Mathematics Hub

Introduction to Trigonometry

Original lesson by Beth Menzie

Objective

In this lesson, students learn to understand the role of triangle similarity in trigonometry and find lengths of sides of triangles using trigonometry.

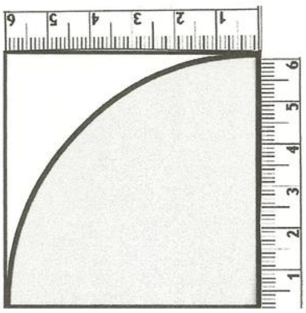
Big Idea

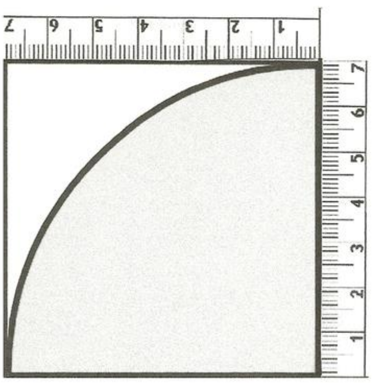
Students learn about right triangle trigonometry by creating similar triangles and producing their own trig tables.

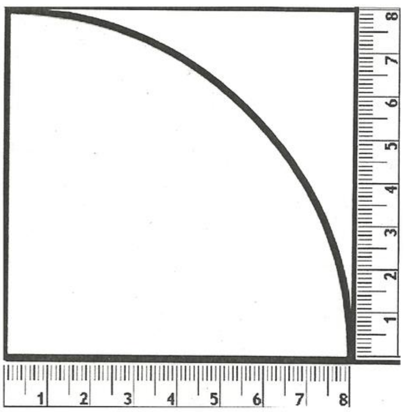
Data Gathering: preparing for a discussion

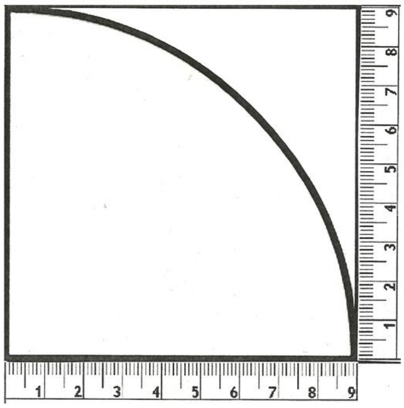
25 MINUTES

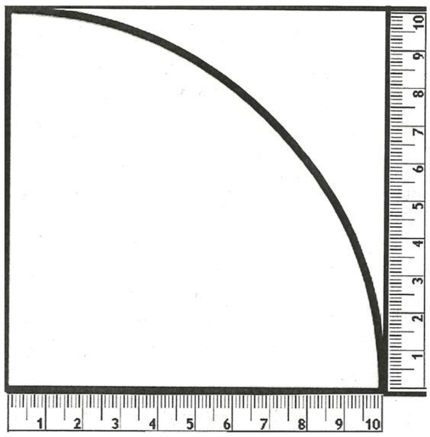
I do not mention anything the term "trigonometry" as we start this exercise.
I divide the students into groups of three (if the number of students does not divide evenly into groups of 3, a group or two of 2 students works fine). I give each member of a group the two handouts (the same sized quarter circles, and a table), a protractor, and an additional straightedge. (This seemed to help when measuring the angles.)





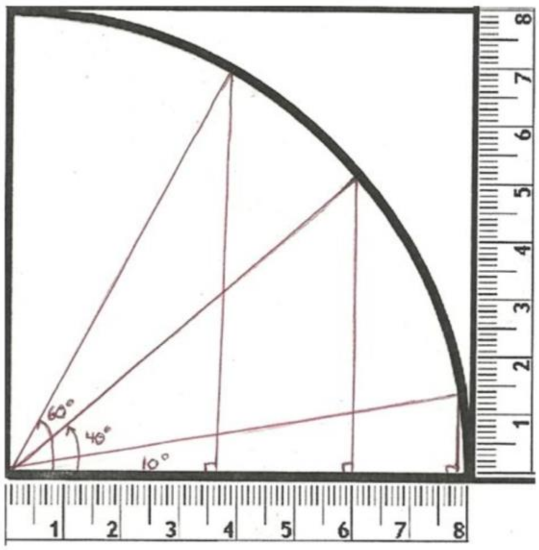






DEGREES	opp	adj	hyp	$\frac{opp}{hyp}$	$\frac{adj}{hyp}$	$\frac{opp}{adj}$
10						
20						
30						
40						
45						
50						
60						
70						
80						

The table contains nine different angle measures. I ask the group to split these angle measures up between the group members, so that each student is responsible for three angle measures. I explain that they are to use the protractor to mark off their angle measures on their quarter circle. Once they have marked off their angle measures, they draw three right triangles, as shown in the example. I provide an example of this on the board, just to make sure that everyone gets off to a good start.



