just a $\frac{1}{4}$ chance you get the peanut.

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|  | C1 | C2 | C3 | P |
| C1 |  | CC | CC | CP |
| C2 | CC |  | CC | CP |
| C3 | CC | CC |  | CP |
| P | PC | PC | PC |  |

* On your first pick you have a $\frac{1}{4}$ chance you get a peanut and $\frac{3}{4}$ caramel, so it is worth taking the risk. If you get a peanut, although that is not nice, you know all the rest are caramels so you should keep going and eat them all! If however your first pick is a caramel, you have a $\frac{1}{3}$ chance of getting a peanut on the next pick as there are only 3 chocolates left, so it may be worth stopping.
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