Next we start the measurement phase. I briefly discuss with the students the meaning of the words “opposite”, “adjacent”, and “hypotenuse” with respect to right triangles. When all seem clear on these concepts, I explain that each student is responsible for measuring and recording the lengths of the opposite and adjacent sides of all three of his or her triangles, accurate to one decimal place. The adjacent side of the triangles is measured easily using the horizontal ruler on the diagram; using their straightedge or protractor, the students can extend the opposite side of the triangle horizontally, connecting to the vertical ruler on the diagram. We also discuss the length of the hypotenuses, which will always be equal to the radius of their quarter-circle.

Once each member of the group has recorded their own data in their table, they share their data with the other members of the group, so that each group member ends the measurement phase with opp, adj, and hyp filled in for all nine angle measurements.

At this point, with opp, adj, and hyp filled in for each angle, my students are ready to find and fill in the ratios. I specified that I wanted the ratios rounded to 4 decimal places, though I must confess I don’t exactly know why I chose ten-thousandths, other than trig tables are often given to this level of accuracy. When everyone’s tables are complete, we end the data gathering stage, and are ready for discussion.

Discussion Phase

10 MINUTES

A Great Discussion

An Unanticipated Extension

sine and cosine curves

I begin our class discussion of the Data by asking the students to observe general trends in the columns of their tables.

- What happens to the value of opp/hyp as the angle measures increase?
- Is the same true for adj/hyp?

Then I ask each group for their values of opp/hyp for 30° and 45°. As the students begin to notice that their values are all very similar, I ask them why they think this is true, hoping that they will begin to see and discuss similar triangles.

At some point in the discussion, we assign the names sine, cosine, and tangent to our columns. We also talk about the use of SOHCAHTOA as an aid to remembering the appropriate ratios.

Calculating Side Lengths

15 MINUTES

After the discussion, we will next put our trig tables to use. I draw a triangle on the board with a 60-degree angle and the length of the hypotenuse indicated and ask the students to find the length of one of the legs of the triangle using a ratio. We talk about the process of deciding which trig function to use, and how to solve a proportion.

I then hand out a trig table that I have photocopied from a textbook. The students are usually unaware that trig tables even exist and can compare the accuracy of their trig values to the values on the table. I ask the students how this table might have been constructed, hoping that they will talk about the role of similar triangles.

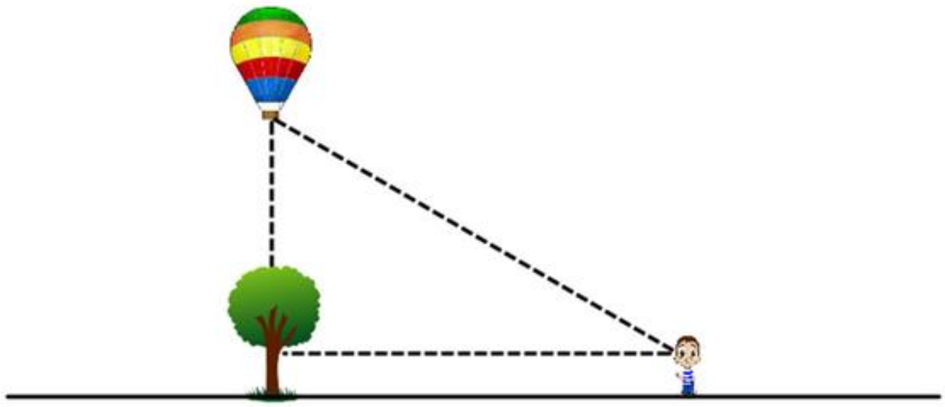
We use the photocopied trig table on a problem, and then move onto locating and using the trig functions on the calculator to find the lengths of sides in a couple other problems.

$m° \angle A$	sin A	cos A	tan A	$m° \angle A$	sin A	cos A	tan A
1	0.0175	0.9998	0.0175	46	0.7193	0.6947	1.0355
2	0.0349	0.9994	0.0349	47	0.7314	0.6820	1.0724
3	0.0523	0.9986	0.0524	48	0.7431	0.6691	1.1106
4	0.0698	0.9976	0.0699	49	0.7547	0.6561	1.1504
5	0.0872	0.9962	0.0875	50	0.7660	0.6428	1.1918
6	0.1045	0.9945	0.1051	51	0.7771	0.6293	1.2349
7	0.1219	0.9925	0.1228	52	0.7880	0.6157	1.2799
8	0.1392	0.9903	0.1405	53	0.7986	0.6018	1.3270
9	0.1564	0.9877	0.1584	54	0.8090	0.5878	1.3764
10	0.1736	0.9848	0.1763	55	0.8192	0.5736	1.4281
11	0.1908	0.9816	0.1944	56	0.8290	0.5592	1.4826
12	0.2079	0.9781	0.2126	57	0.8387	0.5446	1.5399
13	0.2250	0.9744	0.2309	58	0.8480	0.5299	1.6003
14	0.2419	0.9703	0.2493	59	0.8572	0.5150	1.6643
15	0.2588	0.9659	0.2679	60	0.8660	$0.5\overline{0}$	1.7321
16	0.2756	0.9613	0.2867	61	0.8746	0.4848	1.8040
17	0.2924	0.9563	0.3057	62	0.8829	0.4695	1.8807
18	0.3090	0.9511	0.3249	63	0.8910	0.4540	1.9626
19	0.3256	0.9455	0.3443	64	0.8988	0.4384	2.0503
20	0.3420	0.9397	0.3640	65	0.9063	0.4226	2.1445
21	0.3584	0.9336	0.3839	66	0.9135	0.4067	2.2460
22	0.3746	0.9272	0.4040	67	0.9205	0.3907	2.3559
23	0.3907	0.9205	0.4245	68	0.9272	0.3746	2.4751
24	0.4067	0.9135	0.4452	69	0.9336	0.3584	2.6051
25	0.4226	0.9063	0.4663	70	0.9397	0.3420	2.7475
26	0.4384	0.8988	0.4877	71	0.9455	0.3256	2.9042
27	0.4540	0.8910	0.5095	72	0.9511	0.3090	3.0777
28	0.4695	0.8829	0.5317	73	0.9563	0.2924	3.2709
29	0.4848	0.8746	0.5543	74	0.9613	0.2756	3.4874
30	$0.5\overline{0}$	0.8660	0.5774	75	0.9659	0.2588	3.7321
31	0.5150	0.8572	0.6009	76	0.9703	0.2419	4.0108
32	0.5299	0.8480	0.6249	77	0.9744	0.2250	4.3315
33	0.5446	0.8387	0.6494	78	0.9781	0.2079	4.7046
34	0.5592	0.8290	0.6745	79	0.9816	0.1908	5.1446
35	0.5736	0.8192	0.7002	80	0.9848	0.1736	5.6713
36	0.5878	0.8090	0.7265	81	0.9877	0.1564	6.3138
37	0.6018	0.7986	0.7536	82	0.9903	0.1392	7.1154
38	0.6157	0.7880	0.7813	83	0.9925	0.1219	8.1443
39	0.6293	0.7771	0.8098	84	0.9945	0.1045	9.5144
40	0.6428	0.7660	0.8391	85	0.9962	0.0872	11.4301
41	0.6561	0.7547	0.8693	86	0.9976	0.0698	14.3007
42	0.6691	0.7431	0.9004	87	0.9986	0.0523	19.0811
43	0.6820	0.7314	0.9325	88	0.9994	0.0349	28.6363
44	0.6947	0.7193	0.9657	89	0.9998	0.0175	57.2900
45	0.7071	0.7071	1	90	1	0	Undefined

Applying Trigonometry to Solve a Problem

10 MINUTES

I put a sample problem on the SMART board and pose a series of questions.



A little boy standing 200 feet away from a tree sees a hot air balloon hovering directly over the tree. The angle of elevation from the boy's eyes to the balloon is 40°. If the distance from the ground to the boy's eyes is 4 feet, how far above the ground is the hot air balloon?

Can we use trigonometry to solve this problem?

1. What should our first step be when faced with a problem like this?

I am looking for the answer, "Read the problem." A lot of students will begin a task like this – what they perceive as a word problem – by panicking and claiming that they "have no idea what to do." After a brief discussion of this, I ask the students to read the problem silently to themselves.

2. What should our second step be?

Here I am looking for the answer, "Fill in the diagram." I ask a student to read aloud the first sentence of the problem, and then ask the students:

- What is significant in this sentence?
- Where does the 200 feet belong in our diagram?

After the students respond to the question, I fill this value in on the diagram.

I ask another student to read aloud the second sentence.

- What is significant in this sentence?
- What the heck is angle of elevation?

After the students respond and we discuss angle of elevation, I fill the 40° in on the diagram.

I ask a student to read aloud the first part of the third sentence.

- What is the significance of this statement?

I fill in the little's boy's height on the diagram. I have a student read the final part of the problem, and we discuss where x should go, and what exactly x represents. I do not at this time make any explicit remarks about what the question asks for or about the addition of the boy's height.

3. Can we solve this problem using trig?

Here I focus on our diagram and the question.

- Is there a right angle in the diagram?

It is key that the students understand at this point in the unit that they must have a right angle to use trigonometry. When I feel that the students understand the question fully, I ask them to set up a proportion and to solve the problem using their calculators to generate the trig value.

4. What does the problem ask for?

I have purposely not directed attention to the boy's height – I am interested in seeing how many students consider this when they solve the problem. Some will add the 4 feet at the end, some will not. This provides a great opportunity to discuss the importance of reading and re-reading a problem to make sure that a question is answered fully and completely, and to ensure that the answer makes sense.

Independent Problem Solving

15 MINUTES

To begin our closing activity, I hand out the problem set entitled Lengths of Segments and I ask my students work in their groups to answer the questions. In this problem set I include the diagram for each problem. The next lesson will require the students to provide their own diagrams. I also did not specify what level of rounding I wanted for the answers. This is something that the students and I discuss when they get to it – what level of precision is appropriate in the problem? Usually, we agree to round these answers to the nearest whole number.

Problem 7 asks that the students solve for both missing segments. I do not specify whether to use trigonometry or the Pythagorean Theorem, and I encourage discussion among the students, to ensure that the students understand that both methods work and provide similarly precise answers.

Problem 10 involves angle of depression. I discuss this with students as we encounter this and compare it to angle of elevation. This is an opportunity to revisit parallel lines and alternate interior angles.

Problem 12 always generates interesting discussion about the height of the girl and its effect on the problem. This could be an interesting extension of the problem; since the water balloon sails beyond the girl – what would her height have to be for the balloon to hit her head?

I realise that some of these problems may seem slightly violent. The students usually find humour in these (as well as the goofy diagrams) and this helps to make the problems more engaging. Any problems not completed in class today will be assigned for homework.

Acknowledgement

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https://teaching.betterlesson.com/lesson/448228/introduction-to-trigonometry

Find the length of x to the ***nearest tenth***. **Be sure to write out an equation with a trig function in it first.**

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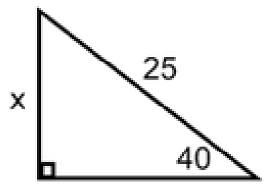

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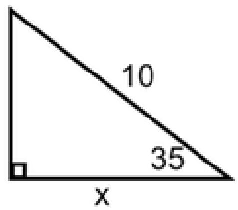

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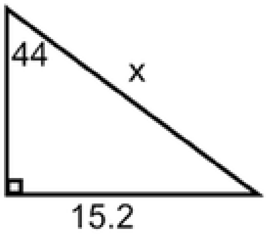

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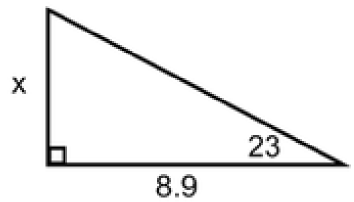

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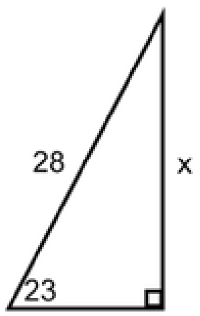

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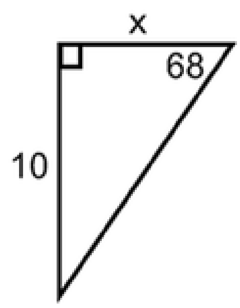
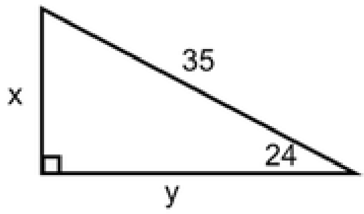
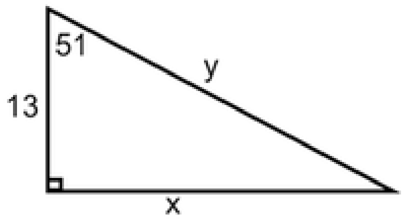


Mathematics Hub

7. Find the values of x and y to the *nearest tenth*:



a.



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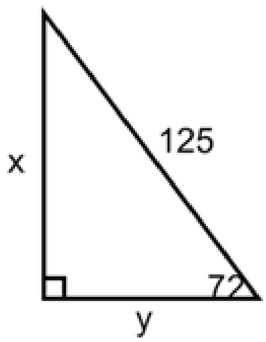
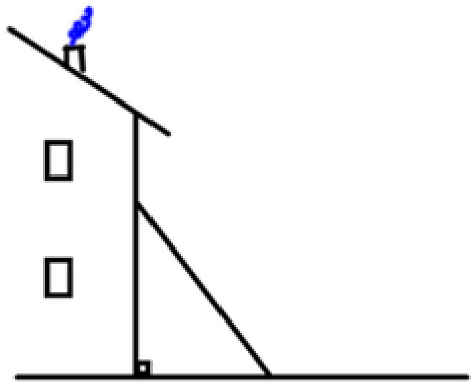
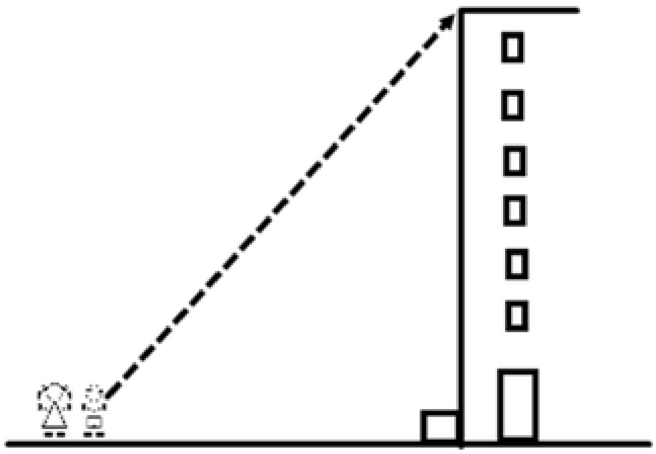


c.

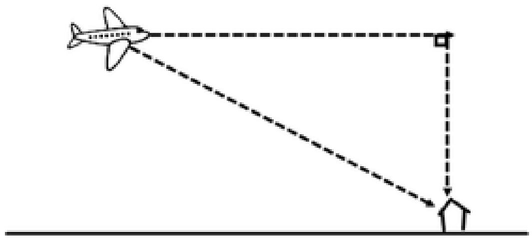
8. A 20 foot ladder leans against a house, making a 75 degree angle with the ground. How far above the ground is the top of the ladder?



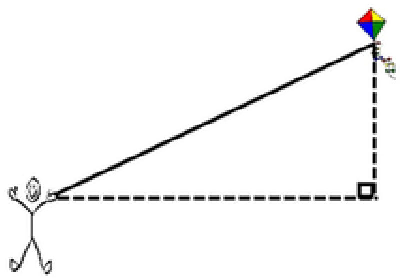
9. A boy stands 50 feet away from a building, and looks up at the top of the building with an angle of elevation of 65 degrees. How tall is the building?



10. A bomber flies 1000 meters above the ground, and sees its target below with an angle of depression of 42 degrees. How many more horizontal meters should the plane travel before dropping its bomb?



11. A child flies a kite. When the angle of elevation of the kite is 28°, the kite is 200 feet above the ground. At this instant, how many feet of string are running from the child's hand to the kite? [Disregard the child's height.]



12. A boy leans out a window 40 feet above the ground and heaves a water balloon out the window, with an angle of depression of 32 degrees. He is aiming for a poor unsuspecting little girl who stands on the ground 20 feet away from the building. Does the balloon hit the girl? If not, did the balloon go too far or did it fall short of the girl